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Accident Forgiveness in the Automobile Insurance Contract

Fan Liu[1]

[1] John L. Grove College of Business, Shippensburg University, Pennsylvania, USA

Correspondence: Fan Liu, John L. Grove College of Business, Shippensburg University, Pennsylvania 17257, USA. Tel: 717-477-1786. E-mail: fliu@ship.edu

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Abstract

I present a two-period asymmetric learning model to study accident forgiveness as an optional insurance policy in the automobile insurance contract. Accident forgiveness, often considered as "premium insurance," protects the insured against a premium increase if an at-fault accident occurs. By offering this feature to all policyholders at the beginning, I design an accident forgiveness policy that charges policyholders higher-than-market premiums based on their risk types in the first period and then experience rates both types in the second period contingent on their previous at-fault accidents. In this model, experience rating still serves as the sorting device although its effectiveness is tempered by the presence of an accident forgiveness policy. Moreover, I find that both individual time and risk preferences contribute to the accident forgiveness purchases.

Keywords: accident forgiveness, asymmetric information, multi-period insurance contract, insurance purchase decision

1. Introduction

In recent years, we hear more about accident forgiveness policies on TV, billboards, and in Internet forums. With accident forgiveness, insurance rates do not go up due to an accident. Insurance companies know that drivers are fearful of the financial consequences of being in even the most minor accidents and that is why insurers are promoting accident forgiveness policies. For example, since 2005, Allstate has successfully launched accident forgiveness as part of its "Your Choice Auto" insurance program which fundamentally changes the type of products traditionally offered by many insurers by presenting consumers with more innovative features (Note 1). Following Allstate's lead, other major auto insurance companies in the U.S. market such as GEICO, Progressive, and Travelers are also offering their existing customers this new feature (see Table 1).

Interestingly, most auto insurers offering accident forgiveness policies in the U.S. auto insurance market provide this feature for "free" to their existing policyholders who have been with them for three to five years and who have maintained an accident-free record for a number of consecutive years (Note 2). For example, Nationwide requires that policyholders must be with Nationwide for five years and be accident free for three years to qualify for the accident forgiveness benefits. With Travelers' accident forgiveness policy, customers who have been with Travelers for four years or more and are accident free for five years do not see a surcharge for their first qualifying accident. However, a few insurers sell accident forgiveness as an optional feature in the insurance contract to all customers (see Table 1). For example, Allstate sells accident forgiveness as part of its "Your Choice Auto Insurance" program and Farmers sells this feature as part of its "Farmers Flex" program (Note 3). In both programs, customers are allowed to purchase accident forgiveness by paying an additional premium.

Table 1. Top 10 writers of PPA insurance in the United States (2009)

Rank	Group	Market Share	Offering Accident Forgiveness
1	State Farm Mutual	18.60%	Guaranteed if accident free for 9 years
2	Allstate	10.50%	As part of ``Your Choice Auto"
3	Berkshire Hathaway	8.20%	Guaranteed if accident free for 5 years
4	Progressive	7.50%	Guaranteed if accident free for 3 years
5	Zurich Financial Services	6.40%	As part of ``Farmers Flex"
6	Nationwide Mutual	4.50%	Guaranteed if accident free for 3 years
7	Liberty Mutual	4.40%	Guaranteed if accident free for 5 years

8	USAA	4.10%	Guaranteed if accident free for 5 years
9	Travelers	2.10%	Guaranteed if accident free for 5 years
10	American Family Mutual	2.00%	Guaranteed if accident free for 3 years

Note. Adapted from SNL Financial LC and insurance companies' websites.

Although accident forgiveness has received considerable attention in the auto insurance industry, there is little or no literature studying accident forgiveness in the auto insurance contract (Note 4). This paper contributes to the existing literature by developing an asymmetric learning model to examine optimal insurance contracts with accident forgiveness provided as an optional feature in the market. My study attempts to improve our understanding of this new policy feature in the insurance contracting.

The model developed in this paper is a two-period model in which insurers compete to attract policyholders. Individuals are risk averse and are subject to a possible income loss in each period. As usual, I employ an asymmetric information assumption in the model by assuming that the probability of loss is not known initially by all insurers and at-fault accidents that occurred in the first period are only observed by the initial insurer (Note 5). When information about previous at-fault accidents is not shared perfectly by the insurers in the market, information asymmetries arise between the initial insurer and the rival insurer, as well as between the insured and the insurer. Regardless of the no commitment assumption in the prior literature (e.g., Nilssen, 2000), insurers in this model are able to commit to the two-period contract, but policyholders always have the option to switch insurers ex post (Note 6). I design an accident forgiveness policy that charges policyholders higher-than-market premiums by their risk types in the first period and experience rates both types in the second period contingent on their previous at-fault accidents. Contrary to the prior literature that elicits competition as the reason to temper the experience rating (e.g., Cooper & Hayes, 1987), this model is built such that accident forgiveness is the device that tempers the experience rating, and, of course, this is the incentive for policyholders to purchase it. This accident forgiveness contract attracts policyholders as it "forgive" the at-fault accident and provides "reward" in terms of coverage and premiums for those accident-free. By offering this feature to all policyholders in the pool, the insurer appears to not only lock in its loyal customers but also attract additional low risk customers.

The other noteworthy contribution of this paper is how I analyze the individual insurance purchase decision. Accident forgiveness, to some extent, is often thought of as "premium insuranc" where consumers purchase premium protection which gives them the right to make at-fault claims without experiencing an increase in their premiums. An insurance purchase decision, as an investment decision under uncertainty, is to be affected by individual risk and time preferences (e.g., Hirshleifer, 1966; Schlesinger & Schulenburg, 1987). By allowing randomization for both risk types of insureds over contracts in the model, my results suggest a nondecreasing effect of the discount factor on the individual accident forgiveness purchase. Further, risk averse individuals become more likely to purchase accident forgiveness if the expected utility provided by the insurance contract is above a threshold. This is surprisingly different from the findings in the prior literature, which suggest that risk averse individuals always purchase more insurance.

The paper is organized as follows. Section 2 discusses the previous studies related to the multi-period insurance contracts. Section 3 introduces the features of accident forgiveness in the current insurance market. Section 4 outlines the basic model and examines the characterization for the optimal contract. Section 5 establishes the impacts of the discount factor and risk aversion on the accident forgiveness purchases. Section 6 contains the conclusions.

2. Literature Review

Multi-period contracting is observed in different markets. In auto insurance market, consumers typically make repeat purchases. For example, in many countries, drivers purchase automobile insurance with the same insurer for many years, and the insurers use bonus-malus systems in order to relate insurance premiums to the individual's past experience (e.g., Dionne & Vanasse, 1992; Dionne et al., 2005; Hey, 1985; Lemaire, 1985). Multi-period contracting is also observed in workers' compensation insurance, unemployment insurance, and many other markets. The introduction of multi-period contracts in the analysis gives rise to many issues such as time discounting, commitment of the parties, myopic behavior, and information asymmetry. Multi-period insurance contracts are set not only to adjust ex-post insurance premiums or insurance coverage to past experience but also as a sorting device. They can be a complement or a substitute to standard self-selection mechanisms (Dionne, 2000, p. 194).

Cooper and Hayes (1987) were the first to consider a repeated insurance problem with adverse selection. They use the Nash equilibrium concept in a two-period game where the equilibrium must be separating (Note 7). Cooper and Hayes introduce a second instrument to induce self-selection: experience rating. Experience rating increases the cost to high risks from masquerading as low risks by exposing them to second-period contingent coverages and premiums. The formal problem consists of maximizing the low-risk policyholder's two-period expected utility under the incentive compatibility constraints, the nonnegative intertemporal expected profits constraint, and the no-switching constraints. By assuming that the insurers commit to a two-period contract but the contract is not binding on the insureds, they show that the presence of a second-period competition limits the use of experience rating as a sorting device. At equilibrium, high risk individuals obtain full insurance coverage and are not experience rated, whereas low risk individuals receive only partial insurance coverage and are experience rated.

Dionne and Doherty (1994) introduce the phenomenon of renegotiation in long-term relationships in insurance markets. In a similar vein to Cooper and Hayes (1987), two-period contracts are considered where the insureds can leave the relationship at the end of the first period and only the insurer is bound by a multi-period agreement. The difference with Cooper and Hayes' model is in the possibility of renegotiation. Indeed, insurers are allowed to make a proposition to alter the contract with their insureds, which could have been accepted or rejected. Dionne and Doherty present an alternative model (extending Laffont & Tirole's, 1990, procurement model) (Note 8), which involves semi-pooling in the first period followed by separation in the second. In their model, two contracts are offered. One contract is selected only by high risks and the other by both risk types, thus only the high risks can randomize over two contracts. Dionne and Doherty conclude that partial coverage is offered in the first-period semi-pooling contract along with full coverage being offered to high risks in the second period. Further, both high risks and low risks are experience rated in the second period.

Other models of multi-period insurance markets are not as closely related to this work as that of Cooper and Hayes (1987) and Dionne and Doherty (1994). For example, Nilssen (2000) focuses on consumer lock-in under a no-commitment assumption and illustrates that an equilibrium may exist with full pooling in the first period and consumer lock-in in the second period.

In this paper, I build a two-period model in a competitive insurance market with separation in both periods. This model, as an extension of Cooper and Hayes (1987) and Dionne and Doherty (1994), reveals that an accident forgiveness policy offered in the market induces policyholder's willingness to be experience rated. My model shares the basic feature of the previous paper (e.g., Dionne & Doherty, 1994) regarding the multi-period insurance contracts; that is two contracts are offered in the market and asymmetric information exists between insureds and insurers. However, my model differs in several respects. I focus primarily on providing a model with full separation in both periods (Note 9). Contracts are allowed to be selected by both high risks and low risks, which means that both types can ``randomize'' over the contracts with accident forgiveness. In the second period, both low risks as well as high risks will be experience rated instead of just low risks, as shown in Cooper and Hayes (1987). Finally, my main difference lies in the analysis of insurance purchase decisions. The findings in this paper indicate that the discount factor between periods and the degree of risk aversion are important determinants to the accident forgiveness purchases.

3. Accident Forgiveness

When one is involved in any at-fault accident (or traffic violation), points against the driving record are added into the insurer's experience rating system depending on the description of the accident (or traffic violation) and the insurer's rating system. Surcharges or discounts on premiums are based on the driving record. The more points one has, the worse the driving record becomes, and the higher the premium is. For example, if one has an at-fault accident, according to the current surchargeable point schedule in Massachusetts the driving record might increase three to four points depending on the claim amount (Note 10). However, with an accident forgiveness policy, the points do not increase as much, if at all. By protecting the driving record, accident forgiveness results in a reduction of the auto insurance premium.

In short, the availability of accident forgiveness varies by company. If available, it is simply a built-in feature of an insurer's regular auto insurance policy, or it is purchased as an option on the policy. Even if this feature is provided, it does not mean that if an accident occurred before the purchase it would be forgiven. Instead, it means that if the accident were to occur in the future it would be forgiven upon the conditions and terms specified in the insurance contract. The number of at-fault accidents allowed to be forgiven varies by the insurer providing this feature (Note 11). Even if one has accident forgiveness in a policy, having an accident “forgiven” by an insurance company does not mean the accident is totally omitted from one's driving record. The accident is on one's driving record even though the insurer offering the "forgiveness" may not currently consider it when calculating the auto insurance premium (Note 12).

4. The Model

4.1 Model Assumptions and the Sequence of the Game

I consider a two-period insurance market. Individuals (or insureds) are assumed to be risk averse, and in each period individuals are subject to a risk of financial loss. Individuals differ solely by their probability of loss. High risk individuals have a higher probability of experiencing a loss (accident) than low risk individuals. The respective proportions of high and low risks are assumed to permit a single-period Nash separating equilibrium to exist (Note 13). Risk type is private information to the individuals, and accidents are out of the individual's control so that no moral hazard arises. Insureds share the same von Neumann-Morgenstern utility function with the same per-period income (Note 14). Two insurers (the initial insurer and the rival insurer) in the market compete to attract insureds and are assumed to be risk neutral. I assume that the incumbent initial insurer is the only one in the market to offer accident forgiveness in the contracts and observes its policyholders' loss experience (there is no underreporting of accidents). Moreover, borrowing or lending by the insureds is not permitted, but the insureds are allowed to switch between insurers at no cost.

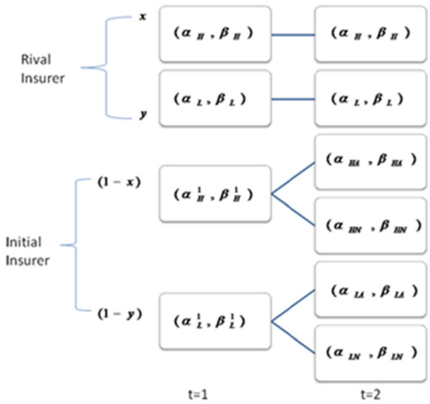

Figure 1. The depiction of two-period model

The game depicted in Figure 1 runs as follows:

1) At the beginning of the first period, the initial insurer offers the accident forgiveness contracts with premium α_H^1 and coverage β_H^1 to high risk individuals and premium α_L^1 and coverage β_L^1 to low risk individuals.

2) The rival insurer simultaneously offers the optimal one-period Rothschild/Stiglitz contracts with α_H and β_H as premium and coverage to high risk individuals and, α_L and β_L to low risk individuals. The optimal one-period Rothschild/Stiglitz contracts can be summarized as: the high risk individual receives full insurance; the low risk individual receives less than full insurance; the high risk individual is indifferent between his contract and that for the low risk (Note 15).

3) Individuals choose among available contracts. (1 - x) refers to the proportion of high risk individuals purchasing accident forgiveness contracts from the initial insurer, and (1 - y) is the proportion for low risk individuals. Premiums are paid, and the first period ends. Wealth losses are realized, and the insureds who experience a loss are compensated according to the first-period component of their contracts by getting the net reimbursement (Note 16).

4) At the beginning of the second period, contingent on having the first-period accident or not, the initial insurer offers the existing insureds four different contracts: α_{HA} and β_{HA} as premium and coverage for high risks having an accident in the previous period, α_{HN} and β_{HN} as premium and coverage for high risks having no accident in the previous period, α_{LA} and β_{LA} as premium and coverage for low risks having an accident in the previous period, and α_{LN} and β_{LN} as premium and coverage for low risks having no accident in the previous period.

5) The rival insurer again offers repeated one-period contracts at the beginning of the second period.

6) Individuals choose to either continue their contracts with the initial insurer or to switch to the one-period contracts provided by the rival insurer. Premiums are paid, and the second period elapses: wealth losses occur and are compensated according to the contracts.

4.2 Model Setup

The derivation of the optimal contract is obtained by maximizing the following problem (Note 17):

$$\begin{aligned}\max_{\alpha,\beta}\ & p_L[U(W-D+\beta_L^1)+\delta[p_LU(W-D+\beta_{LA})+(1-p_L)U(W-\alpha_{LA})]]\\ & +(1-p_L)[U(W-\alpha_L^1)+\delta[p_LU(W-D+\beta_{LN})+(1-p_L)U(W-\alpha_{LN})]]\end{aligned} \tag{1}$$

which is the utility of low risks purchasing an accident forgiveness contract in the first period and remaining with the same initial insurer for the second period. This maximization problem is subject to the randomization constraints as:

$$\begin{aligned}& p_H[U(W-D+\beta_H^1)+\delta[p_HU(W-D+\beta_{HA})+(1-p_H)U(W-\alpha_{HA})]]\\ & +(1-p_H)[U(W-\alpha_H^1)+\delta[p_HU(W-D+\beta_{HN})+(1-p_H)U(W-\alpha_{HN})]]\\ & =(1+\delta)[p_HU(W-D+\beta_H)+(1-p_H)U(W-\alpha_H)],\end{aligned} \tag{2}$$

$$\begin{aligned}& p_L[U(W-D+\beta_L^1)+\delta[p_LU(W-D+\beta_{LA})+(1-p_L)U(W-\alpha_{LA})]]\\ & +(1-p_L)[U(W-\alpha_L^1)+\delta[p_LU(W-D+\beta_{LN})+(1-p_L)U(W-\alpha_{LN})]]\\ & =(1+\delta)[p_LU(W-D+\beta_L)+(1-p_L)U(W-\alpha_L)];\end{aligned} \tag{3}$$

the self-selection constraints as:

$$\begin{aligned}& p_H[U(W-D+\beta_H^1)+\delta[p_HU(W-D+\beta_{HA})+(1-p_H)U(W-\alpha_{HA})]]\\ & +(1-p_H)[U(W-\alpha_H^1)+\delta[p_HU(W-D+\beta_{HN})+(1-p_H)U(W-\alpha_{HN})]]\\ & \geq(1-x)\{p_H[U(W-D+\beta_L^1)+\delta[p_HU(W-D+\beta_{LA})+(1-p_H)U(W-\alpha_{LA})]]\\ & +(1-p_H)[U(W-\alpha_L^1)+\delta[p_HU(W-D+\beta_{LN})+(1-p_H)U(W-\alpha_{LN})]]\}\\ & +x(1+\delta)[p_HU(W-D+\beta_L)+(1-p_H)U(W-\alpha_L)],\end{aligned} \tag{4}$$

$$\begin{aligned}& p_L[U(W-D+\beta_L^1)+\delta[p_LU(W-D+\beta_{LA})+(1-p_L)U(W-\alpha_{LA})]]\\ & +(1-p_L)[U(W-\alpha_L^1)+\delta[p_LU(W-D+\beta_{LN})+(1-p_L)U(W-\alpha_{LN})]]\\ & \geq(1-y)\{p_L[U(W-D+\beta_H^1)+\delta[p_LU(W-D+\beta_{HA})+(1-p_L)U(W-\alpha_{HA})]]\\ & +(1-p_L)[U(W-\alpha_H^1)+\delta[p_LU(W-D+\beta_{HN})+(1-p_L)U(W-\alpha_{HN})]]\}\\ & +y(1+\delta)[p_LU(W-D+\beta_H)+(1-p_L)U(W-\alpha_H)];\end{aligned} \tag{5}$$

the accident forgiveness constraints as (Note 18):

$$p_HU(W-D+\beta_{HA})+(1-p_H)U(W-\alpha_{HA})=p_HU(W-D+\beta_H)+(1-p_H)U(W-\alpha_H) \tag{6}$$

$$p_LU(W-D+\beta_{LA})+(1-p_L)U(W-\alpha_{LA})=p_LU(W-D+\beta_L)+(1-p_L)U(W-\alpha_L) \tag{7}$$

and the zero-profit constraint for the insurer as:

$$\begin{aligned}& (1-x)\{[(1-p_H)\alpha_H^1-p_H\beta_H^1]+\delta[(1-p_H)[(1-p_H)\alpha_{HN}-p_H\beta_{HN}]+p_H[(1-p_H)\alpha_{HA}-p_H\beta_{HA}]]\}\\ & +(1-y)\{[(1-p_L)\alpha_L^1-p_L\beta_L^1]+\delta[(1-p_L)[(1-p_L)\alpha_{LN}-p_L\beta_{LN}]+p_L[(1-p_L)\alpha_{LA}-p_L\beta_{LA}]]\}\\ & \geq 0.\end{aligned} \tag{8}$$

Constraints shown in expression (2) and expression (3) are randomization constraints that ensure both low and high risks are indifferent between their own one-period contracts and the accident forgiveness contracts. With these constraints, the insureds with the same risk type will randomize over different contracts. Expression (4) is the self-selection constraint for high risks and guarantees that high risks will not mimic low risks. High risks only prefer a randomization over the one-period contract and the accident forgiveness contract for high risks to a randomization over contracts for low risks. Expression (5) is the self-selection constraint for low risks. The purpose of purchasing accident forgiveness is to allow policyholders to have at-fault accidents without a premium increase. Expressions (6) and (7) are accident forgiveness constraints and illustrate that the initial insurer providing accident forgiveness contracts must keep its promise to offer its policyholder who has the first-period accident the same second-period expected utility as the one he can obtain from the uninformed rival insurer. Expression (8) as a zero-profit constraint prevents insurers from offering contracts at a loss.

A policy designed for high risks will obviously offer full insurance as partial insurance would simply provide incentives for imitation and offer an opportunity for renegotiation (Note 19). I can rewrite the self-selection constraints for high risk individuals (4) and low risk individuals (5) as

$$\begin{aligned} & U(W-\alpha_H^1)+\delta[p_H U(W-\alpha_{HA})+(1-p_H)U(W-\alpha_{HN})] \\ & \geq (1-x)\{p_H[U(W-D+\beta_L^1)+\delta[p_H U(W-D+\beta_{LA})+(1-p_H)U(W-\alpha_{LA})]] \\ & +(1-p_H)[U(W-\alpha_L^1)+\delta[p_H U(W-D+\beta_{LN})+(1-p_H)U(W-\alpha_{LN})]]\} \\ & +x(1+\delta)[p_H U(W-D+\beta_L)+(1-p_H)U(W-\alpha_L)] \end{aligned} \tag{4'}$$

and

$$\begin{aligned} & p_L[U(W-D+\beta_L^1)+\delta[p_L U(W-D+\beta_{LA})+(1-p_L)U(W-\alpha_{LA})]] \\ & +(1-p_L)[U(W-\alpha_L^1)+\delta[p_L U(W-D+\beta_{LN})+(1-p_L)U(W-\alpha_{LN})]] \\ & \geq (1-y)\{U(W-\alpha_H^1)+\delta[p_L U(W-\alpha_{HA})+(1-p_L)U(W-\alpha_{HN})]\} \\ & +y(1+\delta)U(W-\alpha_H), \end{aligned} \tag{5'}$$

respectively.

Similarly, the randomization constraint (2), the accident forgiveness constraint (6) and the zero-profit constraint (8) can be also rewritten as

$$U(W-\alpha_H^1)+\delta[p_H U(W-\alpha_{HA})+(1-p_H)U(W-\alpha_{HN})]=(1+\delta)U(W-\alpha_H) \tag{2'}$$

$$U(W-\alpha_{HA})=U(W-\alpha_H) \tag{6'}$$

$$\begin{aligned} & (1-x)\{[(\alpha_H^1-p_H D)]+\delta[(1-p_H)(\alpha_{HN}-p_H D)+p_H(\alpha_{HA}-p_H D)]\} \\ & +(1-y)\{[(1-p_L)\alpha_L^1-p_L\beta_L^1]+\delta[(1-p_L)[(1-p_L)\alpha_{LN}-p_L\beta_{LN}]+p_L[(1-p_L)\alpha_{LA}-p_L\beta_{LA}]]\} \\ & \geq 0. \end{aligned} \tag{8'}$$

4.3 Optimal Contracts with Asymmetric Information

4.3.1 Optimal Contracts Characterization

To characterize the optimal contracts by solving the maximization problem, I need to determine the binding of the constraints. Due to the fact that competition and freedom of entry without transaction costs ensure that the profit for each type of contract will be driven to zero, the zero-profit constraint is binding with Lagrangian multiplier $\lambda_z > 0$.

Binding the self-selection constraint with $\lambda_{hss} > 0$ for high risks is necessary to ensure that the high risks do not select the contracts for low risks. As usual, the constraint for low risks cannot possibly be binding because if the high risks are indifferent between two contracts, then the low risks will strictly prefer their own contracts as only the high risks have incentives to mimic the low risks (Note 20). I have $\lambda_{lss} > 0$.

Because the accident forgiveness constraints are binding, it follows

$$\alpha_{HA}=\alpha_H, \alpha_{LA}=\alpha_L, \beta_{HA}=\beta_H, \beta_{LA}=\beta_L \tag{9}$$

With the method of Lagrange multipliers, I substitute these values into the problem to simplify the derivation and take the first-order condition of the maximization problem. Then I get

$$U'(W-\alpha_L^1)=\frac{\lambda_z(1-p_L)(1-y)}{(1-p_L)+\lambda_{lr}(1-p_L)-\lambda_{hss}(1-p_H)(1-x)} \tag{10}$$

$$U'(W-D+\beta_L^1)=\frac{\lambda_z p_L(1-y)}{p_L+\lambda_{lr}p_L-\lambda_{hss}p_H(1-x)} \tag{11}$$

$$U'(W-\alpha_{LN})=\frac{\lambda_z(1-p_L)^2(1-y)}{(1-p_L)^2+\lambda_{lr}(1-p_L)^2-\lambda_{hss}(1-p_H)^2(1-x)} \tag{12}$$

$$U'(W-D+\beta_{LN})=\frac{\lambda_z p_L(1-p_L)(1-y)}{(1-p_L)p_L+\lambda_{lr}(1-p_L)p_L-\lambda_{hss}p_H(1-p_H)(1-x)} \tag{13}$$

Because I have $p_L < p_H$, comparing $U'(W-\alpha_L^1)$ from (10) and $U'(W-D+\beta_L^1)$ from (11) yields

$$U'(W-\alpha_L^1) < U'(W-D+\beta_L^1)$$

and with the assumption of utility function, $U' > 0$ and $U'' < 0$, I have

$$U(W-\alpha_L^1) > U(W-D+\beta_L^1) \tag{14}$$

which identifies the partial coverage for $\alpha_L^1, \beta_L^1)$ as the contract offered in the first period.

Similarly, using $U'(W-\alpha_{LN})$ from (12) and $U'(W-D+\beta_{LN})$ from (13), it shows

$$U(W-\alpha_{LN}) > U(W-D+\beta_{LN}) \tag{15}$$

which also points to the partial coverage for the contract $(\alpha_{LN}, \beta_{LN})$ offered in the second period to low risk individuals having no loss in the first period.

To determine α_{HN} as the premium for high risks with no loss in the first period, I adopt the concept of "rent-constraint contracts," which was proposed in Laffont and Tirole (1990) (Note 21). Here is the idea. Considering that high risks would receive the full-insurance, fair- priced policy in the repeated one-period contracts from the rival insurer, they would deviate to adopt the strategy of accident forgiveness contracts, which involves an "up-front" premium in the first period if the expectation of cost is no higher than the actuarially fair price from the repeated one-period contracts. Thus, to participate in the accident forgiveness contracts, high risks having no first-period loss must receive a rent that is implicitly embodied in the premium α. With competition, the insurer cannot offer a rent to the high risks greater than that corresponding to the transfers paid in the first period, which are determined by the value of x and y. Hence, the premium α_{HN} can be written as $\alpha_{HN}(x,y)$, and the rent given to the high risk individuals who suffer no first-period loss can be derived from:

$$(1-x)(\alpha_H^1 - p_H D) + (1-y)[(1-p_L)\alpha_L^1 - p_L\beta_L^1] + \delta[(1-x)(1-p_H)(\alpha_{HN}(x,y) - p_H D)] = 0.$$

Solving for $\alpha_{HN}(x,y)$ yields

$$\alpha_{HN}(x,y) = p_H D - \frac{[(1-x)(\alpha_H^1 - p_H D) + (1-y)[(1-p_L)\alpha_L^1 - p_L\beta_L^1]]}{\delta(1-x)(1-p_H)} \tag{16}$$

which can be further written as

$$\alpha_{HN}(x,y) = p_H D - \frac{T(x,y)}{\delta(1-x)(1-p_H)} \tag{16'}$$

where

$$T(x,y) = (1-x)(\alpha_H^1 - p_H D) + (1-y)[(1-p_L)\alpha_L^1 - p_L\beta_L^1] \tag{17}$$

is the net transfer paid to the insurer in the first period from both types. Obviously, $T(x,y)=0$ when $x=1$ and $y=1$ (For example, no one purchases the accident forgiveness policy).

Here, I assume that the rent paid to the low risks in the second period is zero (Note 22). It is seen that low risks with no loss in the first period will get an actuarially fair premium α_{LN}. Moreover, to induce them to participate in the accident forgiveness contracts, more coverage β_{LN} will be given in the second period.

Apparently, the rent offered to high risks to participate in this contract is from both intertemporal and cross-sectional subsidization. In other words, only if the insurer makes a profit in the first period by charging a higher-than-market premium would it support the rent given to the policyholders in the second period, which results in $\alpha_H^1 > \alpha_{HA} > \alpha_{HN}$ and $\alpha_L^1 > \alpha_{LA} > \alpha_{LN}$. The high "upfront" premium for the two-period contract may be thought of as an entry cost to the accident forgiveness contract with an experience-rating pricing scheme.

Therefore, for given probabilities x $(0 < x < 1)$ and y $(0<y<1)$ that both high risks and low risks will separate, the optimal two-period contract with accident forgiveness under competitive conditions and with the insureds' switching contracts permitted is characterized as follows.

A separating policy exists for the first period, which is full coverage for high risks and partial coverage for low risks. This is consistent with the one-period Rothschild / Stiglitz contracts. However, a higher-than-market premium is charged for both types. A separating policy exists for the second period, which is full coverage for high risks even if they suffered first-period losses and a partial coverage for low risks.

Pricing, however, is different. An experience rated second-period policy is given for low risks; that is, low risks who suffered no first-period loss receive more coverage with an actuarially fair premium and those who did suffer losses receive the Rothschild/Stiglitz contracts for low risks in the second period. High risks also obtain an experience rated second-period policy; that is, high risks who suffered no first-period loss receive the rent and those who suffered losses receive the Rothschild/Stiglitz contracts for high risks in the second period. The rent is from both intertemporal and cross-sectional subsidization.

4.3.2 Comments

To more fully characterize the optimal contract, I add some comments for three important issues.

Asymmetric information. When information about previous driving records is not pooled across insurers, it means that if an accident or violation occurred it is only observed by the initial insurer responsible for covering it but not by other rival insurers, thus asymmetries of information arise (Note 23). Contrary to what is believed by many to be common practice (e.g., automobile accidents as well as traffic violations are complete and freely available), states vary in the accident reporting regulations (see Table 2), and information maintained by state agencies, such as Motor Vehicle Records (MVRs), is not always available and often is far from complete. More specifically, in some states, driving under the influence of alcohol (DUI) and other major traffic-related convictions can even be erased (see Table 3) from one's motor vehicle record. The source of asymmetries of information might also come from the time lag in the learning process between the initial insurer and its rivals. In other words, the initial insurer, in some sense, might be thought of as the Stackelberg leader in the updating process. It is likely for the initial insurer to have a comparative advantage over rivals in monitoring its own policyholders. Over time, the initial insurer obtains Bayesian updates on its policyholders' loss distributions that are not simultaneously available to the rivals. For example, if the insured vehicle is involved in an accident, it usually takes some time for the rival insurers to access this information while the initial insurer is required to be notified immediately. Moreover, through contractual relationship with its policyholders, the initial insurer may also learn of more relevant risk-related personal information, such as medical history, which is unobservable to other insurers.

Insurer's commitment to the contracts. In the way my model has been set up, it is assumed that insurers could commit to the insurance contract either through enforced legislation or reputational effects. This is consistent with the general provision in the personal auto policy drafted by the Insurance Services Office (Note 24). In the termination provision, the named insured can cancel at any time by returning the policy to the insurer. The insurer also has the right of cancelation but for only three reasons (Note 25): (1) the premium has not been paid, (2) the driver's license of any insured has been suspended or revoked, or (3) the policy was obtained through material misrepresentation. Besides, many states place additional restrictions on the insurer's right to cancel or not renew an auto insurance policy (e.g., the state law may require a longer period of advance notice to the insured).

Table 2. State accident reporting requirements

State	Reported to DMV	Reported to Law Enforcement
Alabama, Alaska, Arkansas, Indiana, Missouri, Nevada, New Hampshire, Ohio, Oklahoma, Pennsylvania, Rhode Island, Vermont, Wyoming	Yes	No
Arizona, Delaware, Georgia, Hawaii, Idaho, Kansas, Kentucky, Louisiana, Maine, Michigan, Montana, Nebraska, North Carolina, North Dakota, South Dakota, Virginia	No	Yes
California, Colorado, Florida, Illinois, Iowa, Maryland, Massachusetts, Minnesota, New Jersey, New Mexico, New York, Oregon, South Carolina, Tennessee, Texas, Utah, Washington, West Virginia, Wisconsin	Yes	Yes
Connecticut, Mississippi, Washington DC	No	No

Note. Adapted from state DMV websites.

The degree of "punishment". The prior literature has already illustrated that the presence of second-period competition for consumers might limit but not destroy the use of an experience rating as a sorting device (e.g., Cooper & Haye, 1987). As I assume semi-commitment settings, policyholders are not bound to the insurer. This results in the punishment for first- period accident being tempered by the presence of rival insurers offering one-period contracts in the second period. Because of this, someone may argue that the insured does not necessarily need to purchase accident forgiveness to be relieved from the previous accident. However, I need to understand that accident forgiveness as an insurance policy feature offered at the beginning of the contracting in this model not only protects insureds from higher future premiums but also rewards insureds with more favorable contract terms. In other words, accident forgiveness is the device to temper the experience rating as well as lower the incentive to switch.

Table 3. State DMV DUI expungement condition

State	DUI Expungement	Condition
Alabama, Alaska, Arizona, Arkansas, Hawaii, Idaho, Illinois, Iowa, Kansas, Kentucky, Louisiana, Maine, Massachusetts, Minnesota, Mississippi, Montana, Nebraska, New Mexico, New York, North Dakota, Ohio, Oklahoma, Oregon, Rhode Island, South Carolina, Tennessee, Texas, Vermont, Washington, West Virginia, Wisconsin, Wyoming, Washington DC	No	No
California	Yes	Complete all the conditions of your DUT sentence.
Colorado	Yes	Only if the DUI happened before you turned 21 and you have no other convictions to be expunged.
Connecticut	Yes	Wait 3 years if the DUI was a misdemeanor; wait 5 years if it was a felony.
Delaware	Yes	You can only expunge the DUI if an acquittal or dismissal terminated the underlying charge.
Florida	Yes	Only an option if the DUI charge didn't involve manslaughter.
Georgia	Yes	You must have no other pending criminal charges and no other convictions of the same or similar crime in the last 5 years.
Indiana	Yes	Only if your case was reversed or dismissed.
Maryland	Yes	Only if your case was dismissed or a judge or jury acquitted you.
Michigan	Yes	The court decides on a case-by-case basis.
Missouri	Yes	Only if it was your first DUI
Nevada	Yes	Only if the DUI was not a felony.
New Hampshire	Yes	After 10 years.
New Jersey	Yes	All expungements are considered as misdemeanors.
North Carolina	Yes	If you're found not guilty or have a criminal charge dismissed.
Pennsylvania	Yes	As long as your license wasn't revoked for being a habitual offender and you weren't a commercial driver at the time.
South Dakota	Yes	Varies by county.
Utah	Yes	After 10 years, as long as the conviction wasn't a felony.
Virginia	Yes	Only if the charges were dropped, you were acquitted, or you received an absolute pardon.

Note. Adapted from state DMV websites.

5. Purchase Decision

One more question worth asking is if there is any essential element that affects the accident forgiveness purchases. The economic explanation of insurance purchase is a story of shifting risk. For consumers, insurance purchases can be conceptualized as decisions in which they are faced with risks that have some distributions of losses across probabilities. To reduce these risks, consumers pay premiums and are compensated by benefits if the losses occur.

Prior literature examining the determinants for consumers' insurance purchase decisions mostly emphasizes how product quality, switching cost, and price affect consumers' decisions (e.g., Cummins et al., 1974; Dahlby & West, 1986; Laury & McInnes, 2003; Schlesinger & Schulenburg, 1993) or argues that distorted beliefs concerning the probability and size of potential losses affect consumers' decisions about insurance (e.g., Johnson et al., 1993; Kunreuther & Pauly 2004, 2005). However, insurance decision-making as behavior under uncertainty might involve time discounting and risk attitude. In this paper, the importance of individual risk and time preferences related to the insurance purchase decision is investigated.

This section completes the derivation of the optimal contracts by determining the probability of purchasing accident forgiveness contracts as a function of the discount factor and the degree of risk aversion.

5.1 Discount Factor

The discount factor is a provocative subject with important implications for many aspects of economic behavior and public policy (e.g., Warner & Pleeter, 2001). In particular, the discount factor is essential in making purchase decisions. The macroeconomics literature has provided evidence showing the relationship between the discount factor and life insurance purchase in a life cycle model (e.g., Fischer, 1973; Yaari, 1965) (Note 26). Articles related to dynamic insurance contracts also illustrate the importance of the discount factor. Rubinstein and Yaari (1983) show that multi-period insurance contracts can increase the welfare of both the insurer and the insured when the number of periods is large and the discount rate is small. Dionne and Doherty (1994) demonstrate the positive relationship between the discount factor and the high risk drivers' participation in the _rst-period pooling insurance. Kunreuther (1996) uses the discount factor to explain why individuals had limited interest in voluntary insurance purchases.

In multi-period contracting models with asymmetric information, the discount factor is very important such that it may affect the optimal allocation in equilibrium. Laffont and Tirole (1990) complete their derivation of the optimal procurement contract by proposing that the proportion of good types' separation from a semi-pooling contract does not increase with the discount factor (Note 27). Dionne and Doherty (1994) share the same feature of the nonincreasing theorem but for an optimal insurance contract. Here, I posit that if both low risks and high risks are introduced to randomize over different contracts there exists a nondecreasing effect of the discount factor on the accident forgiveness purchases for both types.

Proposition 1. The proportion of policyholders, for both high risk and low risk, who purchase contracts with accident forgiveness is nondecreasing with the discount factor.

Proof. Suppose y is the low risks' equilibrium randomizing probability for a given discount factor δ and similarly $\tilde{y}$ is for $\tilde{\delta}$. Assume that $\delta < \tilde{\delta}$, then low risks' utility for two periods with optimal y for δ can be written as:

$$U(y,\delta,\alpha,\beta) = (1-y)\{U(\alpha_L^1,\beta_L^1)+\delta[p_L U(\alpha_{LA},\beta_{LA})+(1-p_L)U(\alpha_{LN},\beta_{LN})]\}+y\{(1+\delta)U(\alpha_L,\beta_L)\}; \tag{18}$$

utility with optimal $\tilde{y}$ for $\tilde{\delta}$ can be written as:

$$U(\tilde{y},\tilde{\delta},\alpha,\beta) = (1-\tilde{y})\{U(\alpha_L^1,\beta_L^1)+\tilde{\delta}[p_L U(\alpha_{LA},\beta_{LA})+(1-p_L)U(\alpha_{LN},\beta_{LN})]\}+\tilde{y}\{(1+\tilde{\delta})U(\alpha_L,\beta_L)\}; \tag{19}$$

utility with $\tilde{y}$ for δ can be written as:

$$U(\tilde{y},\delta,\alpha,\beta) = (1-\tilde{y})\{U(\alpha_L^1,\beta_L^1)+\delta[p_L U(\alpha_{LA},\beta_{LA})+(1-p_L)U(\alpha_{LN},\beta_{LN})]\}+\tilde{y}\{(1+\delta)U(\alpha_L,\beta_L)\}; \tag{20}$$

and utility with y for $\tilde{\delta}$ can be written as:

$$U(y,\tilde{\delta},\alpha,\beta) = (1-y)\{U(\alpha_L^1,\beta_L^1)+\tilde{\delta}[p_L U(\alpha_{LA},\beta_{LA})+(1-p_L)U(\alpha_{LN},\beta_{LN})]\}+y\{(1+\tilde{\delta})U(\alpha_L,\beta_L)\}. \tag{21}$$

As y is an optimum for δ and $\tilde{y}$ is an optimum for $\tilde{\delta}$, I have $U(y,\delta,\alpha,\beta) \geq U(\tilde{y},\delta,\alpha,\beta)$ and $U(\tilde{y},\tilde{\delta},\alpha,\beta) \geq U(y,\tilde{\delta},\alpha,\beta)$. This yields

$$U(y,\delta,\alpha,\beta) + U(\tilde{y},\tilde{\delta},\alpha,\beta) \geq U(\tilde{y},\delta,\alpha,\beta) + U(y,\tilde{\delta},\alpha,\beta).$$

So, adding (18) and (19), and then subtracting (20) and (21) yields

$$\begin{aligned}
&\{U(\alpha_L^1,\beta_L^1) + \delta[p_L U(\alpha_{LA},\beta_{LA}) + (1-p_L)U(\alpha_{LN},\beta_{LN})]\}(\tilde{y}-y)\\
&+\{(1+\delta)U(\alpha_L,\beta_L)\}(y-\tilde{y})\\
&+\{U(\alpha_L^1,\beta_L^1) + \tilde{\delta}[p_L U(\alpha_{LA},\beta_{LA}) + (1-p_L)U(\alpha_{LN},\beta_{LN})]\}(y-\tilde{y})\\
&+\{(1+\tilde{\delta})U(\alpha_L,\beta_L)\}(\tilde{y}-y)\\
&\geq 0,
\end{aligned} \tag{22}$$

and (22) can be further written as

$$(y-\tilde{y})(\tilde{\delta}-\delta)[p_L U(\alpha_{LA},\beta_{LA}) + (1-p_L)U(\alpha_{LN},\beta_{LN}) - U(\alpha_L,\beta_L)] \geq 0 \tag{22'}$$

From the optimal contract characterization, $U(\alpha_{LA},\beta_{LA}) = U(\alpha_L,\beta_L)$, $U(\alpha_{LN},\beta_{LN}) > U(\alpha_L,\beta_L)$, and $\tilde{\delta} > \delta$ as

I assume, it is obvious that $y \geq \tilde{y}$. Because y and $\tilde{y}$ are optimum for β and $\tilde{\delta}$, respectively, $(1-\tilde{y}) \geq (1-y)$ proves the nondecreasing relationship between the discount factor and the proportion of low risks purchasing the accident forgiveness contracts. In a similar vein, using the same procedure, I can prove this nondecreasing relationship for high risk policyholders as well.

This proposition states that when faced with a higher discount factor, the insured becomes more willing to pay extra money to have contracts with accident forgiveness. Here is the favor of intuition. When the discount factor is low, it is costly for the insured to pay a positive transfer in the first period to increase insurance possibilities in the second period. However, when the discount factor is high, compared to the rival insurer's one-period contract offered in the market, accident forgiveness policy not only protects the insured from a higher insurance rate if he experiences losses in the first period but also rewards him in the second period for having no loss. Individual who cares more about his second-period expected utility obviously prefers to purchase this accident forgiveness policy. Thus, it is clear that accident forgiveness purchase decision, to some extent, is driven by the discount factor.

5.2 Risk Aversion

I further turn my attention to investigating how accident forgiveness purchases are affected by the individual risk attitude.

The standard economic theory is that risk averse individuals confronted with sizable hazards are willing to pay a more diversified insurer to bear the risk (e.g., Dionne & Harrington, 1992). Schlesinger and Schulenberg (1987) argue that in the usual insurance literature because a higher degree of risk aversion implies a greater relative emphasis on downside risk, an increase in the level of risk aversion leads to the purchase of a higher level of insurance coverage. Similar logic was employed by Johnson et al. (1993) who state that risk neutral consumers would purchase coverage at an actuarially fair price and risk aversion raises this reservation price. Ganderton et al. (2000) state that all risk neutral or risk averse individuals would purchase insurance and undertake all relevant precautions to the extent that the extra benefits from such actions exceed the marginal costs, less some risk premium in the case of risk aversion. Laury and McInnes (2003) elicit that if one is even slightly risk averse, that person should always purchase insurance.

The question arises as to whether insureds are going to purchase accident forgiveness for more protection if they become more risk averse. In other words, is standard economic theory also applicable to accident forgiveness purchases?

Proposition 2. There exists a threshold in terms of utility for both high risks and low risks above which the proportion of policyholders who purchase contracts with accident forgiveness is nondecreasing in their degree of risk aversion.

Proof. I assume CRRA utility function with $U_r(\alpha,\beta)=\frac{\omega(\alpha,\beta)^{1-\gamma}}{1-\gamma}$, where ω is a function of α and β and $\gamma \neq 1$ for convenience (Note 28). γ is the parameter to measure the risk aversion with $\gamma = 0$ corresponding to risk neutrality, $\gamma < 0$ to risk loving, and $\gamma > 0$ to risk averse. Suppose I have γ_1 and γ_2 with $\gamma_1 > \gamma_2 > 0$. As usual, if γ_1 and γ_2 are assumed to have optimum y_1 and y_2, the low risks' utility for two periods can be written as:

$$\begin{aligned} U(y_1,\gamma_1,\alpha,\beta) = \; & (1-y_1)\{U_{\gamma_1}(\alpha_L^1,\beta_L^1) + \delta[p_L U_{\gamma_1}(\alpha_{LA},\beta_{LA}) + (1-p_L)U_{\gamma_1}(\alpha_{LN},\beta_{LN})]\} \\ & + y_1\{(1+\delta)U_{\gamma_1}(\alpha_L,\beta_L)\}, \end{aligned} \tag{23}$$

$$\begin{aligned} U(y_2,\gamma_2,\alpha,\beta) = \; & (1-y_2)\{U_{\gamma_2}(\alpha_L^1,\beta_L^1) + \delta[p_L U_{\gamma_2}(\alpha_{LA},\beta_{LA}) + (1-p_L)U_{\gamma_2}(\alpha_{LN},\beta_{LN})]\} \\ & + y_2\{(1+\delta)U_{\gamma_2}(\alpha_L,\beta_L)\}, \end{aligned} \tag{24}$$

$$\begin{aligned} U(y_2,\gamma_1,\alpha,\beta) = \; & (1-y_2)\{U_{\gamma_1}(\alpha_L^1,\beta_L^1) + \delta[p_L U_{\gamma_1}(\alpha_{LA},\beta_{LA}) + (1-p_L)U_{\gamma_1}(\alpha_{LN},\beta_{LN})]\} \\ & + y_2\{(1+\delta)U_{\gamma_1}(\alpha_L,\beta_L)\}, \end{aligned} \tag{25}$$

$$\begin{aligned} U(y_1,\gamma_2,\alpha,\beta) = \; & (1-y_1)\{U_{\gamma_2}(\alpha_L^1,\beta_L^1) + \delta[p_L U_{\gamma_2}(\alpha_{LA},\beta_{LA}) + (1-p_L)U_{\gamma_2}(\alpha_{LN},\beta_{LN})]\} \\ & + y_1\{(1+\delta)U_{\gamma_2}(\alpha_L,\beta_L)\}. \end{aligned} \tag{26}$$

Adding (23) and (24), and then subtracting (25) and (26) yields

$$\begin{aligned}&(y_1 - y_2)\{U_{\gamma_2}(\alpha_L^1, \beta_L^1) + \delta[p_L U_{\gamma_2}(\alpha_{LA}, \beta_{LA}) + (1 - p_L)U_{\gamma_2}(\alpha_{LN}, \beta_{LN})]\\ &-U_{\gamma_1}(\alpha_L^1, \beta_L^1) - \delta[p_L U_{\gamma_1}(\alpha_{LA}, \beta_{LA}) + (1 - p_L)U_{\gamma_1}(\alpha_{LN}, \beta_{LN})]\\ &+(1 + \delta)U_{\gamma_1}(\alpha_L, \beta_L) - (1 + \delta)U_{\gamma_2}(\alpha_L, \beta_L)\}\\ &\geq 0.\end{aligned} \tag{27}$$

With $\alpha_{LA} = \alpha_L$and $\beta_{LA} = \beta_L$, (27) can be further rewritten as

$$\begin{aligned}&(y_1 - y_2)\{[U_{\gamma_2}(\alpha_L^1, \beta_L^1) - U_{\gamma_1}(\alpha_L^1, \beta_L^1)] - [U_{\gamma_2}(\alpha_L, \beta_L) - U_{\gamma_1}(\alpha_L, \beta_L)]\\ &+\delta(1 - p_L)\{[U_{\gamma_2}(\alpha_{LN}, \beta_{LN}) - U_{\gamma_1}(\alpha_{LN}, \beta_{LN})] - [U_{\gamma_2}(\alpha_L, \beta_L) - U_{\gamma_1}(\alpha_L, \beta_L)]\}\} \geq 0.\end{aligned} \tag{27'}$$

To predict the sign of $(y_1 - y_2)$ in (27'), we need to discuss the sign of $[U_{r_2}(\alpha_L^l,\beta_L^l)\text{-}U_{r_1}(\alpha_L^l,\beta_L^l)]\text{-}[U_{r_2}(\alpha_L,\beta_L)\text{-}U_{r_1}(\alpha_L,\beta_L)]+\delta(1\text{-}p_L)\{[U_{\gamma_2}(\alpha_{LN},\beta_{LN})(\alpha_{LN},\beta_{LN})]\text{-}[U_{r_2}(\alpha_L,\beta_L)\text{-}U_{r_1}(\alpha_L,\beta_L)]\}$.

To simplify the discussion, we define

$$g(\omega(\alpha, \beta)) = U_{\gamma_2}(\omega(\alpha, \beta)) - U_{\gamma_1}(\omega(\alpha, \beta)) \tag{28}$$

where $\omega(\alpha, \beta)$ represents different contracts (e.g., contract (α_L^1, β_L^1) or (α_L, β_L)). Then, we only need to consider the possible sign of

$$g(\omega(\alpha_L^1, \beta_L^1)) - g(\omega(\alpha_L, \beta_L)) + \delta(1 - p_L)[g(\omega(\alpha_{LN}, \beta_{LN})) - g(\omega(\alpha_L, \beta_L))] \tag{29}$$

The function $g(\omega)$ has the curvature shown in Figure 2(a). To discuss the sign of (29), let us derive both first and second-order derivatives. The first-order derivative $g'(\omega) = \omega^{-\gamma_2} - \omega^{-\gamma_1}$ is positive if ω is smaller than the cutoff point ϖ and negative (or g is decreasing) otherwise, as shown in Figure 2(b). The second-order derivative $-\gamma_2\omega^{-\gamma_2-1} - \gamma_1\omega^{-\gamma_1-1}$ is negative if ω is smaller than the cutoff point $\widehat{\omega}$ and positive otherwise, as shown in Figure 2(c). Although we know that $\omega(\alpha_{LN}, \beta_{LN}) > \omega(\alpha_L, \beta_L) > \omega(\alpha_L^1, \beta_L^1)$, the sign of (29) is also determined by the cutoff point ϖ and $\widehat{\omega}$. Here are three possible cases.

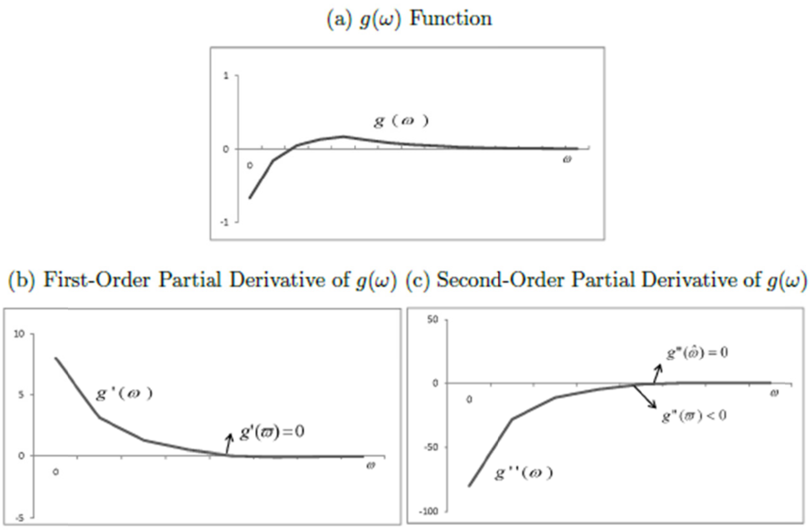

Figure 2. $g(w)$ Function and partial derivates of $g(w)$

Case 1: $\varpi > \omega(\alpha_{LN}, \beta_{LN}) > \omega(\alpha_L, \beta_L) > \omega(\alpha_L^1, \beta_L^1)$ (Note 29). It is straight-forward to see that

$$g(\omega(\alpha_L, \beta_L)) - g(\omega(\alpha_L^1, \beta_L^1)) > g(\omega(\alpha_{LN}, \beta_{LN})) - g(\omega(\alpha_L, \beta_L)) \tag{30}$$

from which

$$\begin{aligned}&[U_{\gamma_2}(\alpha_L^1, \beta_L^1) - U_{\gamma_1}(\alpha_L^1, \beta_L^1)] - [U_{\gamma_2}(\alpha_L, \beta_L) - U_{\gamma_1}(\alpha_L, \beta_L)]\\ &+\delta(1 - p_L)\{[U_{\gamma_2}(\alpha_{LN}, \beta_{LN}) - U_{\gamma_1}(\alpha_{LN}, \beta_{LN})] - [U_{\gamma_2}(\alpha_L, \beta_L) - U_{\gamma_1}(\alpha_L, \beta_L)]\} < 0\end{aligned} \tag{31}$$

And with (27'), $(1 - y_2) \leq (1 - y_1)$. This result illustrates the nonincreasing relation between the proportion of low risks purchasing accident forgiveness contracts and the degree of their risk aversion.

Case 2: $\bar{\omega} > \omega(\alpha_{LN}, \beta_{LN}) > \omega(\alpha_L, \beta_L) > \omega(\alpha_L^1, \beta_L^1) > \bar{\omega}$. With $g'(\omega) < 0$ and $g"(\omega) < 0$,

$$g(\omega(\alpha_L, \beta_L)) - g(\omega(\alpha_{LN}, \beta_{LN})) > g(\omega(\alpha_L^1, \beta_L^1)) - g(\omega(\alpha_L, \beta_L)) \tag{32}$$

which indicates the undetermined sign of $(y_1 - y_2)$.

Case 3: $\omega(\alpha_{LN}, \beta_{LN}) > \omega(\alpha_L, \beta_L) > \omega(\alpha_L^1, \beta_L^1) > \hat{\omega}$. With $g'(\omega) < 0$ and $g"(\omega) > 0$,

$$g(\omega(\alpha_L^1, \beta_L^1)) - g(\omega(\alpha_L, \beta_L)) > g(\omega(\alpha_L, \beta_L)) - g(\omega(\alpha_{LN}, \beta_{LN})) \tag{33}$$

which indicates that $(1 - y_2) \geq (1 - y_1)$. Unlike the previous two cases, this result points to a nondecreasing relation between the proportion of low risks purchasing accident forgiveness contracts and the degree of risk aversion. For those $\omega > \hat{\omega}$, a higher level of γ is associated with a lower level of y (or a higher level of $1 - y$).

Similar results can also be proven for high-risk policyholders.

The economic theory underlying the insurance predicts that more risk-averse drivers (e.g., good drivers with a clean driving history) will be more willing to purchase accident forgiveness policies. However, in practice, it is apparently not that simple. An informal survey among acquaintances shows that some good drivers hesitate to pay for accident forgiveness, even knowing that it protects their future premiums, whereas others consider it a great deal. This proposition identifies a situation where more risk-averse individuals are more likely to purchase accident forgiveness policy if the expected utility provided by this insurance contract is higher than a utility threshold (e.g., the individual's reservation utility).

6. Conclusions

This paper is devoted to studying accident forgiveness policy in the auto insurance contract by developing an asymmetric learning model. Assuming a two-period contract, with a higher-than-market premium charged in the first period, policyholders purchasing an accident forgiveness policy will be protected against an increased second-period premium in the event of an at-fault accident. If there is no accident, the policyholders are rewarded with more favorable contract terms in the second period. In this model, the experience rating can serve the same purpose although its effectiveness is tempered by the presence of accident forgiveness. By offering this feature to all insureds instead of only low risks in the pool, the initial insurer may have an advantage in attracting more customers.

When analyzing the accident forgiveness purchase decisions, I find that higher discount factor provides an incentive to the policyholders to purchase accident forgiveness as the possibility of having any at-fault accident becomes of more concern to the their future premium (or utility). Moreover, the examination of the impact of individual risk aversion on accident forgiveness purchases suggests that, contrary to the previous studies that suggest a significantly positive relationship between the risk aversion and the insurance purchase, there exists a threshold below which accident forgiveness will be less affected by the degree of risk aversion. This interesting finding might be helpful to provide a better understanding of why policyholders who are good drivers may not be willing to purchase accident forgiveness at some point.

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Notes

Note 1. "Your Choice Auto" consumers can choose from three new, optional packages: Platinum Protection package, Gold Protection package, and Allstate Value Plan. Based on their individual needs, consumers can choose among new features like accident forgiveness, new safe driving rewards, or enhanced protection for new cars.

Note 2. Accident forgiveness eligibility is also determined by state laws and regulations.

Note 3. "Farmers Flex" provides customers with a new set of options and features such as accident forgiveness and new car pledge package.

Note 4. Nini (2009) empirically investigates the claim reporting behavior in the auto insurance market that provides accident forgiveness to policyholders.

Note 5. Much effort has been spent on thinking about designing an auto insurance contract in a dynamic market with the existence of asymmetric information (e.g., Cooper & Hayes, 1987; Dionne & Doherty, 1994; Kunreuther & Pauly, 1985; Nilssen, 2000; Rothschild & Stiglitz, 1976).

Note 6. In this paper, I refer to this situation as semi-commitment.

Note 7. They implicitly assume that the conditions to obtain a Nash separating equilibrium in a single period contract were sufficient for an equilibrium to exist in their two-period model.

Note 8. Laffont and Tirole (1990) fully characterize the equilibrium of a two-period procurement model with commitment and renegotiation. They analyze whether renegotiated long-term contracts yield outcomes resembling those under either not renegotiated long-term contracts or a sequence of short-term contracts, and they link the analysis with the multiple unit durable good monopoly problem.

Note 9. Separation in the first period is exactly the phenomena that we observe in the automobile insurance market. For insurers, practically, it is hard for the insurers to pool different types of individuals together and offer them the same contracts.

Note 10. The following is the current surchargeable point schedule in Massachusetts (see Massachusetts official website of the Office of Consumer Affairs and Business Regulation at http://www.mass.gov): major traffic violation (such as DUI): 5 points; major at-fault accident (such as a claim over $2,000): 4 points; minor at-fault accident (claim of $500 to $2,000): 3 points; minor traffic violation (such as speeding): 2 points.

Note 11. In this paper, I simply assume that all at-fault accidents or traffic violations might be forgiven.

Note 12. In the United States, the length of time that an auto accident stays on the driving record varies depending on the state in which you reside. For example, in Illinois, any chargeable claim an individual submits increases the price of insurance for three years following the claim (see Israel, 2004).

Note 13. Rothschild and Stiglitz (1976) prove that a competitive insurance market may have no equilibrium if there are relatively few high risk individuals who have to be subsidized.

Note 14. Utility function $U(.)$ is assumed to be twice continuously differentiable with $U''(.) < 0 < U'$.

Note 15. See Rothschild and Stiglitz (1976).

Note 16. Net reimbursement equals the indemnity paid under the insurance contract in loss state minus the premium paid out.

Note 17. See the Appendix for the notations used in the model.

Note 18. Accident forgiveness as the policy feature binds the contract the initial insurer can offer in the second period to the policyholders having an accident in the first period. I name these two constraints as "accident forgiveness constraints".

Note 19. Full insurance means that the insured will have the same utility regardless of loss experience, for example $U(W-D+\beta)=U(W-\alpha)$ or $\beta=D-\alpha$.

Note 20. In the principal-agent theory, a principal faces two self-selection constraints: one for high risks not to mimic low risks and one for low risks not to mimic high risks. Only one of these constraints is binding, and the other constraint that is indeed satisfied when it is ignored in the principal's optimization program can be verified ex post (Bolton & Dewatripont, 2005, pp. 53-54).

Note 21. Laffont and Tirole (1990) explained the rent-constraint contract as the contract in which the principal would wish to lower the rent but cannot do so because of the existence of the initial contract.

Note 22. Laffont and Tirole (1990) illustrate that there should be no efficiency gain if introducing the additional rent by choosing a different normalization.

Note 23. A consumer's incentive to strategically withhold accident information from his insurer is disregarded here.

Note 24. See 2005 edition of the Personal Auto Policy by the Insurance Services Office and Rejda (2009, Ch. 22 pp. 513-514).

Note 25. See Rejda (2009, Appendix B, pp. 668-669).

Note 26. Yaari (1965) considers the subjective discount rate when he studies the problem of uncertain lifetimes and life insurance in the context of the expected utility hypothesis using a continuous time model; Fischer (1973) includes a discount factor in the utility-of-consumption function and describes it as a measure of the defectiveness of the imagination or of impatience.

Note 27. Laffont and Tirole (1990) refer good types to the firms with lower project costs.

Note 28. The CRRA utility function is widely used in the literature related to insurance purchase decisions (e.g., Brown & Poterba, 2000; Charupat & Milevsky, 2002; Hong & Rios-Rull, 2007). I also discuss the results with the CARA utility function. Please see the Appendix for details.

Note 29. To simplify the discussion, let us assume that $\omega(\alpha_L,\beta_L)-\omega(\alpha_L^1,\beta_L^1)=\omega(\alpha_{LN},\beta_{LN})-\omega(\alpha_L,\beta_L)$.

Appendix A. Notation

The following describe notations used in this model:

$i=L,H$: subscripts for high (H) risk types or low (L) risk types;

$k=N,A$: subscripts for accident (A) or no accident (N) in the first period;

D: size of insurable loss and assumed constant;

W: initial wealth and assumed constant;

p_i: probability of having losses for risk type i;

$U(.)$: individual utility function for the insured;

α: premium payable under insurance contract;

β: net indemnity paid under insurance contract in loss state;

(α_i,β_i): one-period Rothschild/Stiglitz contract for risk type i;

(α_i^1,β_i^1): insurance contract with accident forgiveness in the first period for risk type i;

(α_{ik},β_{ik}): insurance contract with accident forgiveness in the second period for risk type i contigent on the k;

x: the proportion of high risks purchasing one-period contract;

y: the proportion of low risks purchasing one-period contract;

δ: discount factor;

γ: degree of risk aversion;

λ_{hss}: lagrangian multiplier for high risks self-selection constraint;

λ_{lss}: lagrangian multiplier for low risks self-selection constraint;

λ_{hr}: lagrangian multiplier for high risks randomization constraint;

λ_{lr}: lagrangian multiplier for low risks randomization constraint;

λ_{ha}: lagrangian multiplier for high risks accident forgiveness constraint;

λ_{la}: lagrangian multiplier for low risks accident forgiveness constraint;

λ_{z}: lagrangian multiplier for zero-profit constraint.

Appendix B. Proof in the Case of CARA Utility

Proof for Proposition 2 (CARA Utility). Let us assume a CARA utility function with $U(\gamma,\omega) = 1 - e^{-\gamma\omega}$, where $\gamma > 0$ for convenience. If γ is the parameter to measure the risk aversion, suppose two insured customers, one with $\gamma_1$and the other with γ_2, and $\gamma_1 > \gamma_2$. The low-risk individual's utility for two periods can still be written as expressions (23), (24), (25), and (26).

Because γ_1 and γ_2 are assumed to have optima y_1 and y_2, respectively, one still has

$$U(y_1,\gamma_1,\alpha,\beta) + U(y_2,\gamma_2,\alpha,\beta) \geq U(y_2,\gamma_1,\alpha,\beta) + U(y_1,\gamma_2,\alpha,\beta),$$

which can be further written as

$$(y_1 - y_2)\{[U_{\gamma_2}(\alpha_L^1,\beta_L^1) - U_{\gamma_1}(\alpha_L^1,\beta_L^1)] - [U_{\gamma_2}(\alpha_L,\beta_L) - U_{\gamma_1}(\alpha_L,\beta_L)]$$
$$+\delta(1-p_L)\{[U_{\gamma_2}(\alpha_{LN},\beta_{LN}) - U_{\gamma_1}(\alpha_{LN},\beta_{LN})] - [U_{\gamma_2}(\alpha_L,\beta_L) - U_{\gamma_1}(\alpha_L,\beta_L)]\}\} \geq 0.$$

Similar to what has been proven in the case of the CRRA utility, one can derive a new function

$$g(\omega) = U_{\gamma_2}(\omega) - U_{\gamma_1}(\omega) = e^{-\gamma_1\omega} - e^{-\gamma_2\omega} \tag{A1}$$

It is easy to determine that the cutoff point on the first-order derivative curve is

$$\overline{\omega} = \frac{ln\frac{\gamma_1}{\gamma_2}}{\gamma_1 - \gamma_2}. \tag{A2}$$

For $\omega < \overline{\omega}$, we have $g'(\omega) > 0$.

The second-order derivative of the g function also has a cutoff point, which can be expressed as

$$\widehat{\omega} = \frac{ln\frac{\gamma_1^2}{\gamma_2^2}}{\gamma_1 - \gamma_2} \tag{A3}$$

For $\omega < \overline{\omega}$, we have $g'(\omega) < 0$ and for the previous cutoff value $\overline{\omega}$, it is easy to see that $g"(\omega) < 0$ for any $\omega \leq \overline{\omega}$.

It is also assumed that $\omega(\alpha_L^1,\beta_L^1)$, $\omega(\alpha_L,\beta_L)$, and $\omega(\alpha_{LN},\beta_{LN})$ represent $\omega's$ with different insurance contracts. From the design of the contracts, policyholders have $\omega(\alpha_{LN},\beta_{LN}) > \omega(\alpha_L,\beta_L) > \omega(\alpha_L^1,\beta_L^1)$. Further, it is assumed that $\overline{\omega} > \omega(\alpha_{LN},\beta_{LN}) > \omega(\alpha_L,\beta_L) > \omega(\alpha_L^1,\beta_L^1)$ and $\omega(\alpha_L,\beta_L) - \omega(\alpha_L^1,\beta_L^1) = \omega(\alpha_{LN},\beta_{LN}) - \omega(\alpha_L,\beta_L)$. Then it is straightforward to see that

$$g[\omega(\alpha_L,\beta_L)] - g[\omega(\alpha_L^1,\beta_L^1)] > g[\omega(\alpha_{LN},\beta_{LN})] - g[\omega(\alpha_L,\beta_L)]$$

which proves $y_1 - y_2 \leq 0$.

If one assumes that $\widehat{\omega} > \omega(\alpha_{LN},\beta_{LN}) > \omega(\alpha_L,\beta_L) > \omega(\alpha_L^1,\beta_L^1) > \overline{\omega}$, with $g'(\omega) < 0$ and $g"(\omega) < 0$, then

$$g[\omega(\alpha_L,\beta_L)] - g[\omega(\alpha_{LN},\beta_{LN})] > g[\omega(\alpha_L^1,\beta_L^1)] - g[\omega(\alpha_L,\beta_L)]$$

which indicates the undetermined sign of $(y_1 - y_2)$.

If one assumes that $\omega(\alpha_{LN},\beta_{LN}) > \omega(\alpha_L,\beta_L) > \omega(\alpha_L^1,\beta_L^1) > \widehat{\omega}$, with $g'(\omega) < 0$ and $g"(\omega) > 0$, it is obvious that

$$g[\omega(\alpha_L^1,\beta_L^1)] - g[\omega(\alpha_L,\beta_L)] > g[\omega(\alpha_L,\beta_L)] - g[\omega(\alpha_{LN},\beta_{LN})]$$

which proves $y_1 - y_2 \geq 0$.

Apparently, the results from the CARA utility assumption are similar to those proven with the CRRA utility assumption.