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Sinh Inverted Exponential Distribution: Simulation & Application to Neck Cancer Disease

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Abstract

A goal of this research is providing new probability distribution called Sinh inverted exponential distribution. The new distribution was extensively depending on the hyperbolic sine family of distributions with exponential distribution as a baseline distribution. Valuable statistical properties of the proposed distribution including mathematical and asymptotic expressions for its probability density function and Reliability. Moments, quantiles, moment generating function, failure rate function, mean residual lifetime, order statistics and entropies are derived. Actually, the applicability and validation of this model is proved in simulation study and an application to neck cancer disease data.

Keywords: hyperbolic sine, quantiles, moments, mean residual lifetime, entropy, simulation, maximum likelihood estimates

1. Introduction

In many applied areas such as lifetime analysis and other fields, there is strong need to develop the classical distributions. So, different methods to generating new families of distributions are defined. These include; Azzalini's skew family by Azzalini (1985), Marshal- Olkin generated family by Marshall and Olkin (1997), exponentiated generator by Gupta *et al.* (1998), beta-G by Eugene *et al.* (2002). In addition, Cordeiro and de Castro (2011) studied Kumaraswamy-G family, Nadarajah *et al.* (2013) introduced geometric exponential-Poisson generator. Recently, Kharazmi and Saadatinik (2016) proposed hyperbolic cosine family also, Kharazmi and Saadatinik (2018) discussed the hyperbolic sine family of distributions, Chakraborty and Handique (2017) investigated the generalized Marshall-Olkin Kumaraswamy-G family and generalized inverse Weibull family by Hemeda *et. al* (2019) and more.

According to Kharazmi and Saadatinik (2018), the hyperbolic Sine (HS) family with cumulative CDF is

$$F(x)=\frac{2e^{\delta}}{(e^{\delta}-1)^2}\left(\cosh(\delta G(x))-1\right), \tag{1}$$

and probability density PDF is

$$f(x)=\frac{2\delta e^{\delta}}{(e^{\delta}-1)^2}g(x)\sinh(\delta G(x)); x>0, \delta>0 . \tag{2}$$

Where, $G(x)$ and $g(x)$ are the CDF and PDF for any random variable, respectively and the hyperbolic sine function $(sinh(x))$ is defined as

$$sinh(x)=\frac{1}{2}\left(e^{x}-e^{-x}\right). \tag{3}$$

Using series expansion theorem, $sinh\ (x)$ takes the following formula;

$$sinh(x)=\sum_{j=0}^{\infty}\frac{x^{2j+1}}{(2j+1)!} . \tag{4}$$

In our study, we will take $G(x)$ is the CDF of the inverse exponential distribution and $g(x)$ its PDF.

Dey (2007) studied the inverse exponential (IE) distribution with CDF and PDF are given by

$$G(x;\theta)=e^{\frac{-\theta}{x}}, \tag{5}$$

$$g(x;\theta)=\frac{\theta}{x^2}e^{\frac{-\theta}{x}};\ x>0,\ \theta>0. \tag{6}$$

2. The New Model and Its Behaviors

This section contributes the representation of Sinh inverted exponential (SIE) distribution. The CDF, reliability, hazard rate, cumulative hazard rate functions are deduced and discussed analytically. As well as, asymptotic behaviors of SIE Distribution will be acquired.

2.1 Mathematical Representations

By substituting from (5) and (6) into (1) and (2), then the SIE CDF and PDF will be

$$F_{SIE}(x)=\frac{2e^{\delta}}{(e^{\delta}-1)^2}\left(\cosh\left(\delta e^{\frac{-\theta}{x}}\right)-1\right), \tag{7}$$

$$f_{SIE}(x)=\frac{2\delta\theta e^{\delta}}{(e^{\delta}-1)^2x^2}e^{\frac{-\theta}{x}}\sinh\left(\delta e^{\frac{-\theta}{x}}\right);x>0,\theta,\delta>0. \tag{8}$$

Using (1), the PDF will be in the following form

$$f_{SIE}(x)=\frac{\delta\theta e^{\delta}x^{-2}}{(e^{\delta}-1)^2}\left(e^{\delta e^{\frac{-\theta}{x}}-\frac{\theta}{x}}-e^{-\delta e^{\frac{-\theta}{x}}-\frac{\theta}{x}}\right). \tag{9}$$

The survival and hazard rate (hr) functions are

$$S_{SIE}(x)=1-\frac{2e^{\delta}}{(e^{\delta}-1)^2}\left(\cosh\left(\delta e^{\frac{-\theta}{x}}\right)-1\right), \tag{10}$$

$$h_{SIE}(\theta)=\frac{2\delta\theta e^{\delta-\frac{\theta}{x}}\sinh\left(\delta e^{\frac{-\theta}{x}}\right)}{\left[1-\frac{2e^{\delta}}{(e^{\delta}-1)^2}\left(\cosh\left(\delta e^{\frac{-\theta}{x}}\right)-1\right)\right](e^{\delta}-1)^2x^2}. \tag{11}$$

In addition, the cumulative hazard rate function corresponding to (10) is

$$S_{SIE}^{c}(x)=-\ln s(x)=-\ln\left[1-\frac{2e^{\delta}}{(e^{\delta}-1)^2}\left(\cosh\left(\delta e^{\frac{-\theta}{x}}\right)-1\right)\right]. \tag{12}$$

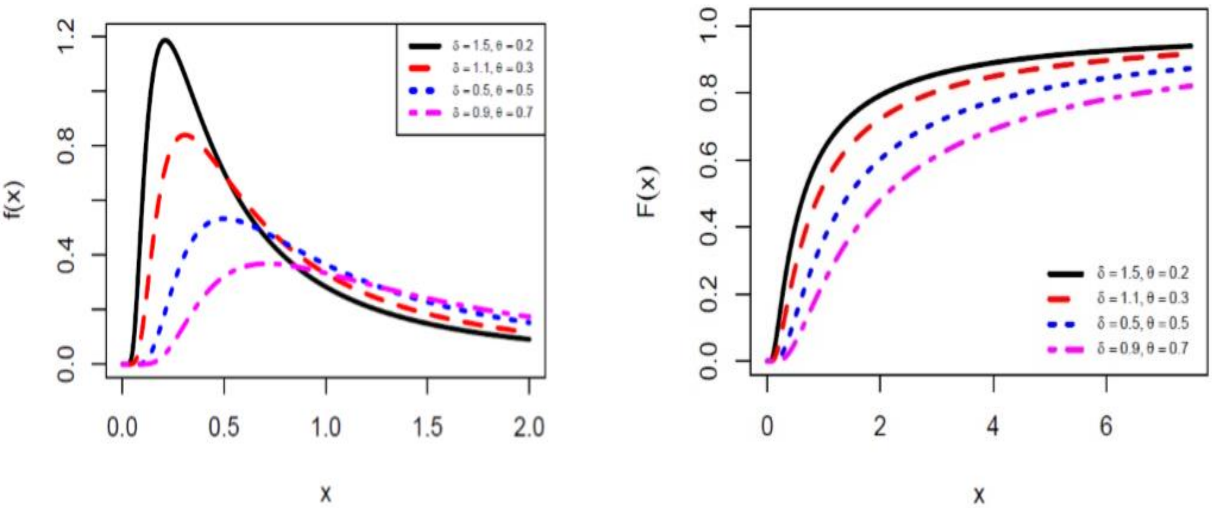

Figure 1. PDF and CDF of SIE distribution

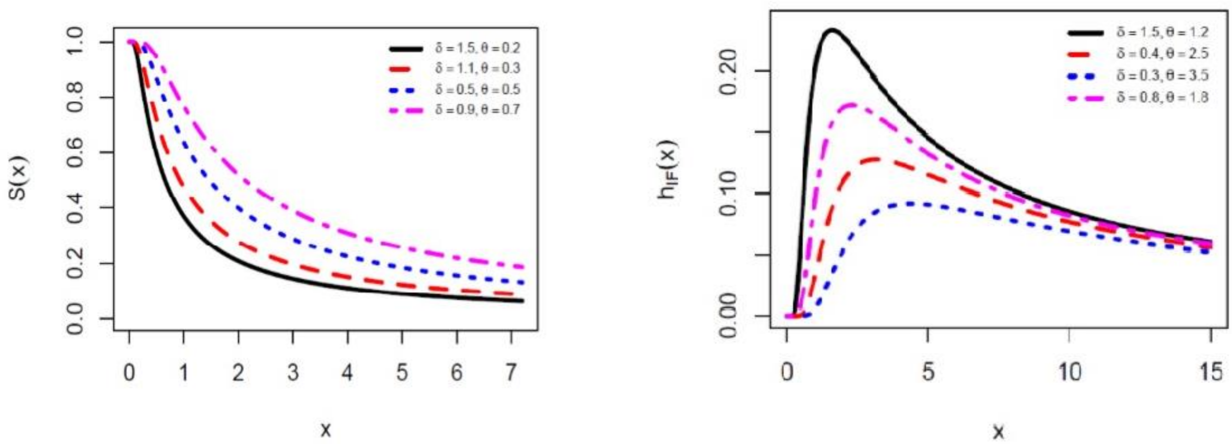

Figure 2. Survival & hr functions of SIE distribution

2.2 Asymptotic Limits

The asymptotic limits of CDF (7) and PDF (8) for SIE distribution are shown in the following remarks:

Remark 1: *Asymptotic of CDF*

i. $\lim_{x\to\infty} F_{SIE}(x) = 1.$

ii. $\lim_{x\to 0} F_{SIE}(x) = 0\,.$

iii. $\lim_{x\to 1} F_{SIE}(x) = \dfrac{2e^{\delta}}{(e^{\delta}-1)^2}\left(\cosh\left(\delta e^{-\theta}\right)-1\right).$

iv. $\lim_{x\to\theta} F_{SIE}(x) = \frac{2e^{\delta}}{(e^{\delta}-1)^2}\left(\cosh\left(\delta e^{-1}\right)-1\right).$

v. $\lim_{x\to\delta} F_{SIE}(x) = \frac{2e^{\delta}}{(e^{\delta}-1)^2}\left(\cosh\left(\delta e^{\frac{-\theta}{\delta}}\right)-1\right)$

Remark 2: *Asymptotic of PDF*

i. $\lim_{x\to\infty} f_{SIE}(x)=0$.

ii. $\lim_{x\to 0} f_{SIE}(x)=0$.

iii. $\lim_{x\to 1} f_{SIE}(x) = \frac{2\delta\theta e^{\delta-\theta}}{(e^{\delta}-1)^2}\sinh\left(\delta e^{-\theta}\right).$

iv. $\lim_{x\to\theta} f_{SIE}(x) = \frac{2\delta e^{\delta-1}}{\theta(e^{\delta}-1)^2}\sinh\left(\delta e^{-1}\right).$

v. $\lim_{x\to\delta} f_{SIE}(x) = \frac{2\theta e^{\delta-\frac{\theta}{\delta}}}{\delta(e^{\delta}-1)^2}\sinh\left(\delta e^{\frac{-\theta}{\delta}}\right).$

3. Useful Statistical Properties

Various statistical measures will be deduced such as moments, moment generating function, incomplete moments and mean residual life time, quantile function, median, mode, entropies, skewness and kurtosis of SIE distribution in this section.

3.1 Moments and Incomplete Moments

Using equations (3), (4), the PDF of SIE distribution takes the following formula;

$$f_{SIE}(x) = \sum_{j=0}^{\infty} A_j\, x^{-2}\, e^{\frac{-2\theta(j+1)}{x}} ; x>0,\delta,\theta>0. \tag{13}$$

Where, $A_j = \frac{2\theta\delta^{2j+2}\, e^{\delta}}{(e^{\delta}-1)^2(2j+1)!}$.

Since the m*th* moments is defined as

$$\mu'_m = \int_0^{\infty} x^m f_{SIE}(x)dx .$$

By substituting from (13) into the last equation, the m*th* moment is written as

$$\mu'_m = \sum_{j=0}^{\infty} A_j \int_0^{\infty} x^{m-2} e^{\frac{-2\theta(j+1)}{x}} dx ,$$

$$\mu'_m = \sum_{j=0}^{\infty} A_j \frac{\Gamma(1-m)}{\left(\theta(2\,\mathrm{j}+1)\right)^{1-m}} . \tag{14}$$

The first moment (mean) can be calculated as follows

$$\mu = \sum_{j=0}^{\infty} A_j = \sum_{j=0}^{\infty} \frac{2\theta \delta^{2j+2} e^{\delta}}{(e^{\delta}-1)^2 (2j+1)!}. \tag{15}$$

The incomplete moment ($\mu^t{}_m$) is computed by the following equation:

$$\mu^t{}_m = \int_0^t x^m f(x;\delta,\theta)dx = \sum_{j=0}^{\infty} A_j \frac{\Gamma(1-m,t)}{\left[\theta(2\,\mathrm{j}+1)\right]^{1-m}}. \tag{16}$$

Where, $\Gamma(s,t) = \int_t^{\infty} x^{s-1} e^{-x}\, dx$ is the incomplete gamma function.

3.2 Moment Generating Function & Mean Residual Life Time

The moment generating function of a probability distribution can be derived as follows

$$M_{SIE}(t) = \sum_{m=0}^{\infty} \frac{t^m \mu'_m}{m!},$$

where, μ'_m is the *mth* moment about origin. Using (13) then $M_{SIE}(t)$ will be

$$M_{SIE}(t) = \frac{t^m}{m!} \sum_{j=0}^{\infty} A_j \frac{\Gamma(1-m)}{\left[\theta(2\,\mathrm{j}+1)\right]^{1-m}}. \tag{17}$$

The mean residual of SIE distribution $m_{SIE}(t)$ is determined by

$$m(t) = \left[\frac{1}{S(t)} \int_t^{\infty} x\, f(x)\, dx\right] - t \quad \text{(Gupta and Gupta (1983)).}$$

Substituting from PDF (13) then $m_{SIE}(t)$ will be calculated by

$$m_{SIE}(t) = \frac{\sum_{\mathrm{j}=0}^{\infty} A_j\, \Gamma(2,t)}{1 - \frac{2e^{\delta}}{(e^{\delta}-1)^2}\left(\cosh\left(\delta e^{\frac{-\theta}{x}}\right) - 1\right)} - t. \tag{18}$$

3.3 Quantile Function & Skewness and Kurtosis

The quantile function of SIE distribution, ($x(p) = F^{-1}(p)$) is determined by converting (7) as follows:

$$x(p) = \frac{-\theta}{\ln\left[\frac{1}{\delta}\cosh^{-1}\left(\frac{p(e^{\delta}-1)^2}{2e^{\delta}} + 1\right)\right]}. \tag{19}$$

Equation (19) can be solved numerically, the SIE random variable X can be generated where p has the uniform distribution on the interval [0,1].

The skewness (Λ) and kurtosis (Υ) coefficients based on quantiles are computed from the following formulas:

$$\Lambda = \frac{q(0.75) - 2Q(0.25) + Q(0.25)}{Q(0.75) - Q(0.25)} \quad \text{(Kenney and Keeping (1962)).} \tag{20}$$

$$\Upsilon = \frac{Q(0.875) - Q(0.625) + Q(0.375) - Q(0.125)}{Q(0.75) - Q(0.25)}, \quad \text{(Moors (1988)).} \tag{21}$$

Substituting from (19) into (20) and (21) respectively, we can get some values to the skewness and kurtosis coefficients of SIE distribution as represented in Figure 3.

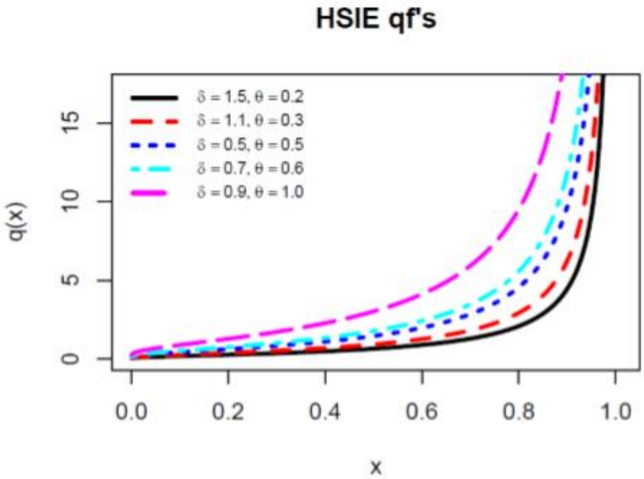

Figure 3. Quantile function of SIE distribution with some values of δ, θ

3.4 Median and Mode

Hence, the median x_{med} of SIE distribution is derived by substituting $p = 0.5$ in equation (19) as follows

$$x_{med} = \frac{-\theta}{\ln\left[\frac{1}{\delta}\cosh^{-1}\left(\frac{(e^{\delta}-1)^2}{4e^{\delta}}+1\right)\right]}. \tag{22}$$

The median x_{med} is determined by solving (22) numerically for some selected values of θ, δ.

The mode x_{mod} is computed by solving the equation $\frac{df_{SIE}(x)}{dx} = 0$ such that $\frac{d^2 f_{SIE}(x)}{dx^2} \geq 0$.

Therefore,

$$\frac{df_{SIE}(x)}{dx} = \frac{2\delta\theta e^{\delta}}{(e^{\delta}-1)^2}\left[-2x^{-3}e^{\frac{-\theta}{x}}\sinh\left(\delta e^{\frac{-\theta}{x}}\right)+\theta x^{-4}e^{\frac{-\theta}{x}}\sinh\left(\delta e^{\frac{-\theta}{x}}\right)+\theta\delta x^{-4}e^{\frac{-2\theta}{x}}\cosh\left(\delta e^{\frac{-\theta}{x}}\right)\right]=0,$$

$$x^{-4}e^{\frac{-\theta}{x}}\left[(\theta-2x)\sinh\left(\delta e^{\frac{-\theta}{x}}\right)+\theta\delta\cosh\left(\delta e^{\frac{-\theta}{x}}\right)\right]=0. \tag{23}$$

The mode x_{mod} is determined by solving (23) numerically for some initial values of parameters θ, δ.

3.5 The Entropy Measures

The entropy of a random variable X is an important measure, it is defined as a measure of variation of the uncertainty (see, Rényi (1961)). In this subsection, we discuss Rényi and w entropy measures.

The Rényi entropy $E_{\text{Ren}}(\xi)$ of a random variable X is defined as

$$E_{\text{Ren}}(\xi) = \frac{1}{1-\xi} Log\left[\int_0^{\infty} f^{\xi}(x)\, dx\right],$$

where $\xi > 0$ and $\xi \neq 1$. Based on PDF (13) using binomial expansion theorem and after some simplifications, we obtain

$$f_{SIE}^{\xi}(x)=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\sum_{k=0}^{\xi}c_{n,m,k}\; x^{-m-2\xi}\, e^{\frac{-\theta(n-m)}{x}}\;, \tag{24}$$

where

$$c_{n,m,k}\;=\frac{(-1)^{k+m}\theta^{m}(k-2\xi)^{n}\delta^{(n-m)}\,\xi!}{k!m!(\xi-k)!(n-m)!}\left(\frac{\delta\,\theta\, e^{\delta}}{(e^{\delta}-1)^{2}}\right)^{\xi}.$$

The Rényi entropy will be

$$E_{\mathrm{Ren}}(\xi)=\frac{1}{1-\xi}Log\left[\sum_{n=0}^{\infty}\sum_{m=0}^{n}\sum_{k=0}^{\xi}c_{n,m,k}\;\frac{\Gamma(m+2\xi-1)}{\left(\theta(n-m)\right)^{m+2\xi-1}}\right];\ \theta\neq 0,\ n\neq m. \tag{25}$$

The w-entropy $E(w)$ is defined by

$$E(w)=\frac{1}{w-1}Log\left[1-\int_{0}^{\infty}f^{w}(x)\;dx\right],\text{ where } w>0 \text{ and } w\neq 1.$$

From equation (25) we can easily contribute that:

$$E(w)=\frac{1}{w-1}Log\left[1-\sum_{n=0}^{\infty}\sum_{m=0}^{n}\sum_{k=0}^{w}\eta_{n,m,k}\;\frac{\Gamma(m+2w-1)}{\left(\theta(n-m)\right)^{m+2w-1}}\right];\ \theta\neq 0,\ n\neq m\cdot \tag{26}$$

Where

$$\eta_{n,m,k}\;=\frac{(-1)^{k+m}\theta^{m}(k-2w)^{n}\delta^{(n-m)}w!}{k!m!(w-k)!(n-m)!}\left(\frac{\delta\theta e^{\delta}}{(e^{\delta}-1)^{2}}\right)^{w}.$$

4. Estimation of Parameters

In this section, the maximum likelihood estimators of the model parameters $\Im=(\delta,\theta)$ of SIE distribution from complete samples are deduced. Assume $X_1,\ X_2,\dots,X_n$ be a simple random sample from SIE distribution with observed values $x_1,\ x_1,\dots x_n$, the log likelihood function of (8) is obtained as follows

$$L(\Im)=\prod_{j=1}^{n}f_{SIE}(x_j)\,,$$

$$ln\,L(\Im)=\sum_{j=1}^{n}\ln f_{SIE}(x_j)\,,$$

Based on the PDF (9), then

$$ln\,L(\Im)=\sum_{j=1}^{n}\ln\left[\frac{\delta\,\theta\, e^{\delta}\,x_j^{-2}}{(e^{\delta}-1)^{2}}\left(e^{\delta e^{\frac{-\theta}{x_j}}-\frac{\theta}{x_j}}-e^{-\delta e^{\frac{-\theta}{x_j}}-\frac{\theta}{x_j}}\right)\right],$$

$$ln\,L(\Im)=n\ln\left(\frac{\delta\,\theta e^{\delta}}{(e^{\delta}-1)^{2}}\right)-2\ln\sum_{j=1}^{n}x_j-\frac{\theta}{\sum_{j=1}^{n}x_j}+\ln\left(\sum_{j=1}^{n}e^{\delta e^{\frac{-\theta}{x_j}}}-\sum_{j=1}^{n}e^{-\delta e^{\frac{-\theta}{x_j}}}\right).$$

Differentiating $ln\,L(\Im)$ with respect to $\theta,\ \delta$ and setting the result equals to zero, the maximum likelihood estimators will be gotten. The partial derivatives of $ln\,L(\Im)$ with respect to each parameter are given as

$$\frac{\partial lnL(\Im)}{\partial\theta}=\frac{n}{\theta}-\frac{1}{\sum_{j=1}^{n}x_j}-\frac{\delta\sum_{j=1}^{\infty}\frac{1}{x_j}e^{\left(\delta e^{\frac{-\theta}{x_j}}\frac{-\theta}{x_j}\right)}}{\sum_{i=1}^{\infty}\left(e^{\delta e^{\frac{-\theta}{x_j}}}-e^{-\delta e^{\frac{-\theta}{x_j}}}\right)}.$$

$$\frac{\partial lnL(\Im)}{\partial\delta}=\frac{n}{\delta}+n-\frac{2ne^{\delta}}{(e^{\delta}-1)}+\frac{\sum_{j=1}^{n}\left(e^{\left(\delta e^{\frac{-\theta}{x_j}}-\frac{\theta}{x_j}\right)}+e^{\left(-\delta e^{\frac{-\theta}{x_j}}-\frac{\theta}{x_j}\right)}\right)}{\sum_{j=1}^{n}\left(e^{\delta e^{\frac{-\theta}{x_j}}}-e^{-\delta e^{\frac{-\theta}{x_j}}}\right)}.$$

The maximum likelihood estimators of the model parameters are determined by solving the non-linear equations $\frac{\partial lnL(\Im)}{\partial\theta}=0$, $\frac{\partial lnL(\Im)}{\partial\delta}=0$ .These equations can be solved simultaneously, numerically using iterative technique. For interval estimation of the parameters, the 2×2 observed information matrix $I(\Omega)=\{I_{uv}\}$ for (θ,δ). Under the regularity conditions, the known asymptotic properties of the maximum likelihood method ensure that: $\sqrt{n}(\hat{\Omega}-\Omega)\xrightarrow{d}N_2(0,I^{-1}(\Omega))$ as $n\to\infty$, where $\xrightarrow{d}$ means the convergence in distribution, with mean $O=(0,0)^T$ and 2×2 covariance matrix $I^{-1}(\Omega)$ then, the $100(1-\beta)\%$ confidence intervals for $\theta\ and\ \delta$ are given, respectively, as follows

$\hat{\delta}\pm Z_{\beta/2}\sqrt{var\ (\hat{\delta})}$ and $\hat{\theta}\pm Z_{\beta/2}\sqrt{var\ (\hat{\theta})}$, where $Z_{\beta/2}$is the standard normal at $\beta/2$. The significance level is $\beta/2$ and the variances of θ,δ are the diagonal elements of $I^{-1}(\Omega)$ corresponding to the model parameters.

5. Simulation Study

A simulation study is carried out to investigate the performance of estimators for SIE distribution in terms of their bias (bias), mean square error (MSE) using maximum Likelihood estimation (MLE) method. Simulated procedures can be described as follows:

Generated samples of sizes n = 30, 50, 100 from SIE distribution are generated and parameters are estimated using the maximum likelihood estimation method. 10000 such repetitions are made to calculate the bias and mean square error (MSE) of these estimates using the formula of estimates for any parameter η by $\text{Bias}_{\psi}(\hat{\psi})=\frac{1}{1000}\sum_{i=1}^{1000}(\hat{\psi}-\psi)$ and $\text{MSE}_{\psi}\ (\hat{\psi})=\frac{1}{1000}\sum_{i=1}^{1000}(\hat{\psi}-\psi)^2$ respectively.

From Table 1, it is observed that;

i. As sample size n increases, bias decreases. That shows accuracy of the MLE of the parameters.

ii. As sample size n increases, MSE decreases. That shows consistency (or preciseness) of the MLE of the parameters as shown in figure 4.

Table 1. Bias and MSE of MLEs for SIE distribution

	$\hat{\theta}=0.3$			$\hat{\delta}=0.1$		
n	**30**	**50**	**100**	**30**	**50**	**100**
BIAS	-0.10987	-0.1078	-0.1006	-0.0963	-0.0712	-0.0160
MSE	0.0142	0.0123	0.0022	0.0693	0.0293	0.0053
	$\hat{\theta}=0.1$			$\hat{\delta}=0.2$		
n	**30**	**50**	**100**	**30**	**50**	**100**
BIAS	-0.1099	-0.1037	-0.0097	-0.1963	-0.1960	-0.1960
MSE	0.1123	0.0184	0.0112	0.0385	0.0274	0.0080
	$\theta=0.2$			$\hat{\delta}=0.2$		
n	**30**	**50**	**100**	**30**	**50**	**100**
BIAS	-0.2099	-0.2099	-0.2098	-0.1961	-0.1959	-0.1959
MSE	0.0443	0.0441	0.0440	0.0384	0.0384	0.0384
	$\hat{\theta}=0.2$			$\hat{\delta}=0.3$		
n	**30**	**50**	**100**	**30**	**50**	**100**
BIAS	-0.2099	-0.2099	-0.2098	-0.2962	-0.2960	-0.2959
MSE	0.0443	0.0441	0.0441	0.0877	0.0876	0.0876
	$\hat{\theta}=0.5$			$\hat{\delta}=0.1$		
n	**30**	**50**	**100**	**30**	**50**	**100**
BIAS	-0.1400	-0.1100	-0.1100	-0.0905	-0.0453	-0.0150
MSE	0.1601	0.1601	0.1600	0.0092	0.0092	0.0092
	$\hat{\theta}=0.1$			$\hat{\delta}=0.5$		
n	**30**	**50**	**100**	**30**	**50**	**100**
BIAS	-0.1109	-0.1107	-0.1107	-0.4953	-0.4952	-0.4949
MSE	0.0123	0.0123	0.0123	0.2453	0.2452	0.2449
	$\hat{\theta}=0.1$			$\hat{\delta}=0.1$		
n	**30**	**50**	**100**	**30**	**50**	**100**
BIAS	-0.1102	-0.1100	-0.1100	-0.0962	-0.0959	-0.0959
MSE	0.0122	0.0122	0.0121	0.0093	0.0092	0.0092
	$\hat{\theta}=0.5$			$\hat{\delta}=0.5$		
n	**30**	**50**	**100**	**30**	**50**	**100**
BIAS	-0.5099	-0.5099	-0.5099	-0.4960	-0.4959	-0.4952
MSE	0.2601	0.2600	0.2600	0.2460	0.2495	0.2453

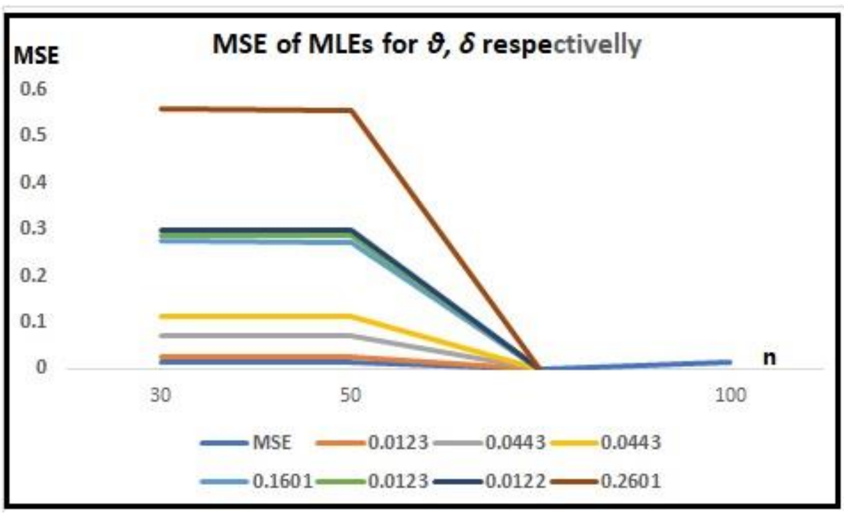

Figure 4. MSEs of parameters estimates for SIE distribution

6. Neck Cancer Disease Application

In this section, the SIE distribution is fitted for a real data. The real data represents the survival times of patients suffering from Neck cancer disease. The patients in this group were treated using a combined radiotherapy and chemotherapy (CT&RT). The data are

12.2 23.56 23.74 25.78 31.98 37 41.35 47.38 55.46 58.36 63.47 68.46

78.26 74.47 81.43 84 92 94 110 112 119 127 130 133

140 146 155 159 173 179 194 195 209 249 281 319

339 432 469 519 633 725 817 776

Kumar *et al.* (2015) fitted this data to the inverse Lindley distribution. We have fitted this data set with SIE distribution compared with Weibull (W) and inverted exponential (IE) probability distributions. The results of estimated values of the parameters (Log-likelihood , AIC, BIC and KS) are listed in Table 2. The Q-Q plot, histogram, fitted PDF and estimated CDF of the SIE curve to this data have been shown in Figures 5 and 6 respectively. The selection criterion is that the lowest Log-likelihood and AIC correspond to the best model fitted. The MLEs, AIC, BIC and KS are shown in Table 2. From the Table, we can observe that the SIE model shows the smaller Log-likelihood, AIC, BIC and KS than other competing distributions.

Table 2. Statistical measures of fitted models using survival times of patients suffering from Neck cancer disease data

Distribution	**Estimators**	**LL**	**AIC**	**BIC**	**KS**
SIE	$\hat{\theta} = 1.53$, $\hat{\delta} = 7.91$	-279.32	564.64	409.13	0.1840
W	$\hat{\beta} = 3.07$, $\hat{\alpha} = 11.26$	-288.79	597.43	532.02	0.1752
IE	$\hat{\delta} = 5.47$	-480.35	862.71	813.00	0.0637

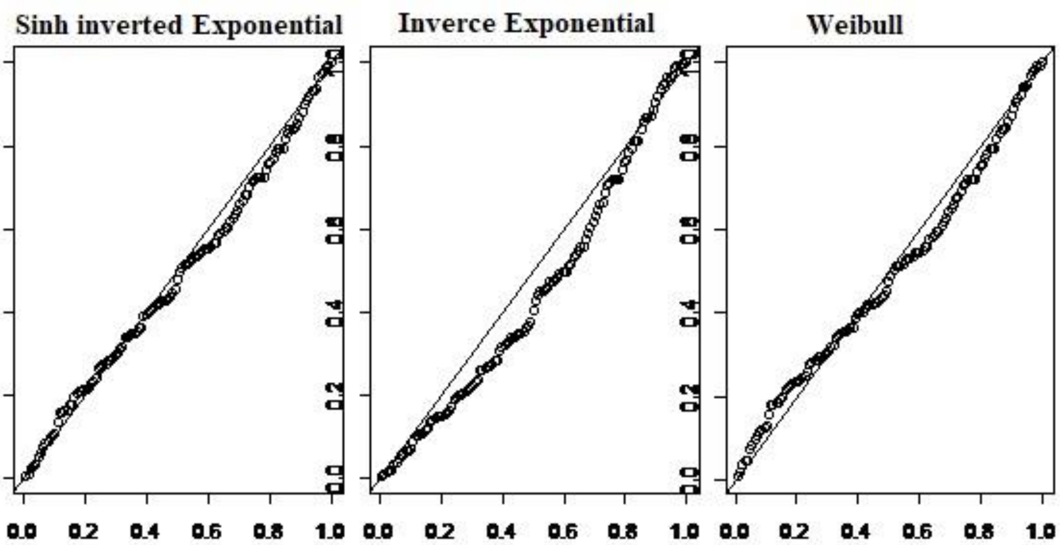

Figure 5. Q-Q plot of SIE, IE and W models for the Neck cancer disease data

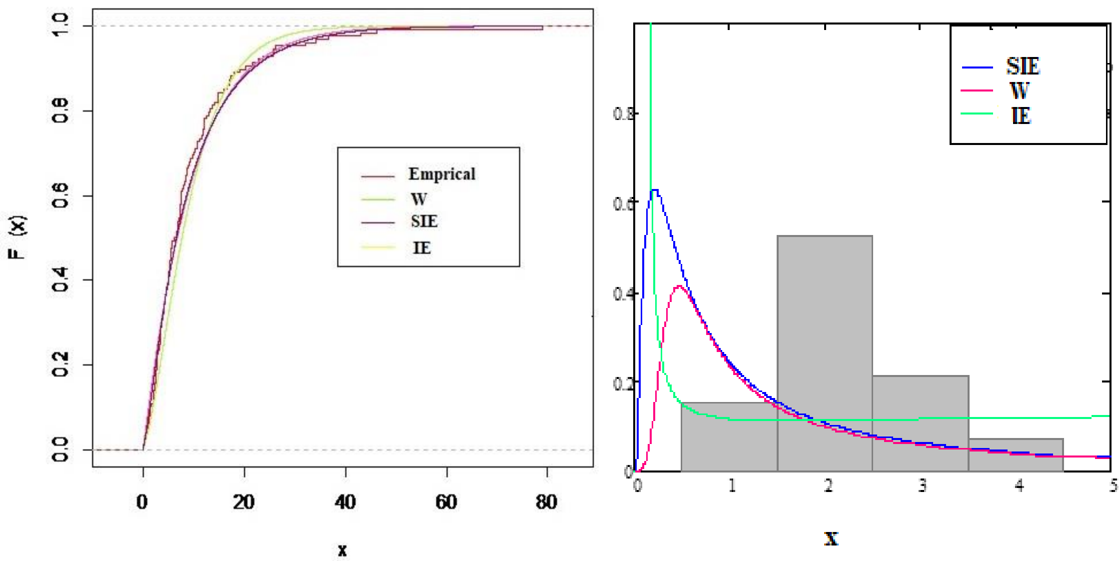

Figure 6. Plots of estimated CDF and histogram fitted PDF of the fitted models for the Neck cancer disease data from left to right

7. Conclusion

In this article, we have introduced and studied a new probability distribution called sinh inverted exponential distribution based on hyperbolic sin generator. The structural and reliability properties of this distribution have been studied and inference on parameters have also been mentioned.The estimation of parameters is approached by maximum likelihood methode.We presented a simulation study to exhibit the performance and accuracy of maximum likelihood estimates of the SIE model parameters. The Neck cancer disease real data application is applied to illustrate the efficience and applicability of the SIE distribution. The application of the SIE distribution shows that it could provide a better fit than other alternative distributions.

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