

## The Multitude of Econometric Tests: Forecasting the Dutsch Guilder

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### Abstract

This paper studies a diversity of exchange rate models, applies both parametric and nonparametric techniques to them, and examines said models' collective predictive performance. We shall choose the forecasting predictor with the smallest root mean square forecast error (RMSE); the empirical evidence for a better type of exchange rate model is in equation (34), although none of our evidence gives an optimal forecast. At the end, these models' error correction versions will be fit so that plausible long-run elasticities can be imposed on each model's fundamental variables.

**Keywords:** efficiency, exchange rate determination, exchange rate policy, forecasting, foreign exchange

### 1. Introduction

Most economic time series exhibit phases of relative stability followed by periods of relatively high volatility, and thus do not display any constant mean. A brief examination of currency exchange rates (among other time-series data) imply that they are heteroscedastic because of the absence of a constant mean and variance, as opposed to being homoscedastic because of the presence of a stochastic variable with a constant variance. For any series with such volatility, the unconditional variance could be constant even though it may be unusually large at certain times.

The trends of some variables may contain either stochastic or deterministic elements, with the analysis of such ingredients influencing the forecasted results of the time series in question.

We can illustrate the behavior of different exchange rates by graphing them, noticing their fluctuation over time, and confirming first impressions through formal testing. For example, one notices that these series are not stationary, in that the sample means do not appear to be constant and there is a strong appearance of heteroscedasticity. This lack of a specific trend makes it difficult to prove that these series have a time-invariant mean. For example, the U.S. dollar-to-British pound exchange rate does not show any particular tendency towards either increasing or decreasing, with the dollar apparently going through long periods of appreciation and then depreciation without a reversion to the long-run average. This type of "random walk" behavior is quite typical of nonstationary time series.

Any shock to such a series displays a high degree of persistence: the dollar/pound exchange rate experienced a tremendous upward surge in 1980, remained at this level into 1984, and was only returning to somewhat near its previous level in 1989. The volatility of these series is not constant and, in fact, some currency exchange rate series have at least a partial correlation with other series; such series are named conditionally heteroscedastic if the unconditional (long-run) variance is constant but with localized periods of a relatively high variance. For instance, large shocks in the U.S. appear at about the same time in both Canada and Great Britain, although these co-movements' existence can be all but predicted because of the underlying forces affecting the economies of the U.S. and other countries.

The disturbance term's variance is assumed to be constant in conventional econometric models, although our series alternates periods of unusually great volatility with spells of relative tranquility. Therefore, our assumption of a constant variance in such cases is incorrect. As an investor holding but one currency, though, one might wish to forecast both the exchange rate and its conditional variance over the life of the investment in such an asset. The unconditional variance -- namely, the long-run forecast of the variance -- would not be important if one plans to buy the asset at time period  $t$  and subsequently sell it at  $t+1$ . Taylor (1995) and Kallianiotis (1985) provide reviews of the literature on exchange rate economics and Chinn and Meese (1995) examine four structural exchange rate models' performance.

This paper is organized as follows. Different trend models are described in section 2. Other linear time-series models are presented in section 3 and multiequation time-series models are discussed in section 4. The empirical results are given in section 5 with a summary of the findings presented at the end of section 6.

## 2. Time-Series Trends

One way to predict the variance of a time series is to explicitly introduce an independent variable that helps forecast its volatility. Consider the simplest case, in which

$$s_{t+1} = \varepsilon_{t+1}X_t \quad (1)$$

where  $s_{t+1}$  = the spot exchange rate (the variable of interest),  $\varepsilon_{t+1}$  = a white-noise disturbance term with variance  $\sigma^2$ , and  $X_t$  = an independent variable that can be observed at time period  $t$ . (If  $X_t = X_{t-1} = X_{t-2} = \dots = \text{constant}$ , then the  $\{s_t\}$  sequence is a standard white-noise process with a constant variance.)

If the realization of the  $\{X_t\}$  sequence is not all equal, then the variance of  $s_{t+1}$  that is conditional on the observable value of  $X_t$  is

$$\text{Var}(s_{t+1}/X_t) = X_t^2\sigma^2 \quad (2)$$

We can represent the general solution to a linear stochastic difference equation with these four components:

$$s_t = \text{trend} + \text{cyclical} + \text{seasonal} + \text{irregular}$$

Exchange rate series do not have an obvious tendency of reversion to any mean. One important function of econometricians is the formation of clear-cut stochastic difference equation models that can simulate trending variables' behavior, with a trend defined by its permanent effect on a time series. Because the irregular component is stationary, its effects will diminish while the trending elements and their effects will persist in long-term forecasts.

### 2.1 Deterministic Trends

One of  $s_t$ 's basic characteristics is its long-term growth pattern despite its short-term volatility. In fact,  $s_t$  may have a long-term trend that is quite apparent and clear-cut. According to Pindyck and Rubinfeld (1981), Chatfield (1985), and Enders (1995), there are eight models that describe this deterministic trend and can be used to extrapolate and forecast  $s_t$ . They are the following:

Linear time trend:

$$S_t = \alpha_0 + \alpha_1 t + \varepsilon_t \quad (3)$$

Exponential growth curve:

$$S_t = Ae^{rt} \quad (4)$$

or

$$\ln S_t = \ln A + rt + \varepsilon_t \quad (5)$$

or

$$s_t = \beta_0 + \beta_1 t + \varepsilon_t \quad (6)$$

Logarithmic (stochastic) autoregressive trend (the only function that can be applied for exchange rates):

$$s_t = \gamma_0 + \gamma_1 s_{t-1} + \varepsilon_t \quad (7)$$

Quadratic trend:

$$s_t = \delta_0 + \delta_1 t + \delta_2 t^2 + \varepsilon_t \quad (8)$$

Polynomial time trend:

$$s_t = \zeta_0 + \zeta_1 t + \zeta_2 t^2 + \dots + \zeta_n t^n + \varepsilon_t \quad (9)$$

Logarithmic growth curve:

$$s_t = 1 / (\theta_0 + \theta_1 \theta_2^t); \theta_2 > 0 \quad (10)$$

or a stochastic approximation:

$$(\Delta s_t / s_{t-1}) = k_0 - k_1 s_{t-1} + \varepsilon_t \quad (11)$$

Sales saturation pattern:

$$S_t = e^{\lambda_0 - (\frac{\lambda_1}{t})} \quad (12)$$

or

$$s_t = \lambda_0 - (\lambda_1 / t) + \varepsilon_t \quad (13)$$

where  $S_t$  = the spot exchange rate,  $t$  = time trend, and the lowercase letters are the natural logarithms of their uppercase counterparts.

## 2.2 Models of Stochastic Trend

We can supplement the deterministic trend models with the lagged values of the  $\{s_t\}$  sequence and the  $\{\varepsilon_t\}$  sequence. These equations thus become models with their own stochastic trends. The models used here are:

### (i) The Random Walk Model

The random walk model appears to imitate the exchange rates' behavior as shown below. These series neither fluctuate over time nor revert to any given mean. (The random walk model is technically a special case of the AR(1) process).

$$s_t = \alpha_0 + \alpha_1 s_{t-1} + \varepsilon_t \quad (14)$$

with  $\alpha_0 = 0$  and  $\alpha_1 = 1$ , where  $s_t - s_{t-1} = \Delta s_t = \varepsilon_t$

$$s_t = s_{t-1} + \varepsilon_t \quad (15)$$

The conditional mean of  $s_{t+\lambda}$  for any  $\lambda > 0$  is

$$E_t s_{t+\lambda} = s_t + E \sum_{i=1}^{\lambda} \varepsilon_{t+i} = s_t \quad (16)$$

The variance is time-dependent:

$$\text{var}(s_t) = \text{var}(\varepsilon_t + \varepsilon_{t-1} + \dots + \varepsilon_1) = t\sigma^2 \quad (17)$$

The random walk process is nonstationary because the variance is not constant. Therefore, as

$$t \rightarrow \infty, \text{var}(s_t) \rightarrow \infty. \quad (18)$$

Therefore, the forecast function will be:

$$E_t s_{t+\lambda} = s_t \quad (19)$$

### (ii) The Random Walk plus Drift Model

The random walk plus drift model adds a constant term  $\alpha_0$  to the random walk model above such that  $s_t$  becomes simultaneously deterministic in part and stochastic in part.

$$s_t = s_{t-1} + \alpha_0 + \varepsilon_t \quad (20)$$

The general solution for  $s_t$  is:

$$s_t = s_0 + \alpha_0 t + \sum_{i=1}^t \varepsilon_i \quad (21)$$

and

$$E_t s_{t+\lambda} = s_0 + \alpha_0 (t + \lambda) \quad (22)$$

The forecast function by  $\lambda$  periods yields

$$E_t s_{t+\lambda} = s_t + \alpha_0 \lambda \quad (23)$$

### (iii) The Random Walk plus Noise Model

The  $s_t$  here is the sum of a stochastic trend and a white-noise component

$$s_t = \mu_t + n_t \quad (24)$$

and

$$\mu_t = \mu_{t-1} + \varepsilon_t \quad (25)$$

where  $\{n_t\}$  is a white-noise process with variance  $\sigma_n^2$  and  $\varepsilon_t$  and  $n_t$  are both independently distributed for all  $t$ .  $E(\varepsilon_t n_{t-\lambda}) = 0$ ; the  $\{\mu_t\}$  sequence represents the stochastic trend, and this model's solution can be written as:

$$s_t = s_0 - n_0 + \sum_{i=1}^t \varepsilon_i + n_t \quad (26)$$

The forecast function is

$$E_t S_{t+\lambda} = s_t - n_t \quad (27)$$

(iv) The General Trend plus Irregular Model

We replace equation (25) above with the so-called “trend plus noise model,”

$$\mu_t = \mu_{t-1} + \alpha_0 + \varepsilon_t \quad (28)$$

where  $\alpha_0$  is a constant and  $\{\varepsilon_t\}$  is a white-noise process.

The solution is

$$s_t = s_0 - n_0 + \alpha_0 t + \sum_{i=1}^t \varepsilon_i + n_t \quad (29)$$

Let  $A(L)$  be a polynomial in the lag operator  $L$ . It is possible to augment a random walk plus drift process with the stationary noise process  $A(L) n_t$ . We thus have the “general trend plus irregular model”:

$$s_t = \mu_0 + \alpha_0 t + \sum_{i=1}^t \varepsilon_i + A(L) n_t \quad (30)$$

(v) The Local Linear Trend Model

We construct the local linear trend model by combining several random walk plus noise processes. Let  $\{\varepsilon_t\}$ ,  $\{n_t\}$ , and  $\{u_t\}$  be three mutually uncorrelated white-noise processes. The equation for the local linear trend model is:

$$\begin{aligned} s_t &= \mu_t + n_t \\ \mu_t &= \mu_{t-1} + \alpha_t + \varepsilon_t \\ \alpha_t &= \alpha_{t-1} + u_t \end{aligned} \quad (31)$$

This is the most detailed out of all the above models because the other processes are special cases of the local linear trend model, which consists of the noise term  $n_t$  and the stochastic trend term  $\mu_t$ . What is most important for our purposes about the model is that the change in its trend yields a random walk plus noise:

$$\Delta \mu_t = \mu_t - \mu_{t-1} = \alpha_t + \varepsilon_t \quad (32)$$

The forecast function of  $s_{t+\lambda}$  equals the current value of  $s_t$  minus the transitory component  $n_t$ , added to  $\lambda$  multiplied by the slope of the trend term in  $t$ :

$$E_t S_{t+\lambda} = (s_t - n_t) + \lambda (\alpha_0 + u_1 + u_2 + \dots + u_t) \quad (33)$$

For future projects, we will estimate all these models and run different tests on the series and the error terms. We will end up with specification and diagnostic tests as a way of gauging the statistical specifications' adequacy and will then compare the forecasting results from the different models.

### 3. Some Linear Time-Series Models

In this section, we define stochastic processes and discuss some of their properties and use in forecasting with an objective of developing models that “explain” the movement of the time series  $s_t$ . However, this will not be done using a set of explanatory variables as in the regression model but by relating it to its own past values and to a weighted sum of lagged and current random disturbances.

The Autoregressive (AR) Model

In the autoregressive process of order  $p$ , the current observation  $s_t$  is generated by a weighted average of past observations going back  $p$  periods, together with the current period's random disturbance. We define this process as  $AR(p)$  and write its equation as

$$s_t = \phi_1 s_{t-1} + \phi_2 s_{t-2} + \dots + \phi_p s_{t-p} + \delta + \varepsilon_t \quad (34)$$

$\delta$  is a constant term which relates to the mean of the stochastic process.

The first-order process  $AR(1)$  is

$$s_t = \phi_1 s_{t-1} + \delta + \varepsilon_t \quad (35)$$

Its mean is:

$$\mu = \delta / (1 - \phi_1) \quad (36)$$

and is stationary if  $|\phi_1| < 1$ . (The random walk with drift is a first-order autoregressive process that is not stationary, however.)

#### 4. Empirical Evidence

We provide here an analysis and summary of the empirical evidence of different models of foreign currency forecasting.

The data given are monthly from March 1973 through December 1994, are coming from Main Economic Indicators of the OECD (the Organization for Economic Cooperation and Development) and International Financial Statistics of the IMF (the International Monetary Fund), and have been applied for the Netherlands. The exchange rate is defined as the U.S. dollar per unit of the Dutch guilder, with direct quotes for the dollar; the lowercase letters denote the natural logarithm of the variables and an asterisk denotes the corresponding variable for the Netherlands.

The first equations estimated are the deterministic trend models in equations (3), (6), (8), (9), (11), and (13). The results appear in Table 1 below and indicate that the exchange rate forecast cannot be supported by models of this type. The second group of equations is the stochastic trend model, from equations (15) and (20); these results, in Table 2, show that this alternative model is much better at both interpreting the data and forecasting the exchange rate. The final model is of a linear time-series, the autoregressive (AR) model of equation (34) shown in Table 3, but its results are also fairly poor. One may infer that time-series models cannot be used to forecast foreign exchange rates with a great degree of faith or assurance for models having such relatively high volatility.

Table 1. Deterministic trends

(i) Linear time trend, eq. (3): $S_t = \alpha_0 + \alpha_1 t + \varepsilon_t$		(ii) Exponential Growth Curve, eq. (6): $s_t = \beta_0 + \beta_1 t + \varepsilon_t$	
$\alpha_0$	37.384*** (1.435)	$\beta_0$	3.559*** (.023)
$\alpha_1$	.037*** (.008)	$\beta_1$	.001*** (.0001)
$R^2$	.073	$R^2$	.288
D-W	.142	D-W	.047
SSR	26,567.54	SSR	6.615
F	20.230	F	102.89
RMSE	10.0699	RMSE	.1607
(iii) Quadratic Trend, eq. (8): $s_t = \delta_0 + \delta_1 t + \delta_2 t^2 + \varepsilon_t$		(iv) Polynomial time trend, eq. (9): $s_t = \zeta_0 + \zeta_1 t + \zeta_2 t^2 + \dots + \zeta_n t^n + \varepsilon_t$	
$\delta_0$	3.776*** (.041)	$\zeta_0$	4.382*** (.528)
$\delta_1$	-.002* (.0006)	$\zeta_1$	-.041 (.038)
$\delta_2$	1.0-05*** (2.0-06)	$\zeta_2$	.0004 (.001)
		$\zeta_3$	8.3-06 (1.5-05)
		$\zeta_4$	-1.8-07 (1.2-07)
		$\zeta_5$	1.3-09** (5.1-10)
		$\zeta_6$	-3.7-12*** (1.2-12)
		$\zeta_7$	4.2-15*** (1.1-15)
$R^2$	.382	$R^2$	.766
D-W	.054	D-W	.140
SSR	5.739	SSR	2.175
F	78.37	F	115.97
RMSE	.1497	RMSE	.0922

(v) Stochastic approximation, eq. (11): $(\Delta s_t/s_{t-1}) = k_0 - k_1 s_{t-1} + \varepsilon_t$		(vi) Sales Saturation Pattern, eq. (13): $s_t = \lambda_0 - (\lambda_1/t) + \varepsilon_t$	
$k_0$	.061 (.043)	$\lambda_0$	3.867*** (.018)
$k_1$	-.016 (.011)	$\lambda_1$	-10.239*** (1.541)
$R^2$	.007	$R^2$	.148
D-W	1.952	D-W	.039
SSR	.307	SSR	7.918
F	1.890	F	44.167
RMSE	.0346	RMSE	.1759

Note.  $S_t$  = the spot exchange rate,  $s_t = \ln(S_t)$ , t = time, D-W = the Durbin-Watson statistic, SSR = sum of squares residuals, RMSE = root mean square error, Data from 1973.03 to 1994.06, \*\*\* = significant at the 1% level, \*\* = significant at the 5% level, \* = significant at the 10% level.  $\Delta$  = change of the variable.

Table 2. Stochastic trends

(i) The Random Walk Model, eq. (15): $s_t = s_{t-1} + \varepsilon_t$		(ii) The Random Walk plus Drift Model, eq. (20): $s_t = \alpha_1 s_{t-1} + \alpha_0 + \varepsilon_t$	
$s_{t-1}$	1.000*** (.0006)	$\alpha_0$	.061 (.043)
		$\alpha_1$	.984*** (.011)
$R^2$	.967	$R^2$	.967
D-W	1.941	D-W	1.925
SSR	.309	SSR	.307
L(.)	496.86	F	7,446.37
RMSE	.0347	RMSE	.0346

Note. See the previous table. L(.) = log of likelihood function.

Table 3. Linear time-series models

The Autoregressive (AR) Model, eq. (34): $s_t = \phi_1 s_{t-1} + \phi_2 s_{t-2} + \dots + \phi_p s_{t-p} + \delta + \varepsilon_t$	
$\delta$	3.828*** (.108)
$\phi_1$	1.006*** (.063)
$\phi_2$	.062 (.089)
$\phi_3$	-.093 (.089)
$\phi_4$	.016 (.090)
$\phi_5$	-.005 (.090)
$\phi_6$	-.105 (.089)
$\phi_7$	.199** (.089)
$\phi_8$	-.071 (.090)
$\phi_9$	-.018 (.090)
$\phi_{10}$	.026 (.090)
$\phi_{11}$	.053 (.090)
$\phi_{12}$	-.092 (.063)
$R^2$	.968
D - W	1.958
SSR	.294
F	619.29
RMSE	.0339

Note. See the previous tables.

## 5. Summary

This paper examines the predictive performance of several foreign currency exchange rate forecast models, namely, linear time-series, the balance of payments approach, the transfer function, the vector autoregression model, and various time-series trends. For every such model, we calculate its root mean square forecast error (RMSE) as follows:

$$RMSE = \text{the square root of } ( \sum_{t=1}^n (A_t - F_t)^2 / n )$$

where  $n$  = the number of observations,  $A$  = the actual value of the dependent variable, and  $F$  = the forecast value. The forecast model with the smallest RMSE is the best predictor we must choose as part of exchange rate forecasting.

An exchange rate is the relative price of two countries' currencies. The most crucial factors that determine a country's currency value relate to the differences in inflation, the relative money supplies, real incomes, and prices, and interest rate, trade balance and budget deficit differentials. The empirical evidence for this approach is not satisfactory in general, however: the combination analysis (i.e., the MARMA model) is more adequate compared to the other approaches and shows that this specific model has a better specification, but there is still room for improvement in foreign currency forecasts' current mathematical models. Exchange rate movements themselves may result from either a parametric change in the above determinants or an artificial intervention by governments.

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