Modeling Volatility in the Gambian Exchange Rates: An ARMA-GARCH Approach

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Abstract
This paper models the exchange rate volatility in the Gambian foreign exchange rates data. Financial time series models that combined autoregressive moving average (ARMA) and generalized conditional heteroscedasticity (GARCH) was explored theoretically and applied to the daily Euro and US dollars (USD) exchange rates against the Gambian Dalasi (GMD) from 2003 through 2013. Based on Akaike information criteria, the ARMA(1,1)-GARCH(1,1) and ARMA(2,1)-GARCH(1,1) were judged the best fitting models to the Euro/GMD and USD/GMD return series respectively. Our empirical results revealed that the distribution of the return series was heavy-tailed and volatility was highly persistent in the Gambian foreign exchange market.

Keywords: exchange rates, Gambia, returns, volatility, ARMA, GARCH

1. Introduction
In the last two decades, modelling exchange rates volatility has drawn much attention from researchers. Exchange rate is one of the salient policy tools for many transitional economies. At the macroeconomic level, exchange rate fluctuations can have significant impact on trade volume. At the microeconomic level it can affect firms and individuals involved in international business. Governments especially in developing countries are continuing to search for mechanisms to cope with the uncertainty that often characterises foreign exchange markets.

According to Antonakakis and Darby (2012), developing countries are increasingly being regarded as alternative destinations for foreign direct investments. This change has been accompanied by a huge increase in international transfers, and in many cases by unexpected changes in exchange rate volatility. Such changes can be very costly for investors as well as governments if they are unforeseen or inefficiently managed. Volatility of an exchange rate can be termed as the variation of the price at which two different countries currencies are traded. It is usually measured as the conditional variance or the conditional standard deviation. Volatility models are important since they can observe the effect of economic factors on foreign exchange rates and, to policymakers and governments in formulating policies related to money supply in the economy and those associated with government expenditures and incomes (Alam & Rahman, 2012).

The Gambian economy is a small open economy in West Africa particularly in terms of basic macroeconomic indicators. In terms of official exchange rate GDP measure, the economy is a total of 896 million US dollars (WDI, 2013). Agriculture, including fisheries, is a dominant activity and contributed about 19.7% percent of GDP in 2013, while industry though small accounts for 12.9% and the main sector of the economic, services (mainly distributive trade, tourism, transportation and telecommunication) accounted for 67.7% of GDP in the same year.

Given the small open and import dependent nature of the Gambian economy, the exchange rates is one of the most important macroeconomic variables. This is manifested as government reserves are kept in foreign currency, most imports and exports are paid in foreign currency and moreover, the remittances received by many
Gambians from abroad show that exchange rate is an important component of the monetary transmission process in The Gambia. The volatility in this price has significant effects on people as well as on policy.

The floating exchange rate system was introduced in the Gambia in 1986 as part of the economic and restructuring package program from the international monetary fund. This allows the exchange rate against international currencies such as the US Dollar to be determined by the forces of demand and supply in the currency market. The Central Bank often intervenes only to maintain the required level of reserves and to smooth out volatility. There are fourteen banks operating in the Gambia of which thirteen are conventional commercial banks and one is an Islamic bank. The major trading currency in the inter-bank foreign exchange market is the US dollar, followed by the Euro and the British Pound. Prolonged volatility in exchange rate is an indication of ineffectiveness of a central bank to perform its mandate of price stability, and the management of the countries foreign exchange reserves (Maana et al., 2010).

This paper models the volatility in the Gambian exchange rate returns data. We explore properties of the Gambian daily exchange rate data and examine ARMA–GARCH models that are suitable to model the returns. Specifically, an autoregressive moving average is specified as the mean equation while the residuals are fitted with a symmetric GARCH process. This paper contributes in two ways. First, to our knowledge, no work has been done on modeling volatility in the Gambian exchange rates, thus the attempt to fill this Gap. Second, we apply the ARMA (P,Q)–GARCH(p,q) model which is different from previous studies. Many studies assumed that returns follow a pure GARCH process with a constant mean. This assumption might not be plausible as it is restrictive that the observed series is a realization of a noise.

A Quasi-maximum likelihood (QML) estimation method is used to estimate our model. Exploratory analysis of the returns indicates that they are heavy-tailed. Our findings suggest that volatility is highly persistent and the estimated model fits the exchange rate returns data well. The remainder of this paper is organized as follows:

Section two discusses relevant literature, section three discusses the theoretical framework of the ARMA–GARCH model considered, section four covers methodology, section five discusses the estimated results, and section six gives the summary and conclusion.

2. Literature Review

Since the seminal works of Engel (1982) and Bollerslev (1986), generalized autoregressive conditional heteroscedastic (GARCH) processes have received considerable attention in the analysis of financial time series. Engel describe the conditional variance by a simple quadratic function of its lagged values, while Bollerslev modeled the conditional variance to be determined by its own lagged values and the square of the lagged values of the innovations or shocks. These time series models are known to capture several essential features of financial series such as leptokurticity and volatility clustering. Empirical studies have shown that such processes are successful in modeling time series. For example in the context of foreign exchange rate markets see earlier works by (Bailie & Bollerslev, 1989; Hsieh, 1989). Many of the drivers of dynamics in exchange rate returns and volatility can best be identified in high frequency data. For more details see (Andersen & Bollerslev, 1998a,b).

According to Choy (2002), knowledge of volatility and its estimation can ensure mitigation of long term risk of any investment which assists in promoting economic growth since investment is the main channel of increasing real output and employment.

The GARCH in mean was used by Ryan and Worthington (2004) to investigate the sensitivity of the Australian Bank stock returns to market interest rate and foreign exchange rate risk. Their results suggest that bank stock returns is mostly determined by market risk, together with short and medium term interest and foreign exchange rates.

In Ghana, Adjasi et al. (2008) investigated the influence of exchange rate volatility on stock market returns by using the exponential GARCH model. They established that there exists a negative relationship between exchange rate volatility and stock market returns. They argue that a depreciation of the local currency results to an increase in stock market returns in the long run. Olowe (2009) examines the volatility of Naira/ US Dollar exchange rates in Nigeria using monthly data over the period 1970 to 2007. Six different univariate GARCH models were fitted to the data. The paper concluded that all the models show that volatility is persistent for both the fixed exchange rate period and the floating regime era, and the best performing models are the Asymmetric Power ARCH and Threshold Symmetric GARCH.

Kamal et al. (2012) modeled exchange rate volatility of the Pakistani Rupee and the US Dollar using three ARCH type models namely: GARCH in mean model, Exponential GARCH and Threshold ARCH Models. According to the results of their study, it was concluded that EGARCH model was the best model in explaining the volatility behavior of exchange rate data of Pakistani Rupee against the US Dollar. A comparative study to
establish whether the univariate volatility models used widely in modelling and forecasting exchange rate volatility in developed countries were equally successful when applied to data from developing countries was done by (Antonakakis & Darby, 2012). Three developing countries were selected and four developed countries. All exchange rates were against the US Dollar. They found that for developed countries the Fractionally Integrated GARCH model was superior to the other models whereas in the case of developing countries the Integrated GARCH model fitted the data better.

All these studies assume that the series follows a GARCH process. This implies the mean equation in their GARCH models is termed as a constant. To the best of our knowledge, no study on exchange rates is done on modelling volatility using ARMA-GARCH models. However, these models have been successfully applied to the energy markets notably the oil and electricity markets. For examples, see (Hickey et al., 2012; Mohammadi & Su, 2010). Therefore, we investigate whether such models can adequately describe exchange rate price behavior in the Gambian foreign exchange market.

3. Theoretical Framework

Empirical research on return distribution has been a subject of discussion among researchers since the 1960s. Badrinath and Chatterjee (1988) and Rachev et al. (2005) have found that the distribution of returns is not characterized by normality but by the stylized facts of fat-tails, high peakedness (excess Kurtosis) and skewness. Although it is generally accepted that distribution of exchange rates are leptokurtic and skewed, there is no unanimity regarding the best stochastic model to capture these empirical studies. We outline the ARMA-GARCH model below.

3.1 Mean and Variance Equation

In this paper, the mean equation is modeled with an ARMA process. The mean equation used serves as a filter for the returns. The residuals are then fitted with a GARCH process. Assuming that the returns, \( r_1, \ldots, r_n \), are generated by a strictly stationary nonanticipative solution of the ARMA (P,Q)–GARCH(p,q) order given by

\[
\begin{align*}
\sigma_i^2 &= \omega + \sum_{j=1}^{q} \alpha_j \varepsilon_{i-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{i-j}^2, \\
r_t - \mu &= \sum_{i=1}^{P} a_i (r_{t-i} - \mu) - \sum_{j=1}^{Q} b_j r_{t-j} + \varepsilon_t, \\
\varepsilon_t &= \sigma_t z_t,
\end{align*}
\]

where \( p \geq 0, q > 0, \omega > 0, \mu \) is the mean of the return series and \( z_t \) is an independent and identically distributed white noise process. Assuming that the orders \( P, Q, p \) and \( q \) are known, the parameter vector is denoted by

\[
\varphi = (\varphi', \theta') = (a_1, \ldots, a_p, b_1, \ldots, b_q, \theta'),
\]

where \( \theta' = (\omega, \alpha_1, \ldots, \alpha_q, \beta_1, \ldots, \beta_p) \).

3.2 Estimation of the ARMA(P,Q)–GARCH (p,q) Model

In the absence of normality, Weiss (1986) and, Bollerslev and Wooldridge (1992) have shown that in GARCH models, maximizing the Gaussian likelihood produces QML estimator that are consistent and asymptotically normally distributed provided that the conditional mean and variance equation are correctly specified. For our case, we use the ARMA-GARCH process of equation (1) under mild conditions and show the QML estimator (Francq & Zakoian, 2004).

The parameter space is given by

\[
\Phi \subset \mathbb{R}^{P+Q+1} \times (0, +\infty) \times (0, +\infty)^{P+Q}.
\]

The true value of the parameter is given by

\[
\varphi_0 = (\varphi_0', \theta_0') = (a_{01}, \ldots, a_{0p}, b_{01}, \ldots, b_{0Q}, \theta_0').
\]

With the Gaussian quasi-maximum likelihood conditional on initial values when, for \( q \geq Q \), then the initial values are obtained as

\[
\begin{align*}
r_1, \ldots, r_{1-(q-Q)+p}, \varepsilon_{q+Q+1}, \ldots, \varepsilon_{1-q}, \sigma_{0}, \ldots, \sigma_{1-p},
\end{align*}
\]

the last \( p \) of these values are positive and may depend on the parameter or on the observations. For any \( \varphi ', \theta' \), the
values \( \tilde{e}_i(\theta) \), \( t = -q + Q + 1, \ldots, n \), and then for any \( \theta \), the values of \( \tilde{\sigma}_i(\theta) \) for \( t = 1, \ldots, n \) is computed from

\[
\tilde{e}_t = \tilde{e}_i(\theta) = r_t - \mu - \sum_{i=1}^{p} a_i (r_{t-i} - \mu) + \sum_{j=1}^{q} b_j \tilde{e}_{t-j}
\]

\[
\tilde{\sigma}_t = \tilde{\sigma}_i(\theta) = \omega + \sum_{i=1}^{p} \alpha_i \tilde{e}_{t-i} + \sum_{j=1}^{q} \beta_j \tilde{e}_{t-j}.
\]

However, when \( q < Q \), the fixed initial values are

\[
\tilde{r}_1, \ldots, \tilde{r}_{1-(Q-q)-p}, \tilde{\epsilon}_1, \ldots, \tilde{\epsilon}_Q, \tilde{\sigma}_0, \ldots, \tilde{\sigma}_{1-p}.
\]

Conditional on these initial values, the Gaussian Log–likelihood is obtained as

\[
\tilde{L}_n(\phi) = n^{-1} \sum_{i=1}^{n} L_i(\phi) = \frac{\tilde{e}_t(\theta)}{\tilde{\sigma}_t(\theta)} + \log \left( \frac{\tilde{\sigma}_t(\theta)}{\tilde{\epsilon}_t(\theta)} \right)
\]

A quasi-maximum likehood estimator of the parameter vector is defined as any measurable solution of the equation

\[
\tilde{\phi}_n = \arg \min \tilde{L}_n(\phi), \phi \in \Phi.
\]

For the consistency and asymptotic normality properties of this estimator see details from (Francq & Zakoian, 2010).

4. Methodology

4.1 Data and Descriptive Statistics

The data used in this paper consists of daily exchange rates of the Gambian Dalasis against the Euro and the US dollar for ten years. The data cover the period from May 2003 to May 2013. It consists of 3653 observations. The data represents the average daily spot prices exchange rates at which the banks buy and sell these foreign currencies. The data were obtained from the Central Bank of The Gambia courtesy of AONDA historical exchange rate database (available at www.oanda.com).

In figure (1), we noticed that the variation in the daily series of the Euro and USD currencies against the Dalasi is not constant over time. This is termed as nonstationarity and it is widely observed in many applied time series data. The movement is an upward trend and indicates that the Dalasi against these international currencies have been depreciating over the last decade. This could be attributed to many factors. One of which include the diminishing nature of the country’s re-export trade due to harmonization of external tariffs in the region and efficiency improvements in competing port facilities, notably in Senegal.

Figure 1. Daily exchange rates plots
Since our daily exchange rate series in this study is non stationary as shown in the Augmented Dickey-Fuller and Phillips-Perron test of stationarity in table 1, we need to transform the original series to render it stationary. This will enable us apply the time series models without violating the underlying theory. We apply the logarithmic transformation to convert the prices to returns. Let the daily exchange rate series be denoted by, \( y_t \), then

\[
    r_t = \ln \left( \frac{y_t}{y_{t-1}} \right) = \ln(y_t) - \ln(y_{t-1}),
\]

where \( r_t \) is the return at time \( t \), \( y_t \) is the exchange rate price at time \( t \) and \( y_{t-1} \) is the exchange rate at time \( t-1 \).

The returns series appear stationary over time and fluctuating around mean zero as shown in figure (2). We can also observe the volatility clustering in the plots. This is evident as period of high volatility is followed by periods of low volatility thus confirming one of the main features of stationary financial time series data.

![Figure 2. Logarithmic returns plots](image_url)

Table 1 gives the stationarity test results for the daily and return series. Both the Augmented Dickey-Fuller and the Phillips-Perron test confirms the presence of unit root in both daily series at the 1% significance level since their p-values are greater than or equal to it. For the returns, both tests suggests stationarity at the 1%, 5% and 10% as the p-values associated with the test are all smaller than the respective significance levels. Therefore, the null hypotheses of the presence of unit root for each of the retuns series is rejected, thus confirming stationarity of the series.

<table>
<thead>
<tr>
<th></th>
<th>Augmented – Dickey Fuller Test</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Daily Series</td>
<td>Returns</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Euro/GMD</td>
<td>USD/GMD</td>
<td>Euro/GMD</td>
<td>USD/GMD</td>
<td></td>
</tr>
<tr>
<td>Test Statistic</td>
<td>-3.7078</td>
<td>-1.818</td>
<td>-15.9198</td>
<td>-15.4906</td>
<td></td>
</tr>
<tr>
<td>P-value</td>
<td>0.0236</td>
<td>0.6554</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Philps – Perron test</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test Statistic</td>
<td>-40.2401</td>
<td>-12.7666</td>
<td>-3733.33</td>
<td>-3714.41</td>
<td></td>
</tr>
<tr>
<td>P-value</td>
<td>0.01</td>
<td>0.3978</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td></td>
</tr>
</tbody>
</table>

The descriptive statistics carried out on the original and return series are shown in table 2. The average daily exchange rate of the Euro and USD against the Dalasi from 2003 to 2013 is 39.391 and 26.947 respectively. The excess kurtosis of the daily series is 0.419 and 1.0645 respectively. This implies the distribution of the Euro and
USD against the Dalasi has approximately the same kurtosis as that of a normal distribution which known to be 3. The excess kurtosis of the returns indicates that they are heavy-tailed (28.973 and 33.319 for the Euro/GMD and USD/GMD respectively). The excess kurtosis tells us by how much the kurtosis of a variable differs from that of a normally distributed variable. Therefore, the exact kurtosis of the variables is the value shown in table 2 plus 3. The mean of both return series is close to zero. The Jarque-Bera test at 1%, 5% and 10% significance rejects the null hypothesis, confirming the departure from normality of the daily and return series for each currency (critical values are 9.21, 5.99 and 4.61 respectively). The Ljung-Box statistics up to lags 5 allows us to conclude lack of randomness in the data, which signifies high presence of serial correlation since the p-values are less than 1%, 5% and 10% significance levels.

Table 2. Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Euro/GMD</th>
<th>USD/GMD</th>
<th>Euro/GMD</th>
<th>USD/GMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>39.391</td>
<td>26.947</td>
<td>0.00012</td>
<td>0.00008</td>
</tr>
<tr>
<td>Median</td>
<td>35.33</td>
<td>27.047</td>
<td>0.00004</td>
<td>0</td>
</tr>
<tr>
<td>Maximum</td>
<td>45.647</td>
<td>33.631</td>
<td>-0.18778</td>
<td>0.18781</td>
</tr>
<tr>
<td>Minimum</td>
<td>24.024</td>
<td>16.999</td>
<td>-0.22725</td>
<td>-0.22975</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>3.213</td>
<td>2.854</td>
<td>0.01589</td>
<td>0.01532</td>
</tr>
<tr>
<td>Excess kurtosis</td>
<td>0.491</td>
<td>1.065</td>
<td>28.973</td>
<td>33.319</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.066</td>
<td>-0.728</td>
<td>-0.57349</td>
<td>-0.657</td>
</tr>
<tr>
<td>Jarque Bera Statistic</td>
<td>39.3786</td>
<td>495.2008</td>
<td>122007</td>
<td>169187.4</td>
</tr>
<tr>
<td>Jarque Bera P-value</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Ljung Box Statistic</td>
<td>17155.91</td>
<td>17621.01</td>
<td>233.959</td>
<td>307.8463</td>
</tr>
<tr>
<td>Ljung Box P-value</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>3653</td>
<td>3653</td>
<td>3652</td>
<td>3652</td>
</tr>
</tbody>
</table>

4.1 Model Selection

The selection of the best ARMA model to fit the returns as the mean equation is based on the Akaike Information Criteria (AIC). Several ARMA models were fitted and evaluated based on this criterion. Therefore, AIC is a measure of the goodness-of-fit of an estimated statistical model. In general, AIC is defined as

\[ AIC = -2 \log(L) + 2k \]

where \( \log(L) \) is the maximized likelihood of the parameters for the estimated model, \( k \) is the number of parameters and the term \( 2k \) is a penalty as an increasing function of the number of estimated parameters. Given any two estimated models, the model with the lower value of AIC is the one to be preferred.

In table 3, nine ARMA models were fitted for each of the returns. The ARMA(1, 1) and ARMA(2, 1) appears to be the best candidates for the mean equation of the Euro/GMD and USD/GMD returns respectively since they have the AIC lowest values. The mean equations are necessary to remove serial dependence and produce independent and identically distributed residuals.

Table 3. AIC of several ARMA model for the mean equation

<table>
<thead>
<tr>
<th>ARMA Model</th>
<th>Euro/GMD Returns</th>
<th>USD/GMD Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,1)</td>
<td>-19548.1</td>
<td>-19937.71</td>
</tr>
<tr>
<td>(0,2)</td>
<td>-19609.1</td>
<td>-20021.65</td>
</tr>
<tr>
<td>(1,0)</td>
<td>-19468.2</td>
<td>-19794.08</td>
</tr>
<tr>
<td>(1,1)</td>
<td>-19614.3</td>
<td>-20023.81</td>
</tr>
<tr>
<td>(1,2)</td>
<td>-19613.2</td>
<td>-20024.09</td>
</tr>
<tr>
<td>(2,0)</td>
<td>-19556.3</td>
<td>-19926.8</td>
</tr>
<tr>
<td>(2,1)</td>
<td>-19613.2</td>
<td>-20024.23</td>
</tr>
<tr>
<td>(2,2)</td>
<td>-19610.9</td>
<td>-20022.11</td>
</tr>
</tbody>
</table>

The ARCH test for heteroscedasticity is conducted on the residuals from the mean equation and the results are shown in table (4). It is concluded that the residuals from the fitted ARMA (1, 1) and ARMA (2,1) models at the
various lags rejects the null hypothesis of no ARCH effects. This is because the p-values obtained are all less than the significant levels at 1%, 5% and 10% respectively. This suggests that a GARCH model may appropriately describe the conditional volatility process.

Table 4. ARCH Test for heteroscedasticity at 5% significance level

<table>
<thead>
<tr>
<th>Lag</th>
<th>Test Statistic</th>
<th>P-value</th>
<th>Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>144.51</td>
<td>&lt;0.001</td>
<td>9.488</td>
</tr>
<tr>
<td>8</td>
<td>241.96</td>
<td>&lt;0.001</td>
<td>15.507</td>
</tr>
<tr>
<td>12</td>
<td>246.64</td>
<td>&lt;0.001</td>
<td>21.026</td>
</tr>
<tr>
<td>4</td>
<td>145.52</td>
<td>&lt;0.001</td>
<td>9.488</td>
</tr>
<tr>
<td>8</td>
<td>224.04</td>
<td>&lt;0.001</td>
<td>15.507</td>
</tr>
<tr>
<td>12</td>
<td>231.2</td>
<td>&lt;0.001</td>
<td>21.026</td>
</tr>
</tbody>
</table>

Note. The ARCH statistic test the null hypothesis of no conditional heteroscedasticity.

From the graphs of the autocorrelation function of the residuals in Figure 3, it is seen that there exists only two significant spikes at around lags 2 and 7 for both series. The PACFs exhibits several significant notably at lags 6 and 15. For the ACFs and PACFs in both residuals it clearly indicates an exponential declining of the spikes. This suggests a GARCH process is an ideal candidate to model the residual. From the graphs, it is observed there is no pattern of seasonal lags being present. Thus, the assumption of no seasonality in the returns is plausible to assume.

Figure 3. ACF AND PACF plots of the residuals from the mean equation

The technique used in selecting the appropriate GARCH among competing models is based on the AIC as well. In empirical applications, only small lag for p and q are often used. Typically, GARCH (1, 1), GARCH (1, 2) and GARCH (2, 1) are adequate in modeling volatilities in financial time series over long sample periods (Bollerslev et al., 1992).

In table 5, we include other GARCH models to check if they could be favourable models in modeling the heteroscedasticity in our data. It is evident from the table that the GARCH (1,1) comes out to be the best in modeling the residuals in both return series.

Table 5. AIC GARCH Model fitted to the residuals

<table>
<thead>
<tr>
<th></th>
<th>(1,0)</th>
<th>(1,1)</th>
<th>(1,2)</th>
<th>(2,0)</th>
<th>(2,1)</th>
<th>(2,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residuals 1</td>
<td>-5.19612</td>
<td>-5.2114</td>
<td>-5.21018</td>
<td>-5.19603</td>
<td>-5.2109</td>
<td>-5.21139</td>
</tr>
<tr>
<td>Residuals 2</td>
<td>-5.20023</td>
<td>-6.22128</td>
<td>-5.21931</td>
<td>-6.20015</td>
<td>-5.22092</td>
<td>-5.22124</td>
</tr>
</tbody>
</table>
Therefore, based on the analysis and selection criteria, we apply the ARMA(1,1)-GARCH(1,1) to the Euro/GMD returns whilst the USD/GMD is fitted with an ARMA(2,1)-GARCH(1,1).

5. Empirical Results

We estimate the selected models using the Quasi-maximum likelihood estimation method. The estimated coefficients for both the conditional mean and variance are contained in Tables (6). In both returns, the autoregressive and moving average terms sum to a number less than 1, which is consistent with a stationary ARMA process. The AR (1) and MA (1) terms are statistically significant at the 1%, 5% and 10% for both exchange rates. The coefficient for AR (2) is not significant at the 1% for the USD/GMD returns.

The sum of the GARCH parameters is approximately equal to one for all the models i.e., \( \alpha_1 + \beta_1 \approx 1 \). This shows that volatility is persistent in our exchange rate data which is consistent with the findings of Beg and Anwar (2012) for the U.K. pound/ U.S dollar daily exchange rates. The coefficient \( \alpha_1 \) captures the influence of new shocks on volatility. Estimates of this parameter are statistically significant for both currencies and positive. The estimate, \( \alpha_1 \), from the fitted model is close to 0.085 for both returns. The parameter \( \beta_1 \), measures persistence of volatility shocks and is positive as well as statistically significant. For both returns, value of \( \beta_1 \) is close to 1 (around 0.93), indicating that old shocks to exchange rate prices tend to persist, instead of dying out quickly. This implies that economic shocks especially those of external have long standing effects on exchange rate volatility in the Gambia.

Table 6. Estimates of the conditional mean and variance equation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Euro/GMD ARMA(1,1) – GARCH(1,1)</th>
<th>USD/GMD ARMA(2,1) – GARCH(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.0579</td>
<td>0.4815</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.001***</td>
<td>(&lt;0.001***</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-0.0764</td>
<td>-0.0764</td>
</tr>
<tr>
<td></td>
<td>(0.0199**)</td>
<td>(0.0199**)</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.7479</td>
<td>-0.7542</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.001***</td>
<td>(&lt;0.001***</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(-0.001***</td>
<td>(-0.001***</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>0.0913</td>
<td>0.0871</td>
</tr>
<tr>
<td></td>
<td>(0.003***</td>
<td>(0.003***</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.9261</td>
<td>0.9262</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.001***</td>
<td>(&lt;0.001***</td>
</tr>
</tbody>
</table>

LM-ARCH Test on Residuals

<table>
<thead>
<tr>
<th>Test Statistic</th>
<th>Euro/GMD</th>
<th>USD/GMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-value</td>
<td>0.9987</td>
<td>0.9806</td>
</tr>
</tbody>
</table>

Note. The values in parenthesis are the p-values of the coefficients. ""'' represent significance at the 1%,5%, 10% levels, while "" denotes significance at 5% and 10% respectively. The ARCH-LM test is up to 20 lags.

The LM-ARCH test results together with AIC and BIC for the residuals is also given. The ARCH test for heteroscedasticity accepts the null hypothesis of no ARCH effects in the residuals because the p-values are all greater than than 1%, 5% and 10% respectively. Moreover, if the model is successful in modeling the return series well, then there should be minimal or no autocorrelation left in the standardized residuals. The graphs in Figure 4, shows that the standardized residuals are white noise and the autocorrelation function of the squared residuals indicates that there is no significant autocorrelation in the residuals of the estimated models. This suggest that the model fits our data well.
The volatilities estimated as the conditional variance which is an unbiased estimate of the true conditional variance from the estimated models are plotted and shown in Figure 5. In all the plots the volatility pattern does not exhibit constant increase or decrease but instead a mixture of periods of high volatility followed by periods of low volatility. This suggests that the Gambian foreign exchange rates during the last ten years have witnessed significant instability.

6. Conclusion

The properties of the Gambian exchange rate data have been explored and a suitable ARMA-GARCH model was formulated and applied to it. Based on AIC criterion, the ARMA(1,1)-GARCH(1,1) was applied to the Euro/GMD returns and for the USD/GMD returns, an ARMA(2,1)-GARCH(1,1) was fitted to it. The theory on Quasi-maximum likelihood estimation of ARMA-GARCH was evaluated before applying the model to estimate volatilities in the Gambian exchange rates. The sum of the GARCH parameters, $\alpha_1$ and $\beta_1$, were found to be very close to 1, suggesting that volatility in the exchange rates is highly persistent.

The volatility in the Gambian exchange rates witnessed significant instability during the last decade in the form of depreciation of the currency. This suggests that exchange rate volatility (which is associated with exchange
rate risk) in the Gambian market is high. This risk is important to understand as it affect transactional account exposure related to receivables (export contracts), payables such as import contracts and repatriation of dividends. It also impact revenues on domestic sales and inputs and also, on operating cost. Therefore, the results of this paper provides an avenue for understanding the volatility associated with the Gambian foreign exchange market which provides a good avenue to relevant authorities and other parties in managing currency risk. It may be of interest to future researchers to use a Multivariate GARCH model that could include fundamental macroeconomic variables such as interest and inflation rates and also, to explore the concept of regime switching to increase the overall fit of the models.

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References


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