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An S-Shaped Crude Oil Price Return-Implied Volatility Relation: Parametric and Nonparametric Estimations

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Abstract

Oil market movements have important implications for portfolio management and hedge strategies for investors who negotiate this commodity. Studies involving the relation of the CBO Crude Oil ETF Volatility Index (OVX) and the United States Oil Fund (USO) return are small in number and do not explore some aspects related to the asymmetry and nonlinearity of this relation. Therefore, this article proposes an analysis about the relation between return and volatility, using parametric and nonparametric methods. To do so, a daily data series from 2007 to 2016, ordinary least squares, quantile regressions and the nonparametric B-splines methods were used. The results indicated a negative, asymmetric and nonlinear contemporary relation between the variables. The effects of negative returns were more pronounced than the positive ones in volatility. In addition, it was found that the relation is not the same for different quantiles. Nonparametric estimates suggested that the positive returns have a convex profile and the negative returns have a concave profile. It indicated the downward-sloping reclined S-curve for the 0.05, 0.90 and 0.95 quantiles of volatility.

Keywords: finance, commodity markets, investment decisions, quantile regression, nonparametric method

1. Introduction

Understanding the relation between return and volatility in the financial market is crucial for risk management. Researches about this relation have been made over time and have been intensified since the creation of implied volatility indices for markets and funds, such as those provided by the Chicago Board Options Exchange (CBOE). These types of index allow us to examine the reaction of players in the face of market dynamics and their expectation about the future, an important factor in the decision making of investments.

Even in the face of the great oscillations that oil prices have shown in recent years (for more information, see Reboredo, 2012) and the construction of the Crude Oil Volatility Index (OVX), based on the values of the options of United States Oil Fund (USO), few studies have been performed to verify the return-implied volatility relation (see, for example, Aboura & Chevallier, 2013; Padungsksawasdi & Daigler, 2014; Agbeyegbe, 2015). Thus, important aspects related to the asymmetry and nonlinearity of the relation between OVX and USO have not yet been explored by evaluation strategies that use flexible methodologies such as nonparametric methods.

Theoretical explanations about the return-volatility relation are mainly based on leverage and feedback hypotheses and on behavioral theory. The first one affirms that negative returns make the leverage of the companies increase, which makes them more risky and the stock price more volatile (Black, 1976; Christie, 1982); The hypothesis of feedback suggests that a variation in volatility causes an opposite change in the share price, due to changes in the risk premium (see French et al., 1987; Campbell & Hentschel, 1992); Finally, behavioral theory explains the return-volatility relation through behavioral concepts, as explained in Hibbert et al. (2008).

This paper aims to investigate the relation between the implied volatility and the returns of the oil fund (USO), little explored in the literature. In particular, it analyses the asymmetric and nonlinear relation of these variables. Thus, a strategical approach of two ways was used, with parametric and nonparametric methods. This was the first time that a nonparametric quantile modeling, the B-splines proposed by Koenker et al. (1994), was applied to the problem in focus. It holds the advantage of not imposing a functional form. It was also used in order to contribute to the analysis of the results of other methods and to the format verification of the relations between variables.

The results suggested that, in general, the nonlinear relation between OVX and USO returns is significant and with different slopes, depending on the quantile evaluated. When this relation is associated with a format, the best description would be an “S-shaped”, with results that change the concavity depending whether negative or positive returns.

The paper is structured in four more sections, in addition to this introduction. In section 2, theories that support empirical studies about return-volatility relation and its applications were presented, with emphasis on those using implied volatility indices. In section 3, the estimated equations and the methods used were described. In section 4, the results for all proposed methods and models were discussed. Finally, in section 5, the final considerations were performed.

2. The Return-Volatilily Relation

The main theories used to explain the return-volatility relation are the leverage hypothesis, the feedback hypothesis and the behavioral theory. The first is based on the articles of Black (1976) and Cristie (1982) and assumes an inverse relation between return and volatility, with variations in prices causing volatility. According to this theory, negative returns cause a greater leverage of the debt for the companies. As a result, these firms become more risky and their prices tend to become more volatile.

The feedback hypothesis has, as main references, the articles of French et al. (1987) and Campbell and Hentschel (1992). It concerns the influence of future volatility on stock prices. Positive volatility shocks cause a higher future rate of return required. Another characteristic associated to the feedback hypothesis is asymmetry. Negative changes in the expected return tend to be intensified, while the positive changes tend to be attenuated, due to changes in the risk premium.

Hibbert et al. (2008) suggested a different explanation for the negative relation between return and volatility: behavioral theory. With daily and intraday frequency data of the S&P500 and Nasdaq100 returns, and their implied volatilities, VIX and VXN, the authors found results that support the asymmetric volatility-return relation as a contemporary phenomenon.

From behavioral finance concepts and Theory of Prospectus of Kahneman and Tversky’s (1979), behavioral theory works with the identification of different behaviors of agents in the face of losses and gains. Low (2004) had already suggested that the cause of the asymmetric effect of the return-volatility relation could be behavioral. In the same sense, other studies provide evidence of this asymmetric relation betwen return and implied volatility (see, for example, Agbeyegbe, 2015; Badshah, 2013).

Low’s article (2004) has another important contribution. He was the first to investigate the nonlinear return-volatility relation by using a quadratic term for returns in the equation describing the contemporary relation between S&P100 return and VXO data. The author identified a convex profile for extreme losses, which contributed to the “S-shaped” analysis of the relation between variables. In addition, it found an asymmetric association in this relation.

Padungsaksawasdi and Daigler (2014) investigated the return-volatility relation of ETF funds of the euro, gold and oil, with daily and intraday data. Their results indicated two points: first, that the euro and gold do not have an asymmetric return-volatility relation. Second, the relation of the variables presents a parabola format, with the concavity upwards, different from the results for Low (2004). On the other hand, Agbeyegbe (2015) tested the nonlinear relation between the variables using conditional copula and quantile regression methods. His results identified an inverted “U” through quantiles for the relation between USO and OVX. These results suggested that the format of the contemporary relation between variables may be different depending on the variable analyzed, whether using market indices or using different ETFs indices.

The first empirical investigation to assess the relation between implied volatility (previously VIX (Note 1), now VXO) and market return (S&P 100) was Fleming et al. (1995). The results indicate a strong temporal relation return-volatility, asymmetry and a high contemporary correlation (negative) between the variables.

In a recent study, Daigler et al. (2014) use quantile regressions to investigate the relation between the return of the dollar/euro ETF and the variations in the implied volatility, the Euro Currency Volatility Index (EVZ). Their results indicated that, regardless of the sign of returns, they cause a significant increase in euro volatility. In addition, they found a negative effect stronger than the positive effect for upper quantile of volatility, while for lower quantile the negative returns do not present significant effects.

Another study that uses quantile regressions in its empirical investigations is Badshah (2013). With daily data from VIX, VSTOXX, VDAX (Note 2) and the Nasdaq Volatility Index (VXN) the results of this study suggest that the behavioral theory is the best to explain the relation between return and volatility. Regarding to asymmetry, the study indicates a raising increase in the ratio from the median to the higher quantiles. Talukdar, Daigler and Parhizgari (2016) also suggest that the behavioral theory best explains the relation between return and volatility, in this case for the CBOE's, VIX, SKEW (Note 3) and Volatility of VIX Index (VVIX) indices, than the other theories.

Agbeyegbe (2016), similarly to Agbeyegbe (2015), also uses the conditional copula and quantile regression methods to evaluate the return-volatility relation of DJA, S&P500, S&P100 and NASDAQ indices and their respective volatility indices, DJIA Volatility Index (VXD), VIX, S&P Volatility Index (VXO) and VXN. With a sample of data from 2001 to 2012, their results suggest that there is an inverted U-shaped relation between the variables.

Aboura and Chevallier (2013) found results that indicated a relation of inverse feedback and leverage for the oil market (WTI and OVX), unlike those found in the literature until then. Thus, the volatility of oil is positively related to previous movements in oil prices. The authors used daily data from 2007 to 2011 and OLS models.

There are other studies that are used to investigate the relation between volatility and oil return, or to forecast volatility, through ARCH (G) models (Note 4) (for example, Sadorsky, 2006; Wang et al., 2008; Kang et al., 2009; Marzo & Zagaglia, 2010; Nomikos & Pouliasis, 2011). However, this is not the aim of this study, which focuses on investigations based on implied volatility.

3. Models and Hypotheses

The proposal of this article in estimate the relation between return and implied volatility involves four different functional forms and three different estimation methods. The estimates are made by OLS method, but due to the particularities of financial data, such as not normality, heteroscedasticity and heavy tails, quantile regression methods are also used.

These models have the advantage of allowing the verification of different results for the different quantiles of the same sample, reducing the problems of poor specification, presence of outliers and heteroscedasticity. Moreover, they provide results that are compatible, for example, with the heterogeneity of the behavior of various investor groups in the financial markets (Talukdar et al., 2016).

As Koenker and Hallock (2001) explain, in summary, the quantile regression method, introduced by Koenker and Bassett (1978), minimizes the sum of the absolute residuals to a conditional quantile (ξ). In this case, the quantile τ of interest can be found by

$$min_{\beta \in R} \sum \rho_\tau (y_i - \xi)$$

where ρ is the absolute value function.

To obtain an estimate of the conditional median function, for example, the scalar (ξ) must be replaced in the first equation by the parametric function $\xi\,(x_i, \beta)$ and $\tau = 0.5$ must be defined. To obtain estimates of other conditional quantiles, absolute values by ρ_τ (.), for $\tau \in (0,1)$ must be replaced and then solved.

$$min_{\beta \in R} \sum_{i=1}^{n} \rho_\tau (y_i - \xi(x_i, \beta)) \tag{1}$$

The result of this minimization problem is obtained by the modification of the Barrodale and Roberts (1973) algorithm, described in detail by Koenker and d'Orey (1987, 1994).

In addition, tests were performed to verify the correct specification of parametric quantile regression models, as described in Racine (2006), which extends the work of Zheng (1998), applied in a similar way in Figueiredo et al. (2011) (Note 5). Another test was the Anova, for adjustments of quantile regression, in order to verify if there was any difference between the results of the different estimated quantiles.

It is likely that there is not a single specific functional form to explain the relation between return and implied volatility. Given the different propositions suggested in recent years and the numerous empirical studies, it is useful to make flexible estimates. In order to avoid poor specification of the parametric structure, an alternative estimation approach is proposed in this study: the nonparametric.

In the specification case of an incorrect functional form in the relation estimated by the quantile regression method, the potential advantages of using this method can be lost (Laurini, 2007). Therefore, to complement the mentioned methods analysis, it is suggested in this paper the use of the nonparametric quantile modeling constrained B-spline smoothing (COBS). This method originates from the quantile spline proposed by Koenker et al. (1994) and has the advantage of not imposing a functional form. In this method, the data structure and estimation of the curves serves as information about the behavior of the regression in each stratum. In general, the aim is to minimize the function:

$$min_{g\in\zeta} \sum_{i=1}^{N} \rho_\tau (y_i - g(x_i))^2 - \lambda \int |g''(x)| dx \quad (2)$$

where ζ corresponds to a Sobolev Space for differentiable functions up to the second order, *g* is a nonparametric function, λ is a smoothing parameter and g" is the second derivative of g regarded to x. The solution of (2) is provided by a linear optimization process and λ was chosen from the Akaike criterion optimization, as oriented by Koenker et al. (1994) (Note 6).

The four equations proposed to evaluate the relation between return and volatility originate from the theories developed by Black (1976), Christie (1982), French et al. (1987), Campbell and Hentschel (1992) and Tversky (1979), in addition to recent applied studies. The first is based on the work of Hibbert et al. (2008), with some adaptations, and can be described as follows:

$$M1 \equiv \%\Delta OVX_t = \alpha_0 + \alpha_1 R_t + \alpha_2 R_{t-1} + \alpha_3 R_{t-2} + \alpha_4 R_{t-3} + \alpha_5 \%\Delta OVX_{t-1} + \alpha_6 \%\Delta OVX_{t-2} + \alpha_7 \%\Delta OVX_{t-3} + \varepsilon_1 \quad (3)$$

where $\%\Delta OVX_t$ is the percentage change of the *OVX* at time *t*; R_t is the contemporaneous daily percentage change in the *USO* index, R_{t-1}, R_{t-2}, and R_{t-3} are one-, two- and three-day lag returns for the *USO*, respectively; $\%\Delta OVX_{t-1}$, $\%\Delta OVX_{t-2}$ and $\%\Delta OVX_{t-3}$ are the one-, two- and three-day lag percentage change in the *OVX* index.

Estimates are also proposed based on the articles by Fleming et al. (1995), with some adaptations, M2 equation, and Low (2004), equations M3 and M4, often used in the literature to set the relation between return and volatility.

In equation M2, it is investigated if the return of the USO, regardless of direction (if positive or negative), results in percentage change of the OVX, using the absolute value of the contemporary return in the model. The sum of the coefficients of absolute contemporary returns measures the asymmetry of this relation. Basically, positive returns are measured as the sum of the absolute contemporary return coefficients, while negative returns are measured as the difference of the coefficients. On the other hand, the equations M3 and M4 describe the impacts of the returns and the quadratic contemporary returns on the percentage change of the volatility. The models are as follows:

$$M2 \equiv \%\Delta OVX_t = \alpha_0 + \alpha_1 R_t + \alpha_2 R_{t-1} + \alpha_3 R_{t-2} + \delta_1 R_{t+1} + \delta_2 R_{t+2} + \delta_3 |R_t| + \varepsilon_t \quad (4)$$

$$M3 \equiv \%\Delta OVX_t = \beta_0 + \beta_1 R_t + \varepsilon_t \quad (5)$$

$$M4 \equiv \%\Delta OVX_t = \beta_0 + \beta_1 R_t + \beta_2 {R_t}^2 + \varepsilon_t \quad (6)$$

where the variables R_{t+1}, R_{t+2} are the one- and two-day lead returns for the *USO*, respectively; $|R_t|$ is the absolute value of the contemporaneous return on the USO and R_t^2 is the square of the contemporaneous return on the *USO*.

As in Hibbert et al. (2008), to determine the adjustment and importance of the contemporary return on the volatility changes, it is proposed to verify the Hypothesis I. According to this author, if this hypothesis is not rejected, the explanation behavioral model for the relation between return and volatility may be more appropriate than the leverage and feedback hypothesis.

Hypothesis I: *Contemporaneous return is the most important factor in determines changes in implied volatility*.

It is suggested a modification in M1, the inclusion of the squares of the contemporaneous returns, which together with the analysis of M4, allows the analysis of the existence of a nonlinear component of the return-volatility relation. With this, it is possible to verify Hypothesis II.

Hypothesis II: *Contemporaneous relation between the return and the volatility changes has a significant nonlinear component.*

If this hypothesis is verified, it indicates that the contemporary behavior of the returns modifies, especially if compared the measures in central positions with extremes. Evidence in this sense has already been found by Low (2004), in its simplified form, for market indices.

A question that arises in the case of hypothesis II is: which nonlinear form best describes this relation? Low (2004) suggests an inclined "S", analyzing the different behaviors between samples for positive and negative returns, for the VIX. The author defines the "fear" of the market as an accelerated increase in VIX (convexity) and "exuberance" as an accelerated decrease in VIX (concavity). On the other hand, Agbeyegbe (2016) identifies as an inverted "U" the format of this relationship, through quantiles, for USO and OVX. Considering this, a verification of Hypothesis III is proposed:

Hypothesis III: *The relation between volatility and contemporaneous return has an inverted "U-shaped"*.

Low's article (2004) presents an evaluation for the 5% extreme of negative and positive VIX returns. Hibbert et al. (2008) expands this analysis to other quantiles of this index. In general, an asymmetry seems to be more pronounced on samples of negative returns. Thus, it is proposed to verify the following hypothesis:

Hypothesis IV: *There is asymmetry in the relation between the volatility and the return of the oil market*.

4. Empirical Results

This paper is based on the USO data and its associated implied volatility measure, OVX. The investigated period is comprised from 05/05/2007 to 12/30/2016, totalizing 2430 observations per variable. The data were obtained from Yahoo Finance database.

4.1 Statistics and Data Analysis

The descriptive statistics of the USO returns and the percentage changes of the *OVX* are reported in Table 1. The changes of the OVX has heavy tails, as can be observed in the kurtosis result, the same does not occur for the return of the USO. The measure of asymmetry presents positive and relatively low values, indicating moderate asymmetry only for the percentage changes of the OVX. Another characteristic to be highlighted is the value of the tests for the normality of the series, the Jarque-Bera (JB). The results of this statistic indicate that the two variables are not normally distributed. These results are common to financial variables, such as that documented in the literature of stylized facts of financial series (for more details, see Silva Filho & Ziegelmann, 2014). The Dickey Fuller (ADF) test rejected the presence of unit root for a lag of 8 periods. Information about this test can be obtained in MacKinnon (1996).

Table 1. Descriptive statistics

	Mean	Median	Min.	Max.	Standard deviation	Asymmetry	Kurtosis	JB	ADF
%*OVX*	0,00126	-0,0031	-0,3559	0,5295	0,0501	1,5176	12,47	16664,3#	-19,30*
R	-0,00032	0,0000	-0,1068	0,0960	0,0225	0,0070	2,177	479,7#	-14,72*

Note. * Significant ADF test result at 1%. # Significant JB test result at 1%.

4.2 OLS Results

The relation between the USO return and the OVX percentage change was examined through the equations M1, M2, M3 and M4, and some adaptations. It started with the contemporary analysis of these indices, based on the equations M3 and M4, similar to those used by Low (2004), and estimates for the whole period and for each year of the sample.

The results in Table 2 show that, for all years, the contemporaneous relation between the return and the volatility change is negative, except for the year 2008 estimates, which was not significant. These results corroborate with the proposition of Black (1976) and Cristie (1982), as well as with most field studies, which describe increases in volatility with decreasing returns. In addition, the nonlinear relationship between the variables is evidenced by the quadratic term of the contemporary return of the estimates for all years, except for the year 2014, which was not significant. The specifications that included the quadratic term of the returns presented a greater explanatory power of the models for the whole period and its annual subsamples, which suggests being a more adequate form to model this relation.

Table 2. Results for the M3 and M4

Periods	N	R^2 adjusted	intercept	R_t	R_t^2
2007-2016	2430	0.1351	0.0010	-0.8174	-
			[1.4524]	[-8.4127]***	-
		0.1881	-0.0047	-0.8119	11.1429
			[-5.2278]***	[-9.2210]***	[5.3122]***
2007	163	0.0514	0.0044	-0.5916	-
			[1.1784]	[-2.9567]**	-
		0.0957	-0.0015	-0.7550	21.8724
			[-0.3399]	[-3.6385]***	[2.3300]*
2008	253	0.0220	0.0042	-0.2621	-
			[1.7376]**.**	[-1.3246]	-
		0.0875	-0.0046	-0.2450	8.3591
			[-1.4855]	[-1.3196]	[2.6825]*

2009	252	0.0986	-0.0014	-0.5768	-
			[-0.7127]	[-5.0692]***	-
		0.1397	-0.0079	-0.5621	7.1494
			[-3.4187]***	[-5.2932]***	[3.7303]***
2010	252	0.3966	0.0001	-1.358	-
			[0.0350]	[-12.884]***	-
		0.4553	-0.0067	-1.3424	21.5551
			[-3.0645]***	[-11.0046]***	[5.0231]***
2011	252	0.2145	0.00378	-1.6695	-
			[1.0975]	[-3.9681]***	-
		0.3830	-0.0122	-1.2163	37.5485
			[-3.9291]***	[-3.5130]***	[4.7759]***
2012	250	0.2241	-0.0010	-1.0651	-
			[-0.5528]	[-4.3353]***	-
		0.3364	-0.0068	-1.1763	22.5476
			[-3.5300]***	[-6.5334]***	[5.4530]***
2013	252	0.2080	0.0000	-2.0819	-
			[-0.0098]	[-3.7558]***	-
		0.3510	-0.0139	-1.9700	108.4391
			[-3.5826]***	[-4.8366]***	[3.6329]***
2014	252	0.1116	0.0028	-1.1302	-
			[1.0575]	[-2.9814]**	-
		0.1194	0.0012	-0.9667	9.7904
			[0.4001]	[-2.9367]**	[0.8290]
2015	252	0.1806	-0.0008	-0.7210	-
			[-0.2782]	[-4.0071]***	-
		0.3544	-0.0132	-0.7930	17.3528
			[-5.7404]***	[-6.9775]***	[9.5676]***
2016	250	0.2787	0.0001	-0.9518	-
			[0.0498]	[-6.7065]***	-
		0.2984	-0.0044	-1.0060	6.4086
			[-1.5484]	[-7.6234]***	[1.9863]*

Note. The Newey - West matrix was used to calculate the t - values presented in parentheses ( ).

(***), (**), (*) represent significant values at 0.1%, 1% and 5% respectively.

In Table 3, the results of the equation M1 and its adapted version are presented, which includes the quadratic term (for the returns). Negative and significant effects of the contemporaneous and lagged-1-day returns are observed, as well as the change of the lagged volatility of one and three days. Compared to the other variables, the contemporary return of USO is the one that had the greatest contribution in determining the daily percentage changes of OVX. These results suggest that the relation between return and volatility of the oil market is more intense in the contemporary relation than in outdated relations, supporting hypothesis 1 of this study. The theory of leverage seems to have a weaker effect, because only the lagged-1-day return is significant and relatively small. In addition, the inclusion of the quadratic term in the regression proved useful, increasing the explanatory power of the model, from 18.3% to 23%.

Table 3. Regression results for the M1

R^2 adjusted	intercept	R_t	R_t^2	R_{t-1}	R_{t-2}	R_{t-3}	$\%\Delta OVX_{t-1}$	$\%\Delta OVX_{t-2}$	$\%\Delta OVX_{t-3}$
0,183	-0.0132	-35.958	-	-10.856	1.1708	1.6448	-0.2224	0.0023	-0.0615
	(-0.37)	(-9.3)***	-	(-2.61)**	(0.38)	(0.86)	(-3.28)**	(0.06)	(-2.66)**
0,230	-0.2519	-35.685	470.20	-9.9851	0.4089	2.5004	-0.2109	-0.0176	-0.0720
	(-6.1)***	(-10.3)***	(5.7)***	(-2.73)**	(0.13)	(1.12)	(-3.34)***	(-0.42)	(-3.01)**

Note. The Newey - West matrix was used to calculate the t - values presented in parentheses ( ).

The number of observations used was 2426.

(***), (**), (*) and (.) represent significant values at 0.1%, 1%, 5% and 10% respectively.

The results for model M2 are shown in Table 4. They indicated significant impacts only for contemporaneous (negative) and module (positive) returns. There appears to be a positive relation between the size of an oil market movement and the contemporary percentage change in the implied volatility of this market. If the oil market return is positive, the coefficient that impacts the percentage change in volatility will be -3.9 (-36.23 + 32.33). Thus, an increase in the stock market is expected to accompany a decline in the volatility index. On the other hand, if the return is negative, the coefficient that impacts will be -68.46 (- 36.23 - 32.33). Hence, a decline in the stock market is expected to accompany an increase in volatility. Negative oil market movements are associated with changes in the volatility index that are much larger than those associated with positive movements of similar size. These results corroborate with hypothesis IV of this study, which claims that there is asymmetry in the relation between implied volatility and return.

Table 4. Regression results for the M2

R^2 adjusted	intercept	R_t	R_{t-1}	R_{t-2}	R_{t+1}	R_{t+2}	$\|R_t\|$
0.1852	-0.5405	-36.23	-2.2854	1.6112	2.4860	1.2467	32.33
	(-9.21)***	(-9.97)***	(-0.90)	(0.68)	(0.91)	(0.47)	(8.33)***

Note. The Newey - West matrix was used to calculate the t - values presented in parentheses ( ).

The number of observations used was 2426.

(***), (**), (*) and (.) represent significant values at 0.1%, 1%, 5% and 10% respectively.

In general, the results of the 4 proposed equations estimated by OLS corroborate with the proposition of Black (1976) and Cristie (1982), and with most studies of the area, which describe increases in volatility with decreasing returns. Besides that, some aspects suggest certain validity of behavioral theory. Furthermore, it was found that the model based on Hibbert et al. (2008), M1, with the addition of the quadratic term of the contemporary returns, is the one that provides the best results for the 2007-2016 sample, with the highest R^2-adjusted values. It suggests that the inclusion of this nonlinear component should be investigated.

However, these results need to be confirmed by other methods, which are more compatible with the characteristics of the financial data.

4.3 Empirical Results Quantile Regression

The equations M3 and M4 were re-estimated by the quantile regression method and also for separate samples of positive and negative returns. It was done to verify if the relation between the return and volatility changes among the quantiles of distribution of conditional volatility. It was also done to verify if it is assimetric. It was chosen not to present the re-estimated results of the M1 and M2 models, due to space saving and the fact that there is an especial interest in exploring the nature of the contemporary relation between volatility and return. In addition, the resultsdid not show any similarity to those disclosed in the previous subsection.

The results of the quantiles estimates (Note 7) for M3 and M4 are shown in Table 5, and the specifications (Note 8) and the Anova tests for quantile regression adjustments are reported in Tables 7 and 8 (Appendix A). It is possible to verify that contemporary returns present negative signs and are significant for all quantiles investigated (0.05, 0.10, 0.25, 0.5, 0.75, 0.90 and 0.95) in M3, according to the explanations of the previous subsection. The Figure 2 (appendix A) illustrates this estimation. However, the null hypothesis of correct specification is not rejected only for low quantiles (0.05 and 0.10), which suggests that the M1 model may not be correctly specified for the median and higher quantiles. In addition, the values of the Anova test indicate that there are no differences between the values of the different estimated quantiles. In general, these results do not give us enough information to do assertions about the asymmetry and nonlinear form of dependence.

The M4 results are positive and significant values for the square of the returns from the quantile 0.25 until the quantile 0.95, suggesting nonlinearity of this relation, except for lower quantiles (0.05 to 0.1). The tests for model specification presented results that indicated correct specification for the lower and extreme quantiles of the distribution (0.05, 0.1, 0.25 and 0.95). Unlike M3, the Anova tests confirm the different values between the different estimated quantiles. The results can be seen in figure 6 (appendix A). In general, these results corroborate with hypothesis II of a significant nonlinear component, but do not confirm the hypothesis III, which mentions the inverted U-shaped relation of the variables.

Table 5. Results of the quantile regression

Quantile		0.05	0.10	0.25	0.5	0.75	0.90	0.95
M3	intercept	-0.05922	-0.04426	-0.02440	-0.00283	-0.02174	-0.04742	-0.06915
		(-27.6)***	(-34.1)***	(-28.7)***	(-3.5)***	(21.6)***	(26.0)***	(19.6)***
	R_t	-0.81473	-0.76774	-0.74561	-0.76241	-0.76627	-0.79902	-0.86354
		(-10.3)***	(-20.4)***	(-20.3)***	(-21.6)***	(-17.5)***	(-9.9)***	(-5.7)***
M4	intercept	-0.05918	-0.04440	-0.02603	-0.00635	0.01270	0.03241	0.04602
		(-24.9)***	(-30.5)***	(-26.8)***	(-7.1)***	(10.6)***	(17.9)***	(20.3)***
	R_t	-0.83722	-0.73925	-0.75744	-0.79835	-0.90715	-0.94276	-100.769
		(-5.7)***	(-8.0)***	(-14.8)***	(-15.9)***	(-13.6)***	(-8.1)***	(-6.6)***
	R_t^2	-0.60225	0.65866	542.820	104.150	193.007	273.028	40,1239
		(-0.1)	(0.2)	(3.5)***	(6.3)***	(8.9)***	(6.3)***	(6.6)**

Note. The t-values are presented in parentheses ( ).

(***), (**), (*) and (.) represent significant values at 0.1%, 1%, 5% and 10% respectively.

The results of the separate data from negative and positive returns seem to have heteroscedastic and asymmetric behavior, as can be seen in Table 6 and in Figures 3 and 4 (appendix A). For the positive returns, the M3 results showed an inverse relation between return and volatility only for low quantiles (0.05, 0.10, 0.25), changing signal in higher quantiles (0.75, 0.90 and 0.95), which diverges from the presented theories. These results can be seen in figure 9 (appendix A). In M4 estimates, the quadratic terms are only statistically significant for the median and low quantiles (0.05.0.25 and 0.5). In the quantile 0.05 the relation is negative between volatility and return, while in quantiles 0.25 and 0.5 the relation is positive, as can be observed in figure 10 (appendix A).

Regarding the sample of negative returns, all estimated quantiles presented a negative relation between contemporary returns and volatility changes, which increases as higher quantiles are analyzed. On the other hand, the nonlinear relation, represented by the quadratic term in M4, does not present significant results for this sample (except quantil 0.9).

Table 6. Results of the quantile regression

Quantile		0.05	0.10	0.25	0.5	0.75	0.90	0.95
					Positive returns			
M3	intercept	-0.0625	-0.0495	-0.0308	-0.0140	0.0040	0.0244	0.0344
		(-16.4)***	(-16.0)***	(-17.5)***	(-8.24)***	(1.96)*	(6.86)***	(9.01)***
	Rt	-0.6359	-0.4514	-0.3484	-0.0023	0.2694	0.4084	0.8711
		(-2.79)***	(-2.39)**	(-2.97)**	(-0.02)	(2.01)*	(1.82).	(2.51)**
M4	intercept	-0.0674	-0.0482	-0.0275	-0.0103	0.0054	0.0282	0.0427
		(-15.2)***	(-9.05)***	(-12.0)***	(-5.33)***	(2.04)*	(5.61)***	(8.57)***
	R_t	0.2127	-0.7720	-0.8141	-0.4777	0.0502	-0.2539	-0.2863
		(0.53)	(-1.14)	(-3.75)***	(-2.31)**	(0.19)	(-0.41)	(-0.32)
	R_t^2	-19,7629	5,4082	9,0391	6,9188	4,2231	13,0683	20,9554
		(-3.04)	(0.32)	(2.56)**	(1.94)*	(0.94)	(0.94)	(0.83)
					Negative returns			
M3	intercept	-0.0550	-0.0427	-0.0268	-0.0094	0.0043	0.0198	0.0299
		(-12.7)***	(-17.7)***	(-15.1)***	(-5.38)***	(2.14)*	(6.54)***	(5.99)***
	R_t	-0.7063	-0.7311	-1,0042	-1,3381	-2,0240	-2,6314	-3,2272
		(-2.89)***	(-7.17)	(-10.1)***	(-12.9)***	(-15.8)***	(-10.3)***	(-7.18)***
M4	intercept	-0.0527	-0.0444	-0.0275	-0.0120	0.0048	0.0247	0.0355
		(-11.0)***	(-13.66524)***	(-10.2)***	(-5.61)***	(2.00)*	(8.23)***	(5.74)***
	R_t	-0.2982	-0.9221	-1,1093	-1,6606	-1,9680	-1,7903	-2,3841
		(-0.62)	(-3.48)***	(-3.62)***	(-6.59)***	(-7.57)***	(-4.16)***	(-2.71)***
	R_t^2	5,7275	-2,9780	-1,5042	-4,8275	0.48333	17,7086	13,7933
		(0.81)	(-0.86)	(-0.23)	(-0.97)	(0.10)	(2.11)*	(0.67)

Note. The t-values are presented in parentheses ( ).

(***), (**), (*) and (.) represent significant values at 0.1%, 1%, 5% and 10% respectively.

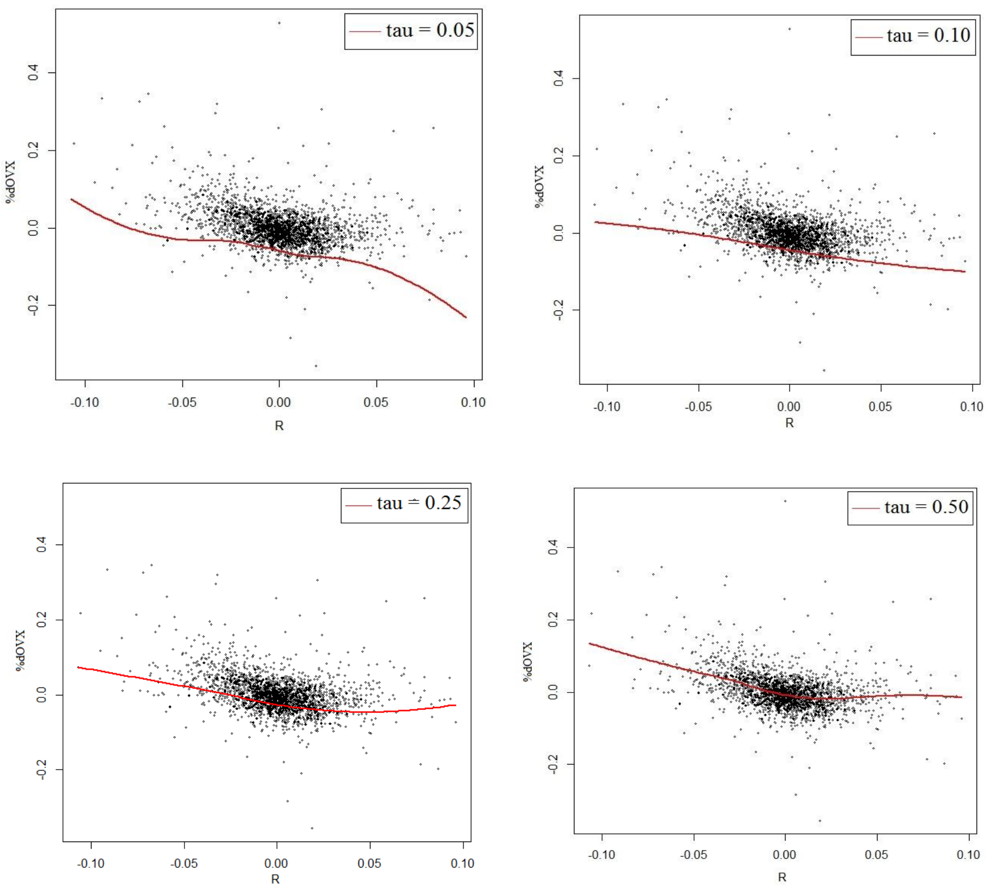
The specification test for the positive and negative sample models had the correct non-rejected specification, except for the 0.5 quantile in M3 of the positive sample. The results of the Anova tests confirmed the difference between the quantiles results, for both positive and negative returns, as can be seen in Tables 7 and 8 (appendix A). In general, these results confirm the hypothesis IV, of asymmetry between the relation of volatility and contemporary returns for the oil market. It is possible to notice the differences between the results estimated by quantile regression and by OLS in Figures 5 to 10 (Appendix A). These results suggest that the OLS estimations may be overestimating or underestimating the values of the measures.

4.4 Constrained B-Splines Smoothing Results

The use of this nonparametric method is complementary to the previous ones. The intention is to verify the nonlinearity of the contemporary relation between returns and the implied volatility without imposing any functional form for the data. In addition, the non-complete adjustment of the parametric structure for the samples, verified by the results of the specification tests, justifies a more flexible analysis on the behavior of these variables. In this estimation stage, the “cobs” package of the R software was used. For more details about the operationalization and the choice of the smoothing parameter used, see Koenker et al. (1994).

The results are presented for the same quantiles evaluated in the previous parametric investigation, shown in figure 1. They suggest that the relation between return and volatility does not appear to be linear for most of the quantiles investigated, except for the 0.10 quantile, which visually appears to be linear behavior. The quantiles 0.05, 0.90 and 0.95 are in an “S-shaped”, or two “U-shaped”, the first with the concavity facing up (for negative returns) and the second with the concavity facing down (for positive returns), similar to the results of “fear” and “exuberance” described by Low (2004). In the other quantiles 0.25, 0.5 and 0.75 the relation is almost linear, being more pronounced of the negative returns to zero.

These results are important and provide new evidence for the investigation of oil market behavior. When the results separated by positive and negative returns were analyzed, the relation between the variables changes depending on the sign of the returns and the quantile evaluated. This indicates that hypothesis III of this study is refuted, and suggest that defining a single form for all returns samples and not considering different quantis of volatility can be wrong.



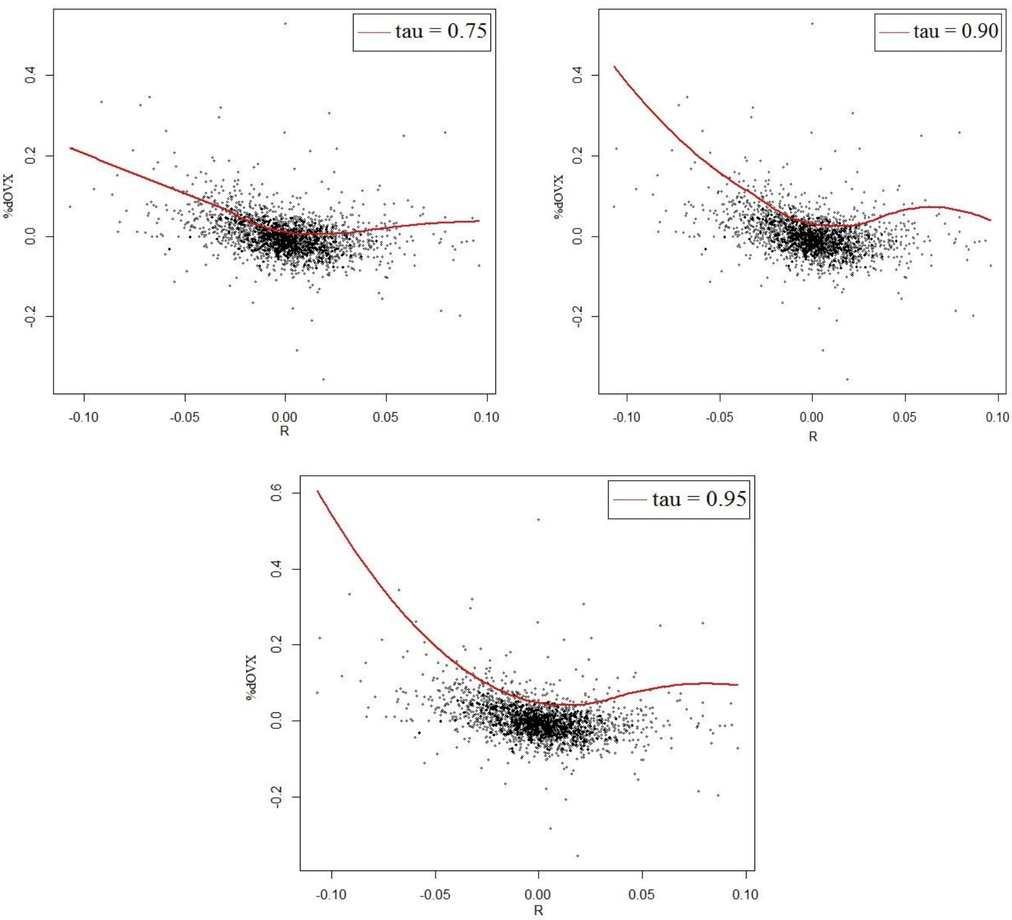


Figure 1. Results of nonparametric estimates for % ΔOVX and USO return

Note. The red line represents the B-splines estimates for each quantile (0.05, 0.10, 0.25, 0.75, 0.9 and 0.95), listed in each image. The estimates were performed in software R, through the package 'cobs'.

Another aspect of interest is that, for higher quantiles, the relation between the variables is more pronounced, with a higher slope for negative returns than for positive ones. In general, it means that for higher quantiles of volatility, the sensitivity to return changes increases. Moreover, the negative returns have different impacts on the percentage changes of volatility. The association between falling prices and increased risk is more sensitive (higher slope) than the association between price increases and risk reduction, similar to that found in Low (2004) for the market index.

Finally, these results have implications for the construction of risk measures and their empirical quantification. Therefore, it is important to consider that the risk is not in accordance with the simple symmetry paradigm. Furtheromore, the return-volatility relation has a contemporary nonlinear component.

5. Conclusions

The volatility-return relation is a widely documented topic in finance literature. Understanding this relationship in the oil market is fundamental for investors and energy policymakers. Knowing characteristics about the form of this relation can avoid losses for investors and inefficiency of energy policies. Theoretical explanations about this relation are mainly based on leverage and feedback hypotheses and on behavioral theory. However, studies involving the relation of the Crude Oil Volatility Index (OVX) and the United States Oil Fund (USO) are in small number and do not explore some aspects regarding to the asymmetry and nonlinearity of this relation.

In this context, the article was made to investigate the asymmetry and nonlinearity of this relation. The empirical research strategy involved parametric and nonparametric methods, in a complementary way, so that possible problems of poor specification of the functional form of this relation could be overcome.

It was found evidence that specifying the contemporaneous relation of return-volatility with linear equations may not be the most appropriate form for all volatility quantiles, especially for the highest ones. The results for samples of negative and positive returns suggested differences in the behavior of volatility, being more pronounced for negatives. The nonlinearity and the asymmetry of this relation were confirmed by the results of the nonparametric method B-splines. The format for describing the volatility-return relation seems to be sloping "S-shaped", with a "U-shaped" on negative returns and an inverted "U-shaped" for positive returns.

In this article, the nonparametric method was used in a complementary way to the results obtained by parametric methods. The next step is the application of tests to verify the rejection of the linear parametric adjustment in favor of the nonparametric adjustment for the different quantiles of the oil volatility-return relation.

References

Aboura, S., & Chevallier, J. (2013). Leverage vs. feedback: Which Effect drives the oil market? *Finance Research Letters, 10*(3), 131-141. http://dx.doi.org/10.1016/j.frl.2013.05.003

Agbeyegbe, T. D. (2015). An inverted U-shaped crude oil price return-implied volatility relationship. *Review of Financial Economics, 27*, 28-45. http://dx.doi.org/10.1016/j.rfe.2015.08.001

Agbeyegbe, T. D. (2016). Modeling US stock market volatility-return dependence using conditional copula and quantile regression. In *The Economics of the Global Environment* (pp. 597-621). Springer International Publishing. http://dx.doi.org/10.1007/978-3-319-31943-8_26

Badshah, I. U. (2013). Quantile regression analysis of the asymmetric return-volatility relation. *Journal of Futures Markets, 33*(3), 235-265. http://dx.doi.org/10.1002/fut.21551

Barrodale, I., & Roberts, F. D. (1973). An improved algorithm for discrete l_1 linear approximation. *SIAM Journal on Numerical Analysis, 10*(5), 839-848. http://dx.doi.org/10.1137/0710069

Black, F. (1976). Studies of stock price volatility changes. In Proceedings of *the 1976 Meeting of the Business and Economic Statistics Section*, American Statistical Association, Washington, D.C.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy, 81*(3), 637-654. http://dx.doi.org/10.1086/260062

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics, 31*(3), 307-327. http://dx.doi.org/10.1016/0304-4076(86)90063-1

Campbell, J. Y., & Hentschel, L. (1992). No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of financial Economics, 31*(3), 281-318. http://dx.doi.org/10.1016/0304-405X(92)90037-X

Christie, A. A. (1982). The stochastic behavior of common stock variances: Value, leverage and interest rate effects. *Journal of Financial Economics, 10*(4), 407-432. http://dx.doi.org/10.1016/0304-405X(82)90018-6

Daigler, R. T., Hibbert, A. M., & Pavlova, I. (2014). Examining the return–volatility relation for foreign exchange: evidence from the euro VIX. *Journal of Futures Markets, 34*(1), 74-92. http://dx.doi.org/10.1002/fut.2158

Figueiredo, E., Silva Júnior, J. C. A., & Jacinto, P. A. (2011). A hipótese de Kuznets para os municípios brasileiros: Testes para as formas funcionais e estimações não-paramétricas. *Revista Economia, 12*, 149-165.

Fleming, J., Ostdiek, B., & Whaley, R. E. (1995). Predicting stock market volatility: A new measure. *Journal of Futures Markets, 15*(3), 265-302. http://dx.doi.org/10.1002/fut.3990150303

French, K. R., Schwert, G. W., & Stambaugh, R. F. (1987). Expected stock returns and volatility. *Journal of Financial Economics, 19*(1), 3-29. http://dx.doi.org/10.1016/0304-405X(87)90026-2

He, X., & Ng, P. (1999). COBS: Qualitatively constrained smoothing via linear programming. *Computational Statistics, 14*(3), 315-338. http://dx.doi.org/10.1007/s001800050019

Hibbert, A. M., Daigler, R. T., & Dupoyet, B. (2008). A behavioral explanation for the negative asymmetric return–volatility relation. *Journal of Banking & Finance, 32*(10), 2254-2266. https://doi.org/10.1016/j.jbankfin.2007.12.046

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society*, 263-291. http://dx.doi.org/10.2307/1914185

Kang, S. H., Kang, S. M., & Yoon, S. M. (2009). Forecasting volatility of crude oil markets. *Energy Economics, 31*(1), 119-125. http://dx.doi.org/10.1016/j.eneco.2008.09.006

Koenker, R. (2012). *Quantreg: Quantile Regression. R package version 5.05*. R Foundation for Statistical Computing: Vienna. Retrieved from http://CRAN. R-project. org/package= quantreg

Koenker, R. W., & d'Orey, V. (1987). Algorithm AS 229: Computing regression quantiles. *Journal of the Royal Statistical Society. Series C (Applied Statistics), 36*(3), 383-393. http://dx.doi.org/10.2307/2347802

Koenker, R., & Bassett Jr, G. (1978). Regression quantiles. *Econometrica: Journal of the Econometric Society,* 33-50. http://dx.doi.org/10.2307/1913643

Koenker, R., & Bassett Jr, G. (1982). Robust tests for heteroscedasticity based on regression quantiles. *Econometrica: Journal of the Econometric Society,* 43-61. http://dx.doi.org/10.2307/1912528

Koenker, R., & d'Orey, V. (1994). Remark AS R92: A remark on algorithm AS 229: Computing dual regression quantiles and regression rank scores. *Journal of the Royal Statistical Society. Series C (Applied Statistics), 43*(2), 410-414. http://dx.doi.org/10.2307/2986030

Koenker, R., Ng, P., & Portnoy, S. (1994). Quantile smoothing splines. *Biometrika, 81*(4), 673-680. http://dx.doi.org/10.1093/biomet/81.4.673

Laurini, M. (2007). A note on the use of quantile regression in beta convergence analysis. *Economics Bulletin, 52*(3), 1-8.

MacKinnon, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of Applied Econometrics,* 601-618. http://dx.doi.org/10.1002/(SICI)1099-1255(199611)11:6%3C601::AID-JAE417%3E3.0.CO;2-T

Marzo, M., & Zagaglia, P. (2010). Volatility forecasting for crude oil futures. *Applied Economics Letters, 17*(16), 1587-1599. http://dx.doi.org/10.1080/13504850903084996

Nomikos, N. K., & Pouliasis, P. K. (2011). Forecasting petroleum futures markets volatility: The role of regimes and market conditions. *Energy Economics, 33*(2), 321-337. http://dx.doi.org/10.1016/j.eneco.2010.11.013

Padungsaksawasdi, C., & Daigler, R. T. (2014). The Return-Implied Volatility Relation for Commodity ETFs. *Journal of Futures Markets, 34*(3), 261-281. http://dx.doi.org/10.1002/fut.21592

Racine, J. S (2014). *Np: Nonparametric kernel smoothing methods for mixed data types R Package* (Version 0.60-2). Retrieved from http://r-project.org/package=np

Racine, J. S. (2006). *Consistent specification testing of heteroskedastic parametric regression quantile models with mixed data.* Unpublished manuscript, McMaster University. Hamilton, Ontario, Canada, L8S 4L8 Retrieved from http://www.mcmaster.ca/economics/racine

Reboredo, J. C. (2012). Do food and oil prices co-move? *Energy Policy, 49*, 456-467. http://dx.doi.org/10.1016/j.enpol.2012.06.035

Sadorsky, P. (2006). Modeling and forecasting petroleum futures volatility. *Energy Economics*, *28*(4), 467-488. http://dx.doi.org/10.1016/j.eneco.2006.04.005

Silva Filho, O. C., & Ziegelmann, F, A. (2014). Assessing some stylized facts about financial market indexes: A Markov copula approach. *Journal of Economic Studies*, *41*(2), 253-271. http://dx.doi.org/10.1108/JES-06-2012-0080

Talukdar, B., Daigler, R. T., & Parhizgari, A. M. (2017). Expanding the Explanations for the Return–Volatility Relation. *Journal of Futures Markets, 37*(7), 689-716. http://dx.doi.org/10.1002/fut.21827

Wang, T., Wu, J., & Yang, J. (2008). Realized volatility and correlation in energy futures markets. *Journal of Futures Markets, 28*(10), 993-1011. http://dx.doi.org/10.1002/fut.20347

Whaley, R. E. (1993). Derivatives on market volatility: Hedging tools long overdue. *The Journal of Derivatives, 1*(1), 71-84. http://dx.doi.org/10.3905/jod.1993.407868

Zheng, J. X. (1998). A consistent nonparametric test of parametric regression models under conditional quantile restrictions. *Econometric Theory, 14*(1), 123-138. http://dx.doi.org/10.1017/S0266466698141051

Notes

Note 1. At the time, the index was still calculated based on the Black and Scholes (1973) formula. For more information on building this version of VIX, see Whaley (1993).

Note 2. The VDAX is based on options in the DAX 30 stock index while the VSTOXX is based on options in the

SX5E stock index comprising the 50 largest shares in the euro zone.

Note 3. Produced by the CBOE, SKEW is an index that captures the perceived risk of a market crash or a sudden drop in stock prices.

Note 4. For more information about (G)ARCH models, see Bollerslev (1986).

Note 5. Tests performed on software R, through Racine's Package 'np' (2014).

Note 6. About the solution to (2), see He and Ng (1999).

Note 7. Estimation and tests performed in software R, with the help of Koenker's 'quantreg' Package (2012).

Note 8. The measure of this specification test is a variant of the Wald test, described in Koenker and Bassett (1982).

Appendix A

Results of quantile regressions

Table 7. Anova test results for quantile regressions

		Quantile	F value	Significance
		Full sample		
M 3	all	0.05-0.95	0.2684	-
	to the left	0.05-0.5	0.3757	-
	to the right	0.5-0.95	0.1629	-
	extremes	0.05 and 0.95	0.0845	-
M 4	all	0.05-0.95	5.9806	***
	to the left	0.05-0.5	3.7588	***
	to the right	0.5-0.95	7.0121	***
	extremes	0.05 and 0.95	13.9960	***
		Positive Returns		
M 3	all	0.05-0.95	4.273	***
	to the left	0.05-0.5	5.078	***
	to the right	0.5-0.95	3.241	*
	extremes	0.05 and 0.95	13.888	***
M 4	all	0.05-0.95	6.8342	***
	to the left	0.05-0.5	9.4539	***
	to the right	0.5-0.95	3.0213	**
	extremes	0.05 and 0.95	4.1005	*
		Negative returns		
M 3	all	0.05-0.95	14.340	***
	to the left	0.05-0.5	8.948	***
	to the right	0.5-0.95	15.836	***
	extremes	0.05 and 0.95	25.402	***
M 4	all	0.05-0.95	8.8690	***
	to the left	0.05-0.5	4.6908	***
	to the right	0.5-0.95	8.4639	***
	extremes	0.05 and 0.95	16.0960	***

Note. "all" is about quantiles, 0.05, 0.10, 0.25, 0.5, 0.75, 0.90 and 0.95; "to the left" are the quantiles 0.05, 0.10, 0.25 and 0.5; "to the right" are the quantiles 0.5, 0.75, 0.90 and 0.95; and the "extremes" compare only the 0.05 and 0.95 quantiles.

***, ** and * denote the rejection of the null hypothesis at the significance level of 1%, 5% and 10% respectively.

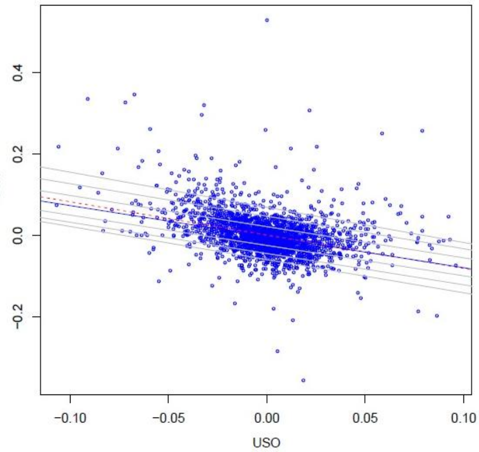


Figure 2. Results of the estimation of the M3, linear model, for %ΔOVX and Return of USO

Note. The dotted red line represents the estimation by OLS. The solid blue line represents the median (quantil 0.5) and the other lines represent, from lower to highier, the quantiles 0.05, 0.10, 0.25, 0.75, 0.9 and 0.95.

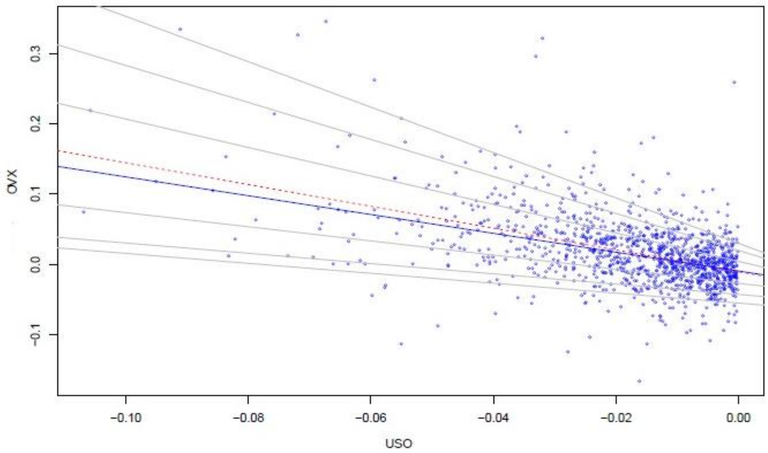


Figure 3. Results of the estimation of the M3, linear model, for %ΔOVX and negative returns of USO

Note. The dotted red line represents the estimation by OLS. The solid blue line represents the median (quantil 0.5) and the other lines represent, from lower to highier, the quantiles 0.05, 0.10, 0.25, 0.75, 0.9 and 0.95.

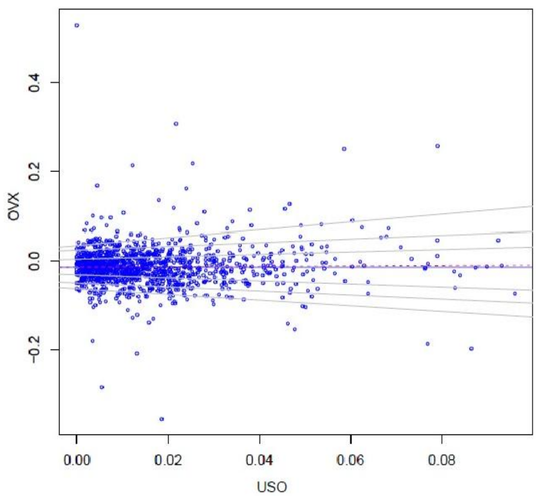


Figure 4. Results of the estimation of the M3, linear model, for %ΔOVX and positive returns of USO

Note. The dotted red line represents the estimation by OLS. The solid blue line represents the median (quantil 0.5) and the other lines represent, from lower to highier, the quantiles 0.05, 0.10, 0.25, 0.75, 0.9 and 0.95.

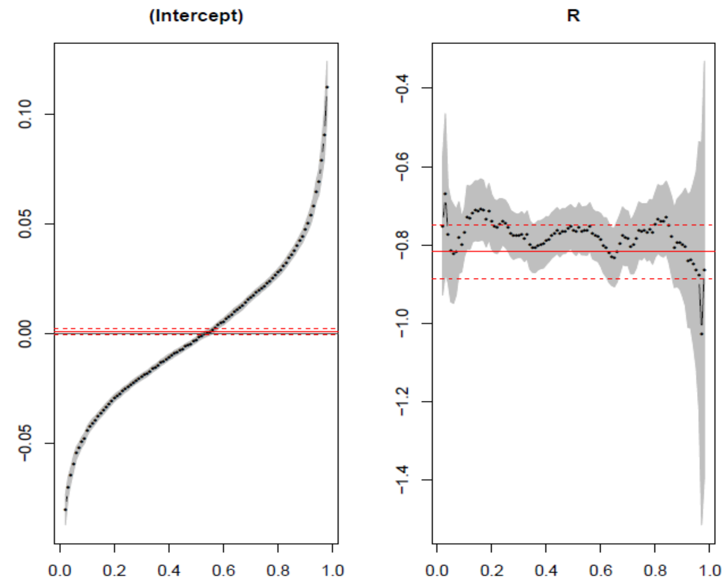


Figure 5. Results of the estimated coefficients in M3, for the entire sample

Note. The solid red line represents the values of the parameters estimated by OLS, the dotted lines are the ranges of these values; R is the USO return.

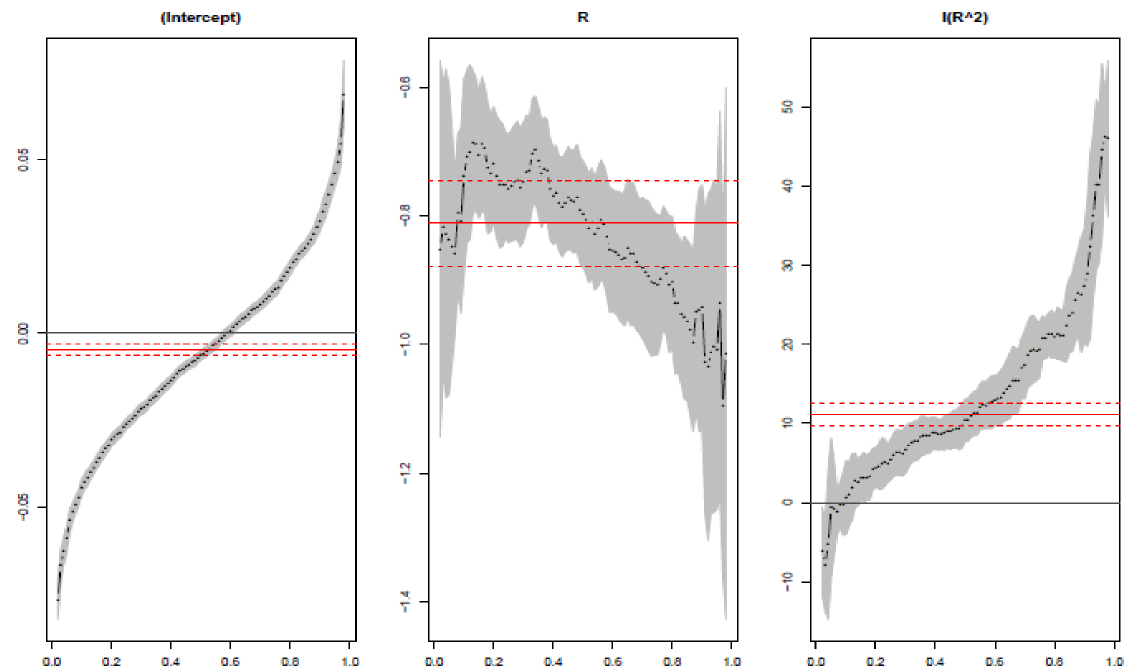


Figure 6. Results of the estimated coefficients in M4, for the entire sample

Note. The solid red line represents the values of the parameters estimated by OLS, the dotted lines are the ranges of these values; R is the USO return and R^2 is the square return of the USO.

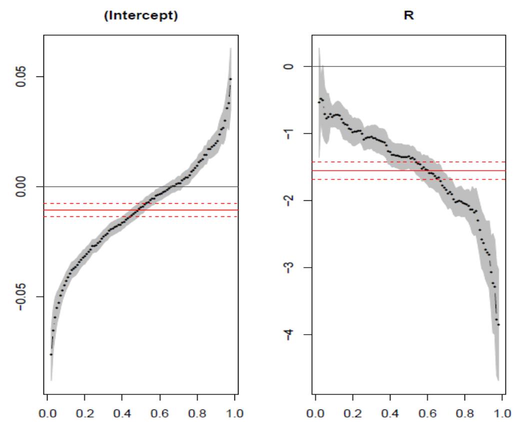


Figure 7. Results of the estimated coefficients in M3, for negative returns

Note. The solid red line represents the values of the parameters estimated by OLS, the dotted lines are the ranges of these values; R is the USO return.

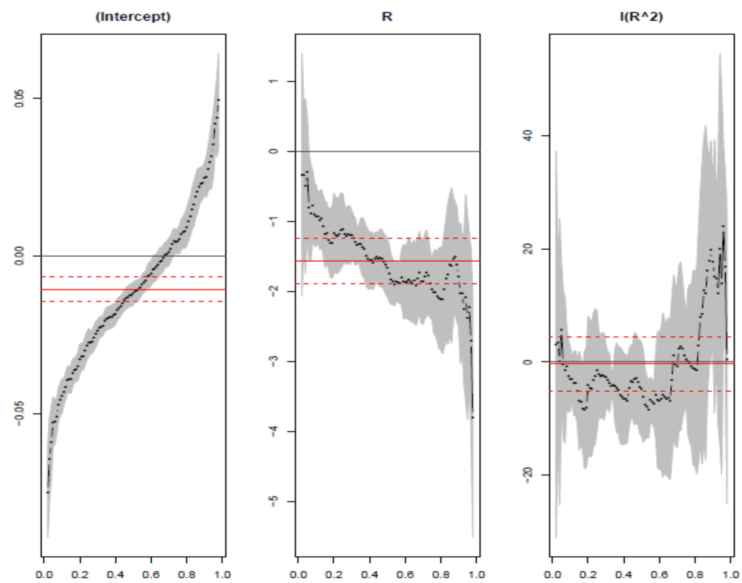


Figure 8. Results of the estimated coefficients in M4, for negative returns

Note. The solid red line represents the values of the parameters estimated by OLS, the dotted lines are the ranges of these values; R is the USO return and R^2 is the square return of the USO.

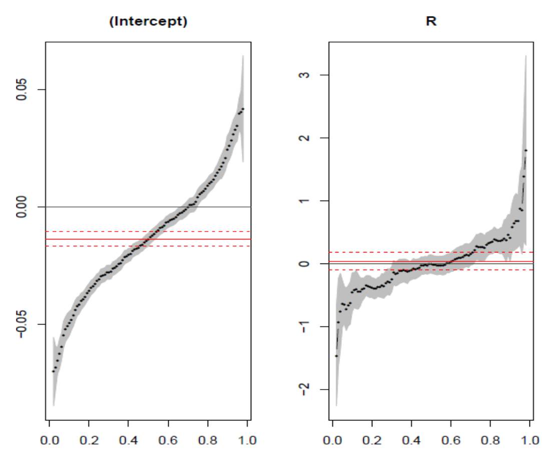


Figure 9. Results of the estimated coefficients in M3, for positive returns

Note. The solid red line represents the values of the parameters estimated by OLS, the dotted lines are the ranges of these values; R is the USO return.

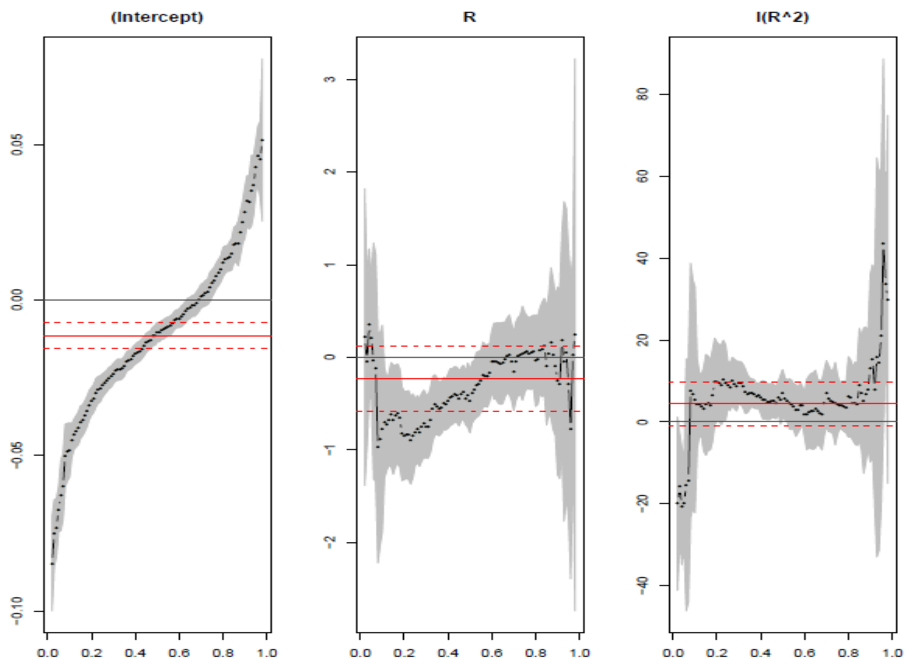


Figure 10. Results of the estimated coefficients in M4, for positive returns

Note. The solid red line represents the values of the parameters estimated by OLS, the dotted lines are the ranges of these values; R is the USO return and R^2 is the square return of the USO.

Table 8. Tests for the quantile specification

			Entire sample				
Quantile	0.05	0.10	0.25	0.50	0.75	0.90	0.95
M 3	-0.4375062	-0.1284453	313.472	21.084	4.085.514	467.285	333.878
	(0.31)	(0.19)	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***
M 4	-0.4505913	-0.006050332	-0.2462416	296.463	279.161	0.06396408	-0.9611806
	(0.27)	(0.12)	(0.18)	(0.00)***	(0.00)***	(0.098)*	(0.67)
			Positive returns				
M 3	-0.6948889	-0.6968786	-0.7046878	-0.7050707	-0.7046878	-0.707142	-0.7119072
	(0.49)	(0.10)	(0.18)	(0.02)**	(0.74)	(0.81)	(0.97)
M 4	-0.7044846	-0.6927087	-0.7046878	-0.7073977	-0.7046759	-0.707142	-0.7119072
	(0.78)	(0.15)	(0.67)	(0.76)	(0.42)	(0.62)	(0.91)
			Negative returns				
M 3	-0.8656327	-0.3183105	-0.5732867	-0.229459	-0.8169341	-0.5343522	-0.6875599
	(0.54)	(0.13)	(0.26)	(0.12)	(0.51)	(0.27)	(0.39)
M 4	-0.8423341	-0.5534052	-0.7976845	-0.8124068	-0.9112352	-0.8697938	-1077409
	(0.39)	(0.15)	(0.31)	(0.36)	(0.49)	(0.44)	(0.78)

Note. *** Null hypothesis of correct specification is rejected at a level of 0.1%. ** Null hypothesis of correct specification is rejected at a level of 5%. The t-values are presented in parentheses ( ).

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