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A New Nonparametric Approach to Price Convertible Bond Based on Random Interest Rate

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Abstract

This paper proposes an idea of combining the following two nonparametric approaches for two-factor convertible bond valuation. One is the stimulation of random rate, the interest rate term structure based on polynomial spine function was attained by only using historical data; Another is Canonical risk-neutral probability, which was attained by observed stock returns, so that the convertible bonds can be valuated by using equivalent martingale measure.

Keywords: Canonical valuation, Maximum entropy principle, Polynomial spine function, Random interest rate model, Risk-neutral probability

1. Introduction

Convertible bond (CB) is an intermediate between the stock and the pure bond, which becomes a more and more popular financial derivative. With the development of theory on pricing CB over 30 years, the CB pricing theory has formed two models including structural-form and reduced-form. The structural-form model aims to valuate the CB by analyzing the corporation's capital structure and taking the corporation value as a variable. Merton (1974) initially gave this method and argued that the corporation value obeyed a diffusion process, thus Merton considered the CB as a option written on corporation value. The method, however, does not work very well for the difficulty in observing the corporation value in the markets, which makes parameter estimators unavailable. Another model is reduced-form put forward firstly by McConnell & Schwartz (1986). This model takes the corporation stock as the underlying asset. Both of these two models can be called single- or multi-factor model according to the number of factors which affect the CB value.

Unfortunately, most of the existing models have their own weak points. For interest rate factor, it is usually assumed to obey a specific model so that its probability distribution is supposed firstly. But no any theory could explain which model is the best, moreover, the model will be verified through the relevant data. To atone for that, Ait-Sahalia (1996), Jiang (1998), Wang &Zhou (2006), Pan (2008) and Song & Lu (2008) offered semi-parameter or nonparametric rate models. For volatility factor, volatility, one of the most important parameters in modern options pricing, is unfortunately the only parameter that cannot be observed directly on the capital market. To bypass the problem of volatility estimation or volatility modeling as well as of making assumption about the distribution of the underlying altogether, Stutzer (1996, 2000) proposed the so-called canonical valuation method, a nonparametric approach, which derives a risk-neutral distribution (Canonical distribution) from the observed time series data of the underlying and then uses the resulted canonical probabilities to average the expiry payoff's directly to obtain the price of a European option. This approach which based on historical data is not subjected to any parameter model.

The paper proposes an idea of combining the two nonparametric approaches above for CB valuation. The remainder is structured as follows, the paper first estimates Government Bond Yields through an interest rate term structure model based on polynomial spine function, then the canonical risk-neutral probability is attained by observed stock returns, finished the convertible bonds can be valuated by using equivalent martingale measure.

2. Nonparametric interest rate model based on polynomial spine function

This section follows closely Wang &Zhou (2006) and uses the cubic spine function to estimate government bond yields.

2.1 Nonparametric interest rate model

Let $B(t,s)$ denote the t-time price of zero coupon bond (ZCB) which pays one unit cash at time s ; $B(s)$ refers to the discount spine function and means the 0-time price of ZCB which pays one unit cash at time s , obviously $B(0,s)=B(s)$; The discount factor on interval $[t_{i-1},t_i]$ is denoted by $B_{t_i}(s)$. The following is cubic spine function:

$$B(s)=\begin{cases} B_{t_1}(s)=1+c_1 s+b_1 s^2+a_1 s^3, s\in[0,t_1] \\ B_{t_i}(s)=B_{t_1}(s)+\sum_{j=2}^{i}(a_j-a_{j-1})(s-t_{j-1})^3, \\ s\in[t_{i-1},t_i], i=2,3,\cdots n \end{cases} \tag{1}$$

Solve the model: $$\min_{a_i,b_i,c_i}\sum_{j=1}^{n}[P_j(t)-\sum_s C_j(s)B(s)]^2 \tag{2}$$

Where $P_j(t)$ is market price of j th coupon bond at time t and $C_j(s)$ denotes the cash flow of j -th coupon bond at time s .

a_i,b_i,c_i in (1) can be calculated using least square method. Lastly, ZCB rate $r(s)$ is given as follows:

$$r(s)=-\frac{\ln B(s)}{s} \tag{3}$$

The following example is to illustrate the solution procedure for nonparametric rate term structure.

2.2 Example for illustrating nonparametric rate term structure

Select 13 coupon bonds (see table 1) from Shanghai Stock Exchange and apply them to model (2) for optimal decision. The sample bonds span from 3 April 2008 to 9 May 2015 and their payment periods are one year. Let $t_1=1,t_2=3,t_6=6$, then the following is obtained using the 13 bonds data and by the approach above:

$$c_1=-.52767589, b_1=.61124586, a_1=-.21910044; a_2=-.01168983, a_3=-.01111107$$

Figure 2 is the zero coupon Government-bond rate term structure.

3. Canonical risk neutral probability

3.1 Equivalent martingale measure

The principle of pricing CB in this paper is Not necessary assuming the asset price obeys some process But only using the observed data to obtain the risk neutral probability. The reason is that information about underlying asset is in historical data and not dependent on any given model.

There are some denotations. Firstly assume CB expires at time T ; the current price of stock with independent returns is S_0 and $S(t)$ at $t\in[0,T]$; $D(t)$ and $r(t)$ denote individually the dividends and instantaneous rate at time t ; π and π^* represent the actual return probability and equivalent martingale probability; $\frac{d\pi^*}{d\pi}$ is Radon-Nykodym derivative(that's the risk neutral probability density); the increment $\Delta S(t)=\frac{S(t+\Delta t)}{S(t)}$.

In discrete time model(see Stutzer, 1996),

$$S_0 = E_{\pi^*}\left(\frac{S(T)+D(T)+\sum_{t=1}^{T-1}[D(t)\prod_{s=t}^{T-1}(1+r(s))]}{\prod_{s=1}^{T}(1+r(s))}\right)$$
$$= E_{\pi}\left(\frac{S(T)+D(T)+\sum_{t=1}^{T-1}[D(t)\prod_{s=t}^{T-1}(1+r(s))]}{\prod_{s=1}^{T}(1+r(s))}\frac{d\pi^*}{d\pi}\right) \quad (4)$$

The continuous one is considered in this paper.

3.2 Gibbs Canonical probability

Consider the non-dividend case: $D(t)=0(\forall t)$, each observed interval is Δt, $n=[\frac{T}{\Delta t}]$, n gross returns in market are denoted by $R(t)(=\frac{\Delta S(t)}{S(t)})$. Then the following can be attained by equivalent martingale measure condition(Longstaff,1995):

$$E_{\pi}[R(t)\cdot\frac{d\pi^*}{d\pi}] = \int_t^{t+\Delta t} r(s)ds \quad (5)$$

$E_{\pi}[\cdot]$ denotes the expectation operator under actual probability.

And formula (6) holds by maximizing entropy principle (Foster,1999):

$$\frac{d\pi^*}{d\pi} = \arg\min_{\frac{d\pi^*}{d\pi}} \int \ln\frac{d\pi^*}{d\pi} d\pi^* \quad (6)$$

Thus, the risk neutral probability can be computed using formulas above(the solution process need the feasible assumption “the returns are independent ”):

$$\frac{d\pi^*}{d\pi} = \frac{\exp(\gamma^* R(t))}{E_{\pi}[\exp(\gamma^* R(t))]} \quad (7)$$

Where γ^* is the value of the Lagrange multiplier that minimizes the following:

$$\min_{\gamma} \frac{E_{\pi}[\exp(\gamma R(t))]}{\exp(\gamma\int_t^{t+\Delta t} r(s)ds)} \quad (8)$$

$\frac{d\pi^*}{d\pi}$ in (7)products a kind of probability distribution called Gibbs Canonical neutral probability.

4. Pricing convertible bond (CB)

4.1 Analysis of CB

Let C_T be the value of CB with expiration date T; F is the CB face value; $P_b = F\cdot e^{iT}$ represents the value of straight bond with coupon rate i; C_c is the conversion price. The following is the analysis process. At expiration,

The stock value after converting into stock $\frac{F}{C_c}S_T$ is smaller than the face value F if stock price S_T is less than conversion price C_c. Then the bond holder would not convert the bonds into stocks and receive payments including principle and coupons from corporate, thus the CB just is the straight bond.

When stock price S_T is more than conversion price C_c but $\frac{F}{C_c}S_T$ less than pure bond value P_b, the holder will do the same as 1) and CB value equals to straight bond value P_b.

If converted stock value $\frac{F}{C_c} S_T$ is more than pure bond value P_b, the bond holder must exercise the option converting bonds into stocks to hold corporate stocks whose value just means the CB value.

Consequently, it can be concluded that from above:

$$C_T = \begin{cases} P_b & S_T < C_c \\ P_b & C_c \le S_T \le \dfrac{P_b \cdot C_c}{F} \\ \dfrac{F \cdot S_T}{C_c} & S_T > \dfrac{P_b \cdot C_c}{F} \end{cases} \tag{9}$$

4.2 Convertible bond valuation

Using Gibbs Canonical density $\frac{d\pi^*}{d\pi}$ and formula(9), the value of convertible bond with expiration date T can be calculated by the following expectation:

$$C_0 = E_{\pi^*}\left[C_T\right] = E_{\pi^*}\left(\frac{\max(P_b, \frac{F}{C_c} S_T)}{\int_0^T r(s)ds} \right) = E_{\pi}\left(\frac{\max(P_b, \frac{F}{C_c} S_T)}{\int_0^T r(s)ds} \frac{d\pi^*}{d\pi} \right) \tag{10}$$

There exist two small questions still to be handled. One is about $r(s)$ in (10), actually it can be attained through section 2 "Nonparametric interest rate model based on polynomial spine function"; Another is about the simulation of Gibbs-Canonical probability density, this also can be easily solved by the following steps: firstly calculate gross return $R(t)$ in proper observed time interval Δt, then simulate probability distribution of $R(t)$ and substitute it into (7)and(8) to compute Gibbs-Canonical probability density, finished with value of convertible bond from(10).

5. Conclusions and further researches

The discussed convertible bond in this paper is based on two factors (random rate and corporate stock) model. The pricing method for CB is nonparametric, that is, not necessary to assume any pricing model since the information from financial market can be reflected in relevant data. This method works totally by real observed data.

This nonparametric pricing method is in two aspects: one is the stimulation of random rate, The interest rate term structure based on polynomial spine function can be drawn by only using historical data, any given model is unnecessary. Another is Canonical risk-neutral probability, in fact, Canonical probability measure can be computed by directly using the historical stock data, thus the volatility estimation is skipped. This is the very "nonparametric" approach. The paper finished with calculating CB valuation by equivalent martingale measure.

Discussions further are as follows. This paper only proposes an idea of pricing convertible bond, does not give an empirical test, although Canonical approach is tested very efficient in pricing option. Besides, an improved Canonical approach with constraint(s) can be considered as Stuzer(1996).

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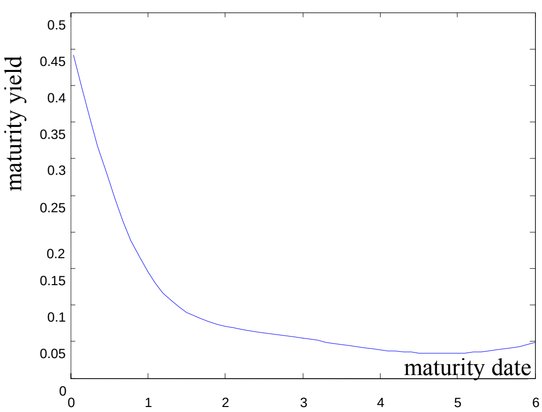
Codes for selected bonds
010103, 010110, 010112, 010203, 010307, 010503, 010308, 010404, 010410, 010505, 010509, 010513, 010605

Table 1.



Zero coupon Government-bonds rate term structure

Figure 2.