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# Geostatistical Analysis with Conditional Extremal Copulas

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**Abstract**

Geostatistics of extremes lies at the intersection of extreme values analysis and modelling the behavior of spatial events. In this paper we present a second-order stationary random field which describes the behavior of geostatistical data. The spatial dependence is characterized using the copula of the multivariate distribution underlying the process in multivariate extreme values context. The spatial copulas underlying the marginal processes are also characterized. The F-madogram of the distribution corresponding to the process is also modeled using a new conditional dependence function.

**Keywords:** geostatistics, copulas, max-stable distributions, stationary process, madograms

**2000 Mathematics Subject Classification:** 60G52, 60G70, 62G20, 62G32

## 1. Introduction

Geostatistics form a domain of statistics focusing on predicting probability distributions of spatial or spatiotemporal datasets. Many environmental statistical analysis are spatial: temperature, precipitation, stream flow, waves, air pollution etc. Considered for a time as statistics applied to mining operations and more generally to problems in the earth sciences, geostatistics became rapidly developing branch of statistics with important applications areas including public health (epidemiology), petroleum geology, hydrogeology, meteorology, oceanography, geochemistry, geography, forestry, agricultural and landscape ecological sciences, econometrics and image processing.

Necessarily multivariate as they are recorded at multiple locations, spatial data can be classified into one of three of basic types: (i) point-referenced or geostatistical data which are described by a random vector $\{Y(x), x \in \chi\}$ defined on the continuum $\chi$ of $\mathbb{R}$ but observed only at fixed sites $S = \{x_1, ..., x_s\} \subset \chi$; (ii) point pattern data, where the observations sites are assumed to be random i.e., independent replications of points of stochastic processes; (iii) areal data, where the spatial domain is a fixed subset which can be partitioned into a finite number of areal units as in atmospheric sciences models (Ribatet, 2011).

In geostatistics fields the variograms, which are inverses of covariance functions, describe how the spatial continuity changes with a given separating distance between two pair of stations h. The classical variogram $\gamma(h)$ provides a framework for modelling and predicting the variability of the stochastic process $\{Y(x),\ x \in \mathbb{R}\}$. In particular, while modelling spatial extreme variablity of an isotropic and max-stable field, Cooley et al. (2006) have introduced the F-madogram $\gamma_F(h)$ which transforms the process via its marginal F such as

$$\gamma(h) = \frac{1}{2}E\left\{[Y(x+h) - Y(x)]^2\right\} \text{ and } \gamma_F(h) = \frac{1}{2}E\left\{|F(Y(x)) - F(Y(x+h))|\right\} \quad (1)$$

Wisely used in quantitative financial analysis and in statistics of economics (Tangho, 2007) and introduced in a pioneer-work of Sklar (Nelsen, 1999), the copulas have until recently been little used in geostatistics analysis. Copulas are multivariate structures which capture the joint dependence without influence of the margins. In a spatial context where data are observed at many stations, it become of great interest to use copula in modelling high dimensional structure by expressing lower marginal dependence with the vector separating two points space. Therefore, copulas present number of opportunities in the spatial sciences by providing efficient means to address dependence structures which vary over locations.

In particular for a second-order stationary random field $\{Y(x)\}$ and in a copulas framework, Bondar et al. (2005) investigated the spatial variability between pairs of points $\left\{x_i, x_j \in \chi\right\}$ that are h-unit distant, that is, $h = \left\|x_i - x_j\right\|$. They defined spatial copulas such as

$$P\left[Y(x_i) < u; Y\left(x_j\right) < v\right] = C_h\left(\left(F_Y(u); F_Y(v)\right)\right) \text{ for all } u, v \in \mathbb{R} \tag{2}$$

where the marginal distribution $F_Y$ is the same for each location x of the domain $\chi$, due to the stationarity. Therefore, the spatial copulas describe the dependence over the whole range of quantiles and not only the mean dependence as variograms. Furthermore, Kazianka and Pilz (2009) suggested the use of models for spatial interpolation focused on copulas.

The main contribution of this study is to describe spatial variability with the copulas of the process under given conditions on subdomains of the space of interest. In particular, we show that the copula of the process is a spatial extremal copula. The copulas of two complementary subsets of the domain of study are also shown to be the marginal extremal copulas of the asymptotic copula of the process. A new spatial measure is constructed to characterize the $\lambda-$madograms of the distribution of the process.

## 2. Materials and Methods

In this study, we consider a random process $\{Y(x),\ x \in S \subset \mathbb{R}^n\}$ which models point-referenced data on a spatial domain S with distribution H. We are interested in modelling the copula of the process and two of marginal copulas related to the process under a max-stability assumption of H. For this purpose, the copulas of multivariate stochastic processes and the some multivariate settings of extremal copulas (Beirlant, 2005) would be useful.

### *2.1 Copulas of Multivariate Stochastic Processes*

Recall that a n-dimensional stochastic process is a collection of random variables defined on a space probability and taking values in $\mathbb{R}^n$, where T is the parameters set (space or time). The following result shows that a collection of copulas and marginal distributions also define a stochastic process, see Schmitz (2003).

**Theorem 1** *Let $C = \{C_{t_1,\ldots,t_n}; t_1 < \ldots < t_n, n \in \mathbb{N}\}$ be a collection of copulas satisfying the consistent condition*

$$\lim_{u_k \to 1^-} C_{t_1,\ldots,t_n}(u_1, \ldots, u_n) = C_{t_1,\ldots,t_n}(u_1, \ldots, u_{k-1}, u_{k+1}, \ldots, u_n)$$

*and $D = \{F_t, t \in T\}$ a collection of one-dimensional distribution functions. Then, there exists a probability space $(\Omega, \mathcal{F}, P)$ and a stochastic processes $\{(Y_x),\ x \in T\}$ such that*

$$P\left(Y_{t_1} < x_1, \ldots, Y_{t_n} < x_n\right) = C_{t_1,\ldots,t_n}\left(F_{t_1}(x_1), \ldots, F_{t_n}(x_n)\right) \tag{3}$$

*and $\{(Y_t),\ t \in T\}$ is measurable for all $t \in T$.*

Assume that the process $\{Y(x)\}$ is a second-order stationary random field defined on a set of observation locations S. That means in particular, that in addition to its stationarity the covariance function do not depend on the direction, that is:

$$Cov\left(Y(x_i), Y(x+h)\right) = C(h) \text{ with } x_i, x_i + h \in S \text{ for any lag h.}$$

Modelling spatial dependence via copulas-based models, Kazianka and Pilz (2009a) define multivariate spatial copulas associated to such as

$$P\left(Y(x_1) \le y_1, \ldots, Y(x_s) \le y_s\right) = C_{\theta,\rho}\left(F_\eta(y_1), \ldots, F_\eta(y_s)\right) \text{ for all } x_i \in \mathbb{R}.$$

where $F_\eta$ is the univariate marginal of the stochastic process, assumed to be the same for all locations $x_i$ with parameter $\eta$, $\theta$ being the specific parameter of the copula and $\rho$ its correlation.

### *2.2 Multivariate Settings of Extremal Copulas*

Multivariate extreme values (EV) analysis is based on interpolating appropriately normalized vectors of the maxima of the observed events. The theory and statistical practice of univariate extremes is well developed and the three possible asymptotic behaviors (Fréchet, Gumbel, Weibull) are summarized by the generalized EV (GEV) distribution. For high dimensional analysis however, the results of extremal models turn out to be more complicated since the joint structure depend both on parametric and not parametric components.

Let $(X_1, ..., X_n)$, $n \in \mathbb{N}$, be a vector of random *i.i.d* variables with a joint distribution F with continuous margins $F_i$. According to Sklar's theorem (Nelsen, 1999), there exists a unique copula $C_F$ providing a canonical parameterisation of F via its univariate marginal quantile functions $F_i^{-1}$ such that:

$$C_F(u_1, ..., u_n) = F\left(F_1^{-1}(u_1), ..., F_n^{-1}(u_n)\right) \text{ with } F_i^{-1}(u) = \inf\{x_i \in \mathbb{R}, F_i(x_i) \geq u\}. \tag{4}$$

If in addition F is an EV distribution, then $C_F$ satisfies the max-stability property, that is,

$$C_F^k(u_1, ..., u_n) = C_F\left(u_1^k, ..., u_n^k\right) \text{ for all } (u_1, ..., u_n) \in [0, 1]^n \text{ and } k > 0. \tag{5}$$

Such a copula is called extremal copula and is represented by a convex function V such as:

$$C_F(u_1, ..., u_n) = \exp\{-V(-(\log u_1)^{-1}, ..., -(\log u_n)^{-1})\}; u_i \in [0, 1] \tag{6}$$

The function V is referred to as the extremal dependence function of $C_F$. It satisfies the following constraint

$$V(x_1, ..., x_n) = D\int_{\Omega_n} \max\left(\frac{w_1}{x_1}, ..., \frac{w_n}{x_n}\right) dH(w);$$

where H is a finite non-negative measure of probability, arbitrary except for the moments constraint $\int_{\Omega_n} w_i dH(w_1, ..., w_n) = 1$. The set $\Omega_n$ is the well-known unit simplex of $\mathbb{R}^n$, that is $\Omega_n = \{(t_1, ..., t_n) \in [0, 1]^n;\ \sum_1^n t_i = 1\}$ (see Beirlant).

## 3. Main Results

### *3.1 Extremal Copulas in a Geostastical Field*

Let $S = \{x_1, ..., x_s\} \subset \mathbb{R}^2$, be the set of locations where the process is observed. Suppose that $Y_{k,1}$; ...; $Y_{k,s}$ denote independent copies from the second-order stationary random field where $k = 1, ..., n$. In all the study, the key-assumption is that the s-dimensional distribution H underlying the process $\{Y(x)\}$ is of the same type that a parametric max-stable distribution. Therefore, every spatial univariate marginal laws lies in the domain of attraction of the real-value parametric GEV distribution, defined spatially on the subdomain $D_\xi = \{x_i \in S;\ \sigma_i(x_i) + \xi_i(x_i)(y_i(x_i) - \mu_i(x_i)) > 0\} \subset S$ by

$$G_i(y_i(x_i)) = \begin{cases} \exp\left\{-\left[1 + \xi_i(x_i)\left(\frac{y_i(x_i)-\mu_i(x_i)}{\sigma_i(x_i)}\right)\right]^{\frac{-1}{\xi_i(x_i)}}\right\} \text{ if } \xi_i(x_i) \neq 0 \\ \exp\left\{-\exp\left\{-\left(\frac{y_i(x_i)-\mu_i(x_i)}{\sigma_i(x_i)}\right)\right\}\right\} \text{ if } \xi_i(x_i) = 0 \end{cases} \tag{7}$$

where the parameters $\{\mu_i \in \mathbb{R}\}$, $\{\sigma_i > 0\}$ and $\{\xi_i \in \mathbb{R}\}$ are referred to as the location, the scale and the shape parameters for the site $x_i$ respectively. In particular, if H is continuous the asymptotic copula the process derives from the following result.

**Theorem 2** *Let $C_H$ be the spatial copula of the process $\{Y(x)\}$. Then, under the key assumption, the copula $C_H$ converge to a spatial extremal copula $C_G$.*

*Proof.* The key assumption insures that the distribution of the process $\{Y(x)\}$ lies in the domain of attraction of a multivariate EVdistribution G. In particular, marginally there exist appropriate spatial coefficients of normalization $\{\sigma_{n,i}(x_i) > 0\}$ and $\{\mu_{n,i}(x_i) \in \mathbb{R}\}$ such as

$$\lim_{n \longrightarrow +\infty} H_i^n(\sigma_{n,i}(x_i) y_i(x_i) + \mu_{n,i}(x_{n,i})) = G_i(y_i(x_i)) \text{ for all } i = 1, .., n. \tag{8}$$

More generally, in one hand, applying (7) to the joint dependence structure, it follows that

$$\lim_{n \longrightarrow +\infty} H^n(\sigma_{n,1}(x_1) y_1(x_1) + \mu_{n,1}(x_{n,1}); ...; \sigma_{n,s}(x_s) y_s(x_s) + \mu_{n,s}(x_{n,s}))$$

$$= G(y_1(x_1); ...; y_s(x_s)) = C_G(G(y_1(x_1)); ...; G_s(y_s(x_s))).$$

On the other hand however,

$$H^n(\sigma_{n,1}(x_1) y_1(x_1) + \mu_{n,1}(x_{n,1}); ...; \sigma_{n,s}(x_s) y_s(x_s) + \mu_{n,s}(x_{n,s}))$$

$$= C_H^n(H_1(\sigma_{n,1}(x_1) y_1(x_1) + \mu_{n,1}(x_1)); ...; H_s(\sigma_{n,s}(x_s) y_s(x_s) + \mu_{n,s}(x_s))). \tag{9}$$

Moreover, the copula $C_H$ verifies the property of max-stability given by the relation (5).

Then, it results an asymptotical copula such as

$$\lim_{n\longrightarrow+\infty} H^n\left(\sigma_{n,1}(x_1)\, y_1(x_1)+\mu_{n,1}(x_{n,1})\,;...;\sigma_{n,s}(x_s)\, y_s(x_s)+\mu_{n,s}(x_{n,s})\right)$$
$$= \lim_{n\longrightarrow+\infty} C_H^n\left(H_1\left(\sigma_{n,1}(x_1)\, y_1(x_1)+\mu_{n,1}(x_1)\right);...;H_s\left(\sigma_{n,s}(x_s)\, y_s(x_s)+\mu_{n,s}(x_s)\right)\right)$$
$$= C_H\left((Gy_1(x_1))\,;...;G_s(y_s(x_s))\right). \tag{10}$$

Therefore, using simultaneously (9) and (10) it follows that, for all realization $y(x)$ of $\{Y(x)\}$

$$C_G\left(G(y_1(x_1))\,;...;G_s(y_s(x_s))\right) = C_H\left((Gy_1(x_1))\,;...;G_s(y_s(x_s))\right).$$

Therefore, the uniqueness of the copula associated to the continuous distribution H (Sklar, 1959) allows us to conclud that $C_H$ is max-stable. Finally, the max-stability implies that $C_H$ is an extremal copula.

**Corollary 3** *Let $C_H$ be the spatial copula associated to H. Then, there exists a space-varying function $A_h$ convex, defined on the unit simplex $\Omega_S$ such as*

$$C_H(u) = \exp\{-r_h(u)\, A_h(v_1;...,v_{s-1})\} \text{ for all } u=(u_1,...,u_s)\in[0,1]^s\,; \tag{11}$$

*where $r_h(u) = -\sum_1^n \log u_i$ and $v_i(h) = -\left(r_h^{-1}\right)\log u_i$, h being the mean value of the separating distance.*

*Proof.* In multivariate EV analysis, many tools have been developed to describe properties of joint dependence structures. Pickands (1981) reduced extremal dependence modelling to the unit simplex $\Omega_S$ and introduced a new convex function A such as:

$$A(t_1,...,t_s) = \textstyle\sum_1^s z^{-1} V(z_1,...,z_s) \text{ where } t_i = \frac{z_i}{\sum_1^s z}$$

Then, using the relation (6), it follows that

$$C_H(u_1,...,u_s) = \exp\left\{\left(\textstyle\sum_1^s \ln u_i\right) A\left(\frac{\log u_1}{\sum_1^s \log u_i};...,\frac{\log u_s}{\sum_1^s \log u_i}\right)\right\} \text{ for all } u_i\in[0,1].$$

In spatial context, let h denote the mean value of the separating distances $h_{ij}$ between the pairs of sites $\{x_i,x_j\}$; $1\le i,j\le s$, in the domain S. It results that,

$$h = \frac{1}{2s(s-1)}\textstyle\sum_{i=1}^s\left(\sum_{j=1}^s \left\|x_i,x_j\right\|\right) \text{ for all } x_i,x_j\in S.$$

Then, the Pickands dependence function of the spatial copula depends on h too. Finally, by setting $r_h(u) = -\sum_1^n \log u_i$ and $v_i(h) = -\left(r_h^{-1}\right)\log u_i$, it results (11).

*3.2 Margins of Spatial Extremal Copulas*

Climate statistical analysis is often concerned with areal data on the domain of interest. In such a case the domain is partitioned into finite numbers of areal units (regions, zip codes, counties etc.). So, any given sub-domain of S may contain a number of areal units. Let's be interested by characterizing the stochastic behavior of climate data under a distortional contrast in this behavior for two given complementary sub-domains $S_A$ and $S_{\bar{A}}$ of $S$. In such a context, let's define a spatial version of the conditional partition of a stochastic based on the Gibbs sampler Beirlant et al. (2004) and referred in Barro (2009) or in Dossou-Gbété et al. (2009) to as the discordant measure.

**Definition 4** The stationary process of second-order $\{Y(x)\}$ is said to have distortion on a sub-domain $S_A$ if , for all vector of location sites $x=(x_1,...,x_s)$;

$$\{Y_A(x) > y_A(x)\} \text{ given } \{Y_{\bar{A}}(x) > y_{\bar{A}}(x)\} \tag{12}$$

where $\{Y_A(x)\}$ and $\{Y_{\bar{A}}(x)\}$ denote the restrictions of the process $\{Y(x)\}$ to $S_A$ and $S_{\bar{A}}$ respectively.

This following result characterize the spatial copulas on the marginal processes.

**Theorem 5** *Let $C_H$ be the spatial copula of the process $\{Y(x)\}$ and $S_A$ a given subdomain of S. Then, under the key assumption, the copulas $C_A$ and $C_{\bar{A}}$ associated to the marginal processes $\{Y_A(x)\}$ and $\{Y_{\bar{A}}(x)\}$ respectively are spatial extremal copulas. Moreover, they are margins of $C_H$.*

*Proof.* According to Sklar's theorem, the copula $C_H$ is canonically associated to the distribution H by the spatial relation:

$$H\left((y_1(x_1)), ..., y_s(x_s)\right) = C_H\left((H_1(x_1)), ..., H_s(y_s(x_s))\right).$$

In the same way, the copula $C_A$ of the marginal process $\{Y_A(x)\}$ is such that for all $u = (u_1, ..., u_s) = (u_A, u_{\bar{A}}) \in [0,1]^s$;

$$C_A\left(u_{A,1}; ...; u_{\bar{A},n_A}\right) = H_A\left(H_{\bar{A},1}^{-1}(u_{A,1}); ...; H_{A,n_A}^{-1}(u_{A,n_A})\right) = \lim_{u_{\bar{A}} \longrightarrow 1^-} C_H(u_1, ..., u_s), \tag{13}$$

where $n_A = |S_A|$, the number of areal units in the sub-region $S_A$ while $u_A$ is the $n_A$−componentwise vector with index in $S_A$. Then, using simultaneously relation (12) and the max-stability in (5), it follows that

$$C_A^k\left(u_{A,1}; ...; u_{\bar{A},n_A}\right) = \lim_{u_{\bar{A}} \longrightarrow 1^-} C_H(u_1, ..., u_s) = \lim_{u_{\bar{A}} \longrightarrow 1^-} C_H\left(u_1^k, ..., u_s^k\right) = C_A\left(u_{A,1}^k; ...; u_{\bar{A},n_A}^k\right).$$

Then, the copula $C_A$ is also max-stable and then is a spatial extremal copula. Similarly, the marginal copula of the process, defined by

$$C_{\bar{A}}\left(u_{\bar{A},1}; ...; u_{\bar{A},s-n_A}\right) = H_{\bar{A}}\left(H_{\bar{A},1}^{-1}\left(u_{\bar{A},1}\right); ...; H_{\bar{A},s-n_A}^{-1}\left(u_{\bar{A},s-n_A}\right)\right) = \lim_{u_A \longrightarrow 1^-} C_H(u_1, ..., u_s),$$

is also shown to be a spatial extremal copula.

### *3.3 Characterization of Conditional $\lambda - Madogram$*

A generalized form of the F-madogram in (1) has been introduced by Cooley et al. (2006). It is the $\lambda - madogram$ associated to the distribution underlying the stochastic process $\{Y(x)\}$ and defined by

$$\gamma_F(h) = \tfrac{1}{2}E\left\{\left|[F(Y(x))]^{\lambda} - [F(Y(x+h))]^{1-\lambda}\right|\right\}; \lambda \in [0,1]$$

where $h$ is the average value of the separating distance between the two points (see Cooley). The following result gives a property in bivariate spatial study.

**Proposition 6** *Suppose H is a bivariate distribution satisfying the key assumption. If its associated multivariate EV distribution marginal are unit-Fréchet distributed, then, the $\lambda - madogram$ is given by*

$$\gamma_\lambda(h) = \frac{1}{D_h(\lambda, 1-\lambda) + \lambda} - c(\lambda) \ \ with\ c(\lambda) = \frac{2\lambda(1-\lambda)+1}{2(\lambda+1)(2-\lambda)} \tag{14}$$

*where $D_h$ is a conditional spatial measure convex defined on the unit simplex of $\mathbb{R}^2$.*

*Proof.* Let's consider in Defintion 4 the simplest case of distorsion where the sub-region $S_A$ contains only one areal unit. So, the quantification of the occurrence of events (12) are summarized by the probability $p_A$ such as

$$p_A = P(\{Y_A(x) > y_A(x)\} / \{Y_{\bar{A}}(x) \le y_{\bar{A}}(x)\}) = 1 - \frac{H(y_1(x_1), ..., y_s(x_s))}{H_{\bar{A}}\left(y_{\bar{A},1}\left(x_{\bar{A},1}\right), ..., y_{\bar{A},s}\left(x_{\bar{A},s}\right)\right)}; \tag{15}$$

where every $y_{\bar{A},i}\left(x_{\bar{A},i}\right)$represents a component of the spatial realization $(y_1(x_1), ..., y_s(x_s))$.

In terms of copulas (13) is equivalent to

$$p_A^+ = 1 - \frac{C(u_1, ...; u_s)}{C_{\bar{A}}(u_2, ...; u_s)} with\,(u_1, ...; u_s) \in [0,1]^s \tag{16}$$

where the marginal spatial copula $C_{\bar{A}}$ is such as

$$C_{\bar{A}}\left(u_{\bar{A},1}, ...; u_{\bar{A},s-1}\right) = P\left\{\left(H_{\bar{A},1}\left(y_{\bar{A},1}\left(x_{\bar{A},1}\right)\right) \le u_{\bar{A},1}; ..., H_{\bar{A},s-n_A}\left(y_{\bar{A},s-n_A}\left(x_{\bar{A},s-n_A}\right)\right) \le u_{\bar{A},s-n_A}\right)\right\}$$

Furthermore, under the key assumption, the two copulas in (16) are space-varying extremal copulas. So, using their canonical parameterizations via their extremal dependence functions yields

$$p_A^+ = 1 - \exp\{-V_{h,C_A}\left((\tilde{u}_1)^{-1}, ..., (\tilde{u}_s)^{-1}\right) + V_{h,\bar{C}_A}\left((\tilde{u}_2)^{-1}, ..., (\tilde{u}_s)^{-1}\right)\}$$

where $\tilde{u}_i = -\ln u_i$ for $i = 1, ..., s$, h being the mean value of the separating distance.

Therefore, by setting

$$D_h(t_1,\ldots,t_s) = V_H(t_1,\ldots,t_s) - V_{H_{\bar{A}}}\left(t_{\bar{A},1},\ldots,t_{\bar{A},s-1}\right);$$

one obtains a convex h-parametric dependence function defined on the unit simplex.

In particular, in bivariate case, it is easy to chek that

$$D_s(t_1,t_2) = V_{h,C_A}(t_1,t_2) - V_{H_{\bar{A}}}(t_2).$$

Finally, using the relationship between the $\lambda$-madogram and the extremal dependence measure $V_h$ given by Cooley et al. (2006), it results (14) as disserted.

## 4. Discussion

The results of the study show that under the assumption of max-stability of a stochastic process, the copula underlying a spatial process converge to an extremal copula. These results differ from the previous studies on spatial modelling because they focus specificly on max-stable GEV-marginally distributed models. The extremal results in this study are not on Gaussian patterns like in many others papers. Moreover, the spatial measure $D_h$ presents a number of opportunities both in the spatial dependence modelling and in extreme values theory because its domain $\Omega_{\mathbb{R}^2}$ can be reduced of one dimension.

## 5. Conclusion

In this study, we have investigated about properties and characterization of a multivariate copula associated to the distribution of a max-stable process. In particular, the results show that if the distribution of the process is a spatial max-stable model its copula also lies in the max-domain of attraction of spatial extremal copula. Specifically, it has been shown that the copulas of two complementary sub-processes are the marginal extremal copulas of the asymptotic copula of the process. A new spatial measure is constructed to characterize the $\lambda - madograms$ of the distribution of the process under a distortional constraint.

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