

# Analyzing the Downside Risk of Exchange-Traded Funds: Do the Volatility Estimators Matter?

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## Abstract

This paper aims to propose the augmented GJR-GARCH (GJR-GARCH<sub>M</sub>) model that extends the GJR-GARCH model by comprising overnight returns volatility (ONV), daily high-low prices range (PK), and fear index (VIX) as explanatory variables for the GJR's variance equation, respectively. The proposed models are used to estimate the daily value-at-risk values and evaluate their downside risk management performance for the SPDRs covering the period from 2009 to 2014. Empirical results show that the GJR-GARCH<sub>M</sub> model outperforms the GJR-GARCH model for most cases, suggesting that the GJR-GARCH-based VaR forecasts can be moderately improved with the additional information embodied in the ONV, PK and VIX volatility estimators. In addition, daily high-low prices range and VIX are far more informative than the overnight volatility estimator for improving the GJR-GARCH-based VaR forecasts. Risk managers can employ the proposed models for estimating and controlling the potential loss of ETFs in the face of financial catastrophes.

**Keywords:** exchange-traded funds, SPDRs, value-at-risk, volatility estimator, GJR

## 1. Introduction

Exchange-traded funds (ETFs) are very prevalent and have become popularly adopted investment tools among common investors over recent years. ETFs are attractive as investments because they have many advantages, such as low fee ratios, tax efficiency, diversified-portfolio and stock-like characteristics. The American Stock Exchange introduces the Standard and Poor's Depositary Receipts (SPDRs, or called Spider) in the early 1990s, which are backed by an equity portfolio that closely traces the S&P 500 index. By far, the Spider is the most actively traded and the largest passive ETF worldwide, with US\$215.91 billion market value as at January, 2015.

Researchers have long been observed that volatility of financial assets returns are often described by several stylized facts, such as time-varying, clustering, and persistence features. The ARCH (autoregressive conditional heteroskedastic, ARCH) model proposed by Engle (1982) and the GARCH (generalized autoregressive conditional heteroskedastic, GARCH) model advocated by Bollerslev (1986) respond to deal with these stylized phenomena. Since then, volatility forecasting technique has been dominated by a variety of the GARCH genre of models, especially for the asymmetric GARCH model. Glosten et al. (1993) propose the so-called GJR-GARCH model which is a simple class of GARCH-type models which can capture leverage effects of positive news and negative news on conditional volatility.

To the best of our knowledge, a large volume of recent studies have been investigated and written about the value-at-risk (VaR) issue for various financial markets by using GARCH techniques, such as Angelidis et al. (2004), Huang and Lin (2004), Liu and Hung (2010), Orhan and Köksal (2012) and So and Yu (2006) for stock markets, Al Janabi (2006), Bams et al. (2005) and So and Yu (2006) for foreign exchange rate markets, Chan and Gray (2006), Sadeghi and Shavvalpour (2006) and Sadorsky (2006) for energy markets. However, despite the importance of VaR on financial risk management and the popularity of ETFs for common investors, there seems to have been relatively little work endeavored on ETFs.

Brooks et al. (2000) advocate the overnight return volatility (ONV) in order to capture accumulated overnight information that would be beneficial for capturing the persistence in the conditional heteroscedasticity of stock returns. Motivated by the daily price range, on the one hand, Parkinson (1980) exploits the scaled high-low price ranges to construct the daily PK estimator based on the assumption which intraday trading prices follow a random walk process. On the other hand, Garman and Klass (1980) propose the so-called GK estimator by including opening and closing trading prices in addition to price range, with similar assumptions to the PK. In addition, Rogers and Satchell (1991) develop the RS volatility estimator by considering the drift in the price process. In 1993, Chicago Board Options Exchange develops the fear index (VIX, or called implied volatility) that is acquired from the S&P 500 index option prices data via an option pricing model.

In recent years, the wide availability of intraday data has encouraged researchers to explore their information value in modeling and predicting the volatility of financial assets, such as Blair et al. (2001), Corrado and Truong (2007), Fuertes et al. (2009), Koopman et al. (2005), Vipul and Jacob (2007) and the reference therein. However, despite a large volume of existing literature on volatility forecasting, none of them investigates the prices information which is embodied in the ONV, PK and VIX volatility estimators for improving predictive accuracy of daily value-at-risk forecasts in ETF. Thus, this paper aims to propose the augmented GJR model that extends the traditional GJR-GARCH model by comprising three volatility estimators, overnight volatility (ONV), daily prices range (PK), and fear index (VIX) as explanatory variables for the variance equations in GJR model. The proposed models are used to estimate their daily VaR values and evaluate their downside risk management performance for the SPDRs returns spanning from 2009 to 2014.

The remainder of this paper is organized as follows. The data and econometric methodology are provided in Section 2, followed in Section 3 by the empirical results of daily VaR forecasts performance for SPDRs across alternative confidence levels. The final Section summarizes the conclusions drawn from this paper.

## 2. Methodology

### 2.1 Data and Descriptive Analysis

The data analyzed in this paper comprises of the daily open, high, low, and closing prices data on SPDRs as well as the VIX data obtained from the Yahoo Finance website. The sample period for these daily data covers from 2 January 2009 to 31 December 2014 for a total of 1,510 trading days. The first four years (1,006 observations) are used as the in-sample period for estimation purpose, while the remaining two years (504 observations) are left for out-of-sample forecast evaluation.

Table 1. Descriptive statistics of daily returns for the SPDRs

Mean (%)	Std.	Min	Max	Skew	Kurt	J-B	Q <sub>s</sub> (12)
0.053	1.142	-6.734	6.960	-0.270*	4.484*	1283.019*	796.501*

Note. This table presents the descriptive statistics of daily returns for the Standard & Poor's Depository Receipts. J-B represents the statistics of Jarque and Bera (1987)'s normal distribution test. Q<sub>s</sub>(12) refers to the Ljung-Box Q test statistic of the squared return series for up to the 12th order serial correlation. \* indicates significance at the 1% level.

Table 1 provides the descriptive statistics of the daily returns for the Standard & Poor's Depository Receipts. As showed in Table 1, the daily average return of SPDR is 0.053%, and very closes to zero. The skewness and kurtosis of returns series exhibit significant evidence, meaning that the returns have a left skewness, and the distribution of the returns is much fat-tailed and high-peaked than normal distribution. The Jarque and Bera (1987) test statistic also shows that the daily SPDR returns are not normal-distributed. Moreover, the Q<sub>s</sub>(12) test statistic exhibits linear dependence for the squared returns and exists significant ARCH effects.

### 2.2 Augmented GJR Model

We propose the augmented GJR model which extends the GJR-GARCH model of Glosten et al. (1993) by including various volatility estimators (ONV, PK and VIX), respectively, for its variance equation as follows:

$$R_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t | \Omega_{t-1} \sim NID(0,1) \quad (1)$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \delta v_{t-1} \quad (2)$$

where  $R_t$  is daily SPDRs return;  $\mu$  is the conditional mean of returns;  $\varepsilon_t$  represents the innovation process;  $z_t$  denotes the standardized residual with zero mean and unit variance;  $\sigma_t^2$  is the conditional variance.  $d_{t-1}$  denotes the indicator function that takes the value of unity if  $\varepsilon_{t-1} < 0$ , and 0 otherwise. The indicator variable differentiates between good news and bad news impacts, so that leverage effects are captured by  $\gamma$ . Thus, in the augmented GJR model, good news has an impact of  $\alpha$ , and bad news has an impact of  $(\alpha + \gamma)$ , with bad (good) news having a greater shock on volatility if  $\gamma > 0$  ( $\gamma < 0$ ). Finally,  $v_{t-1}$  denotes a volatility estimator made at day  $t-1$ , including ONV (overnight volatility), PK (daily high-low price range), and VIX (fear index). Table 2 provides a synopsis of these volatility estimators:

Table 2. The synopsis of various volatility estimators

Abbreviation of volatility estimators	Studies	Formula or explanation
ONV	Brooks et al. (2000)	$\hat{\sigma}_{ONV,t}^2 = (\ln(O_t / C_{t-1}))^2$ (3)
PK	Parkinson (1980)	$\hat{\sigma}_{PK,t}^2 = (4 \ln 2)^{-1} \cdot (\ln(H_t / L_t))^2$ (4)
VIX	-	VIX is a popular measure of the fear index of S&P 500 index options, which represents one measure of the market's expectation of stock market volatility over the next 30 day period. For consistent scaling with ONV and PK, the VIXs are squared and divided by 252.

Note. This table presents the various volatility estimators employed in this paper.  $O_t$ ,  $H_t$ ,  $L_t$  and  $C_t$  denote the opening, high, low, and closing prices at day  $t$ , respectively.

### 2.3 Downside Risk Measurement and Performance Evaluation

The GJR-based VaR forecasts for a one-day holding period can be calculated as follows:

$$VaR_t = Z_{\alpha_1} \cdot \hat{\sigma}_t + \mu \quad (5)$$

Where  $Z_{\alpha_1}$  denotes the corresponding quantile of the standard normal distribution at  $\alpha_1$ , while  $\hat{\sigma}_t$  is the volatility forecast generated from either GJR, GJR-ONV, GJR-PK or GJR-VIX model.

In order to backtesting, this paper uses a likelihood-ratio test of Kupiec (1995) to examine whether the true failure rate is statistically consistent with theoretical failure rate of the VaR model. The null hypothesis of the failure rate  $P$  is tested against the alternative hypothesis that the failure rate is different from  $P$ . The  $LR_{uc}$  statistic can be formulated as follows:

$$LR_{uc} = 2 \ln \left[ \frac{\hat{\pi}^{n_1} (1 - \hat{\pi})^{n_0}}{P^{n_1} (1 - P)^{n_0}} \right] \sim \chi^2(1) \quad (6)$$

where  $\hat{\pi} = n_1 / (n_0 + n_1)$  denotes the maximum likelihood estimate of  $P$ , and  $n_1$  is a Bernoulli random variable that represents the total number of VaR violations. (Note 1)

Christoffersen (1998) constructs the sophisticated conditional coverage test ( $LR_{cc}$ ) which jointly examines whether the total number of violations is equal to the expected one, and the VaR violations are independently distributed. Given the realizations of the SPDRs returns series  $R_t$  and the set of VaR estimates, the indicator variable  $I_t$  can be formulated as follows:

$$I_t = \begin{cases} 1 & \text{if } R_t < VaR_t \\ 0 & \text{if } R_t \geq VaR_t \end{cases} \quad (7)$$

Since accurate VaR estimates exhibit the property of correct conditional coverage, the  $I_t$  series must display both correct unconditional coverage and serial independence. The  $LR_{cc}$  test is a joint test of these two properties, and the related test statistic is  $LR_{cc} = LR_{uc} + LR_{ind}$  as we condition on the first observation. Thus, under the null hypothesis that the failure process is independent and the expected proportion of violations equals  $P$ , the corresponding likelihood ratio can be formulated as follows:

$$LR_{cc} = -2 \ln \frac{(1-P)^{n_0} P^{n_1}}{(1-\hat{\pi}_{01})^{n_{00}} \hat{\pi}_{01}^{n_{01}} (1-\hat{\pi}_{11})^{n_{10}} \hat{\pi}_{11}^{n_{11}}} \sim \chi^2(2) \quad (8)$$

where  $n_{i,j}$  = the number of observations with value  $i$  followed by value  $j$  ( $i, j = 0, 1$ ),

$$\pi_{ij} = P\{I_t = j | I_{t-1} = i\} (i, j = 0, 1), \quad \hat{\pi}_{01} = n_{01} / (n_{00} + n_{01}), \quad \hat{\pi}_{11} = n_{11} / (n_{10} + n_{11}).$$

### 3. Empirical Results and Analysis

Table 3 presents out-of-sample daily VaR forecasts performance across the various models by reporting mean VaR, violation, failure prob.,  $LR_{uc}$  and  $LR_{cc}$  statistics, under 90%, 95% and 99% confidence levels.

As shown in Table 3, the GJR model generates the highest average absolute VaR estimates at every confidence level, and followed by the GJR-ONV, GJR-PK and GJR-VIX models. Thus, the GJR and the GJR-VIX models generate the lowest and highest numbers of VaR violations, respectively.

Panel A of Table 3 provides daily VaR forecasts results for SPDRs at the 90% confidence level. We observe that either the GJR or the GJR-ONV model fails to pass the unconditional coverage test ( $LR_{uc}$ ), indicating that both traditional GJR and GJR-ONV models tend to over-predict VaR values for SPDRs returns. Moreover, the GJR model has been rejected by the conditional coverage test ( $LR_{cc}$ ), indicating that clustered violations were generated. That is, the GJR model is very slow at updating the VaR value when market volatility changes rapidly. By contrast, both the GJR-PK and the GJR-VIX models pass the coverage tests, suggesting that the empirical failure probability is statistically consistent with the prescribed one for each of them, especially for the latter model. Meanwhile, with any sudden change in market volatility, the GJR-PK and the GJR-VIX models are beneficial for rapidly updating the VaR value. Thus, the trading prices information which is implied in PK and VIX volatility measures is crucial for producing adequate daily VaR forecasts for SPDRs returns at the 90% confidence level.

For the case of 95% confidence level, we observe that the  $LR_{uc}$  test statistic is insignificant for the GJR, GJR-ONV and GJR-PK models, indicating that the sample point estimate is statistically consistent with the prescribed confidence level of these three VaR models. The  $LR_{cc}$  statistic further shows that the aforesaid three models also can pass the conditional coverage test, indicating that these models' performance is quite stable over time during the out-of-sample forecasting period 2013~2014. However, the GJR-VIX model fails to offer adequate VaR forecasts according to the  $LR_{uc}$  test statistic.

The VaR forecasts results at the 99% confidence level are very similar to those obtained at the 95% confidence level. That is, the  $LR_{uc}$  and  $LR_{cc}$  statistics reported in Panel C of Table 3 are all insignificant, except for the GJR-VIX model, indicating that the GJR, GJR-ONV and GJR-PK models are able to produce adequate Daily VaR forecasts for SPDRs returns.

Table 3. Daily VaR forecasts results

Model	Mean VaR	Violation	Failure prob.	$LR_{uc}$	$LR_{cc}$
Panel A: 90% Confidence Level					
GJR	-1.0192	36	7.14%	<b>5.02</b>	<b>5.15</b>
GJR-ONV	-0.9864	37	7.34%	<b>4.32</b>	4.51
GJR-PK	-0.9142	42	8.33%	1.63	2.52
GJR-VIX	-0.8268	53	10.51%	0.14	0.63
Panel B: 95% Confidence Level					
GJR	-1.3166	25	4.96%	0.00	0.05
GJR-ONV	-1.2744	26	5.15%	0.02	0.10
GJR-PK	-1.1838	26	5.15%	0.02	0.19
GJR-VIX	-1.0701	34	6.74%	<b>2.92</b>	2.95
Panel C: 99% Confidence Level					
GJR	-1.8745	5	0.99%	0.00	3.73
GJR-ONV	-1.8146	5	0.99%	0.00	3.73
GJR-PK	-1.6897	7	1.38%	0.68	3.27
GJR-VIX	-1.5265	11	2.18%	<b>5.32</b>	<b>6.48</b>

Note. This table presents daily VaR forecasts results for SPDRs at three confidence levels. The critical values of the  $LR_{uc}$  and  $LR_{cc}$  statistics at the 10% significance level are 2.71 and 4.61, respectively. Figures in bold text indicate rejection of the null hypothesis of correct VaR estimates at the 10% significance level.

#### 4. Conclusions

This paper proposes the augmented GJR-GARCH model that extends the GJR-GARCH model of Glosten et al. (1993) by respectively comprising overnight volatility, daily high-low prices range, and fear index as explanatory variables for the GJR-GARCH's variance equation. These VaR models are used to estimate their daily VaR values and evaluate their downside risk management performance for the SPDRs returns under 90%, 95% and 99% confidence levels. Empirical results show that the augmented GJR model outperforms the GJR model for most cases, suggesting that the GJR-GARCH-based VaR forecasts can be moderately improved with the additional information embodied in the ONV, PK and VIX volatility estimators. In addition, daily high-low prices range and fear index are far more informative than the overnight volatility for improving the GJR-GARCH-based VaR forecasts. Risk managers can employ the proposed VaR models for estimating and controlling the potential loss of ETFs in the face of financial catastrophes.

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## Note

Note 1. If the forecasted VaR can not cover the realized dollar loss, this is defined as a violation.

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