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# On the Effect of Subprime Crisis on Value-at-Risk Estimation: GARCH Family Models Approach

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**Abstract**

A survey of the risk management literature shows that few studies have attempted to take into account financial crisis in market risk measurement, in particular when using a Value-at Risk (VaR) analysis. In this paper, we use models to investigate the effects of subprime crisis on the Value-at-Risk estimation. In this framework, we investigate GARCH family models such as, GARCH, IGARCH, and GJR-GARCH. Each is adjusted based on three residuals distributions; normal, Student and Skewed Student-t. Using American stock market data, we show that dynamic volatility is different between the stability and during crisis periods. The estimation results indicate that the amount of VaR is different during these two time periods. This finding could be explained by the volatility clustering effect. The empirical results show also that GJR-GARCH model performs better in both sub-sample periods, in comparison with GARCH and IGARCH models. Moreover, we conclude that Student-t and Skewed Student-t distributions are preferred in the stable period while the normal distribution is recommended during the turbulent period.

**Keywords:** Value-at–Risk, Subprime crisis, Risk management, Market risk, Risk measure, GARCH, Volatility asymmetry.

## 1. Introduction

In the last few years, risk management has known an important advance. In response to the financial crisis of the early 1990's, risk management was started by Value-at-Risk which has become one of the key measures of financial market risk. VaR is defined as an estimate of the maximum potential loss to be expected by a financial institution over a given period with a fixed probability. The Basle Committee prescribes its use as an internal management for financial institutions and risk managers. This technique of risk measurement has known such advances in the field of risk management and being now used to control and manage risk actively. Statistically, VaR, for a given portfolio, is simply an estimate of a specified percentile of the probability distribution of the portfolio's value change over a given holding period. The specified percentile is usually taken in the lower tail of the distribution (for e.g., 95th percentile or the 99$^{th}$ one).

In the literature, several approaches have been used in Value-at-Risk estimation and prediction, such as RiskMetrics model, known as the benchmark approach. Alternatives have been developed and have outperformed the benchmark RiskMetrics model in the VaR prediction. Indeed, Danielson and de Varies (1997), McNeil and Fray (2000) and Ho et al (2000) use extreme value theory to estimate Value-at-Risk. Later, Engle and Manganelli (2004), utilize a quantile regression method. Other approaches, based on Markov switching techniques (Billio and Pelizon, 2000) and high frequency data (Beltratti and Morana, 1999) are also used. In addition, the concept of ''realized volatility'' has emerged as a new technique using the intra-day data as a better alternative to standard VaR model (Andersen, et al, 1999, Moosa and Bollen, 2002)(Note 1).

As far as VaR estimation is concerned, the GARCH family models constitute one of the most interesting framework for VaR prediction which could capture the time-varying volatility feature and provide an efficient variance prediction (Wong and So, 2003; So and Yu, 2006). In the ARCH specification by Engle (1982) and later its generalization (GARCH), by Bollerslev (1986), the residual series was supposed to be normally distributed. However, this assumption was often criticized and comprises at least three drawbacks. First, the normal distribution for asset returns can not describe the extreme events. Second, recent empirical studies find that normal distribution for residual series driven from GARCH type models may generate substantial bias in VaR estimation which mainly concerns the tail properties of this series distribution. Third, return distribution has usually a heavier tail than a normal distribution (Pollitis, 2004). This is mainly due to asymmetry and leptokurtosis of the data distribution. Therefore, several propositions have been made as alternatives to the normal distribution, such as the Student t-distribution (Bollerslev, 1987; Hansen, 1994), generalized error distribution (Nelson, 1991), generalized hyperbolic distribution (Eberlein and Keller, 1995; Barndorff-Nielsen, 1997), stable distribution (McCulloch, 1996), non-central Student t distribution (Harvey and Siddique, 1999), Gram–Charlier distribution (Rockinger and Jondeau, 2001), Pearson's Type IV distribution (Premaratne and Bera, 2001; Yan, 2005; Bhattacharyya et al, 2007), skewed t distribution (Jondeau and Rockinger, 2003), Johnson's SU distribution (Yan, 2005), and mixture of normal distributions (Alexander and Lazar, 2006).

To the best of our knowledge, most empirical studies dealing with VaR calculation have focused on stock market risk (Brooks and Persand, 2002; Giot and Laurent, 2003b, 2004; Huang and Lin, 2004; Chiu et al., 2005). A very few studies have attempted to take into account financial crash and crisis in market risk measurement, especially in VaR context. In this paper, we use GARCH models in order to investigate the effects of subprime crisis, which have affected the American stock markets in July 2007, on VaR estimation. For this purpose, we investigate three GARCH specifications, GARCH, IGARCH and GJR-GARCH models. Each will be adjusted by three residuals distributions; namely normal, Student and Skewed Student distribution.

The remainder of this paper is organized as follows. The VaR concept is described in section 2. Section 3 presents the GARCH family models used for VaR prediction. Section 4 investigates the GARCH family estimation which depends on three residual distributions. Section 5 focuses on model based VaR evaluation techniques. The last section presents the data set, preliminary studies and empirical findings. Finally, we conclude the paper.

## 2. General concept of long and short position Value-at-Risk

The VaR technique was introduced by JP Morgan in 1990 to measure the risk of declining values of financial assets. As in Giot and Laurent (2003), we focus on modelling VaR for portfolios defined on *long* and *short* trading positions.

### *2.1 Long trading positions VaR*

Following Giot and Laurent(2003), traders or portfolio managers have long trading positions if they bought the traded asset and are concerned when the price of the asset falls. To define the long position VaR, let $X_t$ be the price of a financial asset on day *t*. A *k*-day long position VaR on day *t* is defined as the amount for which the probability, that it exceeds the loss $X_{t-k} - X_t$, is equal to a confidence level $1-\alpha$. This could be formulated as follows:

$$Pr\left(X_{t-k} - X_t \leq VaR_{t,k}\right) = 1-\alpha. \qquad (1)$$

For one-day $VaR$, this could be rewritten very simply:

$$Pr\left(X_{t-1} - X_t \leq VaR_{t,1}\right) = 1-\alpha. \qquad (2)$$

Financial time series of returns are defined as:

$$r_t = Log\left(X_t\right) - Log\left(X_{t-1}\right). \qquad (3)$$

Under the assumption, that asset's scaled returns follow a given distribution noted $D$ (Note 2), then the long position one-day VaR at level $\alpha$ is given by

$$VaR_{t,1,Long} = \mu + q_{\alpha}^{D}\sigma_t, \qquad (4)$$

where $\mu$ is the mean of return series, $q_{\alpha}^{D}$ denotes the $\alpha^{th}$ percentile according to the statistical distribution $D$ and $\sigma_t$ indicates the standard deviation or volatility on day t, which could be obtained via GARCH family models.

### *2.2 Short trading positions VaR*

The short position VaR is defined when traders incur losses when stock prices increase. Giot and Laurent (2003) indicate that in the case of short position VaR, the trader loses money when the price increases because he would have to buy back the asset at a higher price than the one he got when he sold it. Thus, we consider also a short position in the investment which is defined as follows:

$$\Pr\left(X_t - X_{t-1} \leq VaR_{t,1}\right) = 1 - \alpha. \tag{5}$$

We obtain:

$$VaR_{t,1,\text{Short}} = \mu + q_{1-\alpha}^{D}\sigma_t, \tag{6}$$

Where $q_{1-\alpha}^{D}$ denotes the $(1-\alpha)^{th}$ percentile according to the statistical distribution $D$.

From equations (4) and (6), we note that VaR depends on three components: the mean $\mu$, the quantile $q_{\alpha}^{D}$ and conditional standard deviation $\sigma_t$. In the next sections, we will attempt to model these components. Indeed, for the mean and conditional standard deviation, we use GARCH type models, and for the quantile, we adjust residual series by three distributions such as normal, student t and skewed student t distributions.

**3. GARCH family models and volatility dynamics**

We consider the return series defined by equation (3). Let the information up to time t be designed *by* $\Omega_t$. The standard GARCH(*p,q*) model developed by Bollerslev (1986) is defined by

$$\begin{aligned} r_t &= \mu + \varepsilon_t, \quad \varepsilon_t \backslash \Omega_{t-1} \sim D(0, \sigma_t^2) \\ \sigma_t^2 &= w + \phi(L)\varepsilon_t^2 + \theta(L)\sigma_t^2, \end{aligned} \tag{7}$$

where $w > 0, \phi(L) = a_1 L + a_2 L^2 + \cdots + a_p L^p$ denotes the lag polynomial with $p$ order according to ARCH part, with $a_i \geq 0,\ i = 1,2,\ldots,p$. $\theta(L) = b_1 L + b_2 L^2 + \cdots + b_q L^q$ is the lag polynomial with $q$ order according to GARCH part, with $b_j \geq 0,\ j = 1,2,\ldots,q$.

$L$ is the lag operator such as $L^i\varepsilon_t^2 = \varepsilon_{t-i}^2,\ i = 1,2,\ldots,p$ and $L^j\sigma_t^2 = \sigma_{t-j}^2,\ j = 1,2,\ldots,q$. $D$ is a conditional density with zero mean and variance $\sigma_t^2$ which can be supposed taking many specifications. In this work, we consider three conditional distributions, namely the normal, student t and skewed student t.

*3.1 The standard GARCH(1,1) model*

Bollerslev et al. (1992) show that GARCH(1,1) specification yields to better results in most applied situations. More recent empirical research illustrates that GARCH (1,1) model could adjust financial asset returns with very successful variance prediction. In this study, we apply the GARCH(1,1) family models and their extensions to estimate the Value-at-Risk. The GARCH(1,1) could be presented as follows:

$$\begin{aligned} r_t &= \mu + \varepsilon_t, \quad \varepsilon_t \backslash \Omega_{t-1} \sim N(0, \sigma_t^2) \\ \sigma_t^2 &= w + a\varepsilon_{t-1}^2 + b\sigma_{t-1}^2, \end{aligned} \tag{8}$$

where $w$, $a$ and $b$ are non-negative parameters with the restriction of $a + b < 1$ to ensure the positivity of conditional variance $\sigma_t^2$ and stationarity as well (Bollerslev,1986).

*3.2 The IGARCH(1,1) model*

Engle and Bollerslev (1986) introduce the IGARCH (Integrated GARCH model) in order to take into account the existence of a unit root in the variance. Therefore, the IGARCH (1, 1) model is defined as follows:

$$\begin{aligned} r_t &= \mu + \varepsilon_t, \quad \varepsilon_t \backslash \Omega_{t-1} \sim D(0, \sigma_t^2) \\ \sigma_t^2 &= w + (1-a)\sigma_{t-1}^2 + a\varepsilon_{t-1}^2. \end{aligned} \tag{9}$$

This model is a better alternative to GARCH (1, 1) specification. When $\mu = 0$ and $b = \lambda$ (smoothing parameter), the IGARCH (1, 1) model reduces to the so called "RiskMetrics" model of JP Morgan (1996) which is defined by:

$$\begin{aligned} r_t &= \varepsilon_t, \quad \varepsilon_t \backslash \Omega_{t-1} \sim D(0, \sigma_t^2), \\ \sigma_t^2 &= \lambda\sigma_{t-1}^2 + (1-\lambda)\varepsilon_{t-1}^2, \end{aligned} \tag{10}$$

where $0 < \lambda < 1$. To improve the performance of RiskMetrics model, we need to set the smoothing parameter equal to 0.94 for daily data and to 0.97 for monthly data. It was shown that $\lambda$ = 0.94 produces very good forecasts for 1-day volatility (RiskMetrics, 1996; Fleming et al., 2001).

*3.3 The GJR-GARCH model*

An alternative approach allows for capturing the effect of asymmetry of the disturbances on the conditional variance based on GJR-GARCH model. This model was introduced by Glosten, Jagannathan and Runkle (1993). This specification is equivalent to the GARCH one, with the only difference being the incorporation of a dummy variable multiplied by the squared of residual terms in the variance equation. Formally, the GJR-GARCH (1.1) model is given by:

$$\begin{aligned} r_t &= \mu + \varepsilon_t, \quad \varepsilon_t \backslash \Omega_{t-1} \sim D(0, \sigma_t^2) \\ \sigma_t^2 &= w + \alpha \varepsilon_{t-1}^2 + \varphi I_{\varepsilon-1<0} \varepsilon_{t-1}^2 + b\sigma_{t-1}^2, \end{aligned} \tag{11}$$

where $I_{\varepsilon<0} = \begin{cases} 1 & if \quad \varepsilon_t < 0 \\ 0 & if \quad \varepsilon_t \geq 0 \end{cases}$ and $a + b + 0.5\varphi < 1$.

The GJR-GARCH is a model with threshold where the dummy variable is equal to 1 if the residual of the previous period is negative and equal to zero otherwise. Thus, the conditional variance follows two different processes according to the sign of the error terms.

## 4. Modeling residual series and GARCH family models estimation

Various probability distributions could be used in the framework of the MLE procedures to estimate the parameters of GARCH family models. In this paper, we investigate the normal, the Student t and the skewed t distributions.

*4.1 The normal distribution*

The use of normal density offers the advantage of simplicity. Recent studies recognize that the properties of the normal distribution are not compatible with the stylized facts (leptokurtic and asymmetrical conditional distribution) generally observed in financial asset return series.

For instance, under normality hypothesis, the residual terms is normally distributed, and we write:

$$f(\varepsilon_t) = \frac{1}{\sigma_t\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma_t^2}\varepsilon_t^2\right). \tag{12}$$

Under this hypothesis, the Log-likelihood function for $GARCH$ family models is defined by

$$L(r_t \backslash \theta) = \sum_{t=1}^{T} -\frac{1}{2}\left[\ln(2\pi) + \ln(\sigma_t^2) + z_t^2\right], \tag{13}$$

Where $\theta$ and $\sigma_t^2$ are respectively the parameters vector and conditional variance corresponding to each GARCH specification, developed previously, and $z_t = \frac{\varepsilon_t}{\sigma_t} \sim N(0,1)$.

Under the normality hypothesis, the one-day VaR, for each GARCH specification is obtained by replacing the percentile $q_\alpha^D$, in equation (4), by the $\alpha^{th}$one relevant to the standard normal distribution $Norm_\alpha$. Then, the long and short position $VaR$ under the normality hypothesis are given by:

$$\begin{aligned} VaR_{1,\alpha,Long} &= \hat{\mu} + \hat{\sigma}_t Norm_\alpha, \\ VaR_{1,\alpha,Short} &= \hat{\mu} + \hat{\sigma}_t Norm_{1-\alpha}, \end{aligned} \tag{14}$$

with $\sigma_t$ is specific to each $GARCH$ model described previously and $Norm_\alpha$ and $Norm_{1-\alpha}$denote respectively the $\alpha^{th}$ and $(1-\alpha)^{th}$ percentile according to the standard normal distribution. We note that the normality hypothesis could lead to convergent estimates of the parameters of GARCH model (principle of the pseudo-maximum likelihood). Nevertheless, the specification of the conditional distribution does not only relate to the problem of parameters estimation of GARCH models, but also and in more direct way to the determination of the fractile of the conditional distribution. The choice of a normal specification may have significant effect on the $VaR$ estimates and forecasts.

*4.2 The Student distribution*

The Student distribution offers the possibility of modeling tails of distribution thicker than those of the normal one (leptokurtic distribution). More precisely, the Kurtosis of the Student distribution is determined by its degree of freedom $\eta$. Consequently, within the GARCH framework, this estimated parameter allows to capture the excess of Kurtosis which could not be explained even by GARCH model itself. Under this hypothesis, the standardized residual series $z_t$ follows the standard Student t distribution defined by

$$g(z_t, \eta) = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi\eta}\,\Gamma\left(\frac{\eta}{2}\right)}\left(1+\frac{z_t^2}{\eta}\right)^{-\frac{\eta+1}{2}} \tag{15}$$

With $\Gamma(k) = \int_0^{+\infty} e^{-x}x^{k-1}dx$ and $z_t = \frac{\varepsilon_t}{\sigma_t} \sim N(0,1)$. In this case, the Log-likelihood function for GARCH family models is defined by:

$$L(\eta_t \backslash \theta) = \sum_{t=1}^{T} \ln\left(\frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\,\Gamma\left(\frac{\eta}{2}\right)}\right) - \frac{1}{2}\ln(\sigma_t^2) - \frac{\eta+1}{2}\ln\left(1+\frac{z_t^2}{\eta-2}\right), \tag{16}$$

where $\theta$ and $\sigma_t^2$ are respectively the parameters vector and conditional variance corresponding to each GARCH specification developed previously.

The standardized Student distribution is symmetric and the skewness is null if $\eta > 3$. Moreover, this distribution is leptokurtic if $\eta > 4$. Under the hypothesis that the residual series follows the Student distribution , the one-day $VaR$, for each GARCH specification is obtained by replacing the percentile $q_\alpha^N$, in equation (4), by the $\alpha^{th}$one relevant to the standard Student t distribution $Stu_\alpha$. Then, we obtain

$$\begin{aligned} VaR_{1,\alpha,Long} &= \hat{\mu} + \hat{\sigma}_t Stu_\alpha, \\ VaR_{1,\alpha,Short} &= \hat{\mu} + \hat{\sigma}_t Stu_{1-\alpha}, \end{aligned} \tag{17}$$

with $\sigma_t$ is the conditional standard deviation relevant to each GARCH specification described previously and $Stu_\alpha$ and $Stu_{1-\alpha}$ denote respectively the $\alpha^{th}$ and $(1-\alpha)^{th}$percentile of a Student distribution with $\eta$ degree of freedom.

*4.3 The skewed Student t distribution*

Fernandez and Steel (1998) attempt to extend the Student-t distribution by adding a skewness parameter (Note 3) in order to accommodate the excess of skewness and kurtosis. They allow the introduction of skewness in continuous unimodal and symmetric distribution $g(.)$ by changing the scale at each side of the mode. The main drawback of this density is that it is expressed in terms of the mode and the dispersion. Lambert and Laurent (2001) re-expressed the skewed student-t density in such a way that the innovation process has zero mean and unit variance. The conditional mean equation could be written as follows: $r_t = \mu_t + \varepsilon_t$ with $\mu_t = \mu$.

where $\varepsilon_t = \sigma_t z_t$ follows the GARCH(1, 1), IGARCH(1,1) and GJR-GARCH(1,1) processes. It is widely observed that the distribution of residuals tends to appear asymmetric and leptokurtic. To capture excess kurtosis and skewness, we use the skewed Student-t distribution of Lambert and Laurent (2001).

Following the work of Giot and Laurent (2003) (Note 4), we use a standardized version of the skewed Student-t distribution introduced by Fernandez and Steel (1998). According to Lambert and Laurent (2001), and provided that $v > 2$, the innovation process $z_t$ is said to be (standardized) skewed Student-t distributed, i.e. $z_t \sim SKST(0,1,\xi,v)$, if:

$$f(z_t/\xi, v) = \begin{cases} \frac{2}{\xi+1/\xi}\, g(\xi(sz_t+m)/v) & \text{if } z_t < -\frac{m}{s} \\ \frac{2}{\xi+1/\xi}\, g((sz_t+m)/\xi / v) & \text{if } z_t \geq -\frac{m}{s}, \end{cases} \tag{18}$$

where $g(./v)$ is the symmetric (unit variance) Student-t density (Note 5) and $\xi$ is the asymmetry parameter (Note 6). The parameters $m$ and $s^2$ are respectively the mean and the variance of the non-standardized skewed student-t distribution:

$$m = m(\xi, v) = \frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(\xi - \frac{1}{\xi}\right) \tag{19}$$

and $$s^2 = \left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2 \qquad (20)$$

$s = \sqrt{s^2(\xi, v)}.$

The sign of $\ln(\xi)$ indicates the direction of skewness, and represents the degree of asymmetry of residual distribution. Hence, if $\ln(\xi) > 0$ $(\ln(\xi) < 0)$, the skewness is positive (negative) and the probability density function is skewed to the right (left). When $\xi = 1$, the skewed Student-t distribution is equal to the standard student-t distribution.

The functions $skst_\alpha(v, \xi)$ and $skst_{1-\alpha}(v, \xi)$ are respectively the left and the right quantile at $\alpha\%$ for the skewed Student-t distribution with $v$ degrees of freedom and asymmetry coefficient $\xi$. The log-likelihood function of a standardized (zero mean and unit variance) skewed Student-t distribution is given by:

$$L_{Skst} = T\left\{\ln\Gamma\left(\frac{v+1}{2}\right) - \ln\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}\ln[\pi(v-2)] + \ln\left(\frac{2}{\xi+\frac{1}{\xi}}\right) + \ln(s)\right\} - \frac{1}{2}\sum_{t=1}^{T}\left\{\ln(\sigma_t^2) + (1+v)\ln\left[1 + \frac{(sz_t+m)^2}{v-2}\xi^{-2I_t}\right]\right\}, \qquad (21)$$

where $$I_t = \begin{cases} 1 & if \ z_t \geq -m/s \\ -1 & z_t < -\frac{m}{s} \end{cases} \qquad (22)$$

with $\xi$ is the asymmetry parameter, $v$ is the degree of freedom of the distribution and $h_t = \sigma_t^2$ satisfies the equations of the GARCH volatility models considered in our study.

The one-step-ahead forecast of the conditional mean $\hat{\mu}_t$ and conditional variance $\hat{\sigma}_t^2$ is computed based on past information. Hence, the one-day-ahead $VaR$ computed at time $(t-1)$, under the skewed Student-t distribution, for long and short trading positions are:

$$\begin{cases} \widehat{VaR}_{t,Long} = \hat{\mu} + skst_\alpha(v, \xi)\hat{\sigma}_t \\ \widehat{VaR}_{t,Short} = \hat{\mu} + skst_{1-\alpha}(v, \xi)\hat{\sigma}_t \end{cases} \qquad (23)$$

Lambert and Laurent (2001) show that the quantile function $skst_\alpha(v, \xi)$ of a non standardized skewed student density is:

$$skst_\alpha(v, \xi) = \begin{cases} \frac{1}{\xi}st_{\alpha,v}\left(\frac{\alpha}{2}(1+\xi^2); v\right) & if\ \alpha < \frac{1}{1+\xi^2} \\ -\xi\, st_{\alpha,v}\left(\frac{1-\alpha}{2}(1+\xi^{-2}); v\right) & if\ \alpha \geq \frac{1}{1+\xi^2}. \end{cases} \qquad (24)$$

where $st_{\alpha,v}$ is the quantile function of the (unit variance) Student-t density. We simply obtain the quantile function of the standardized Skewed student-t:

$$skst_\alpha(v, \xi)^* = \frac{skst_\alpha(v,\xi) - m}{s}, \qquad (25)$$

The value of parameter $v$ measures the degree of fat tails in the $VaR$ density. If $v > 2$, the density has fat tails. The value of $\xi$ determines the degree of asymmetry in the $VaR$ density. If $\xi < 1$ $(\ln(\xi) < 0)$, $|skst_{\alpha,v,\xi}| > |skst_{1-\alpha,v,\xi}|$ and we will get a bigger $VaR$ for long position than short position, i.e. the VaR for long trading positions will be larger for the same conditional variance than will the $VaR$ for short trading positions. When $\xi > 1$, $(\ln(\xi) > 0)$, $|skst_{\alpha,v,\xi}| < |skst_{1-\alpha,v,\xi}|$ and we will get the reverse result.

**5. Evaluation methods of model-based VaR**

There are several methods for determining accuracy and efficiency of model-based $VaR$ measurement. These methods are based on risk management loss functions such as the binary and quadratic loss functions (Lopez, 1999), on LR test of unconditional and conditional coverage (Kupiec, 1995; Christoffersen, 1998) (Note 7) and on Mean relative scaled bias (MRSB) proposed by Hendricks (1996). In this paper, we use two approaches to evaluate model's capacity and accuracy in Value-at-Risk estimation, namely the *Sample coverage* used by So and Yu (2006) and $LR$ test of Kupiec (1995).

*5.1 Sample coverage approach*

The sample coverage is used in the literature to evaluate Value-at-Risk estimation. This approach is based on the computation of the empirical failure rate which is defined by $\hat{\alpha} = k/T$, where $T$ is the total number of observations, $k = \sum_{t=1}^{T} I_t$ denotes the number of exceptions; i.e. the number of times returns exceed (in absolute value) the forecasted $VaR$ in the sample, and $I_t$ is a Bernoulli random variable defined by

$$I_t = \begin{cases} 1 & if\ r_t < VaR_t \\ 0 & if\ r_t \geq VaR_t \end{cases}, \tag{26}$$

In a risk management context, $I_t$ represents the *binary loss function*. If the predicted $VaR$ estimate is unable to cover the realized loss, this is called an *exception*. Equal weight is accorded to each loss that exceeds the $VaR$ estimate; and all other profits and losses have a zero weight. We expect that $\hat{\alpha}$ is close to $\alpha$ for a good $VaR$ estimation method. Therefore, the smaller the discrepancy between $\hat{\alpha}$ and $\alpha$, the better performance is the estimation method. To assess the overall performance of each model, we rank the methods according to $|\alpha - \hat{\alpha}|$ for each case.

*5.2 Kupiec test for unconditional coverage*

The Kupiec test is the second method used to test the accuracy of the computed $VaR$ values. A likelihood-ratio test is proposed by Kupiec (1995) in order to test if the sample point estimate is statistically consistent with the $VaR$ model's prescribed confidence level. Statistically, testing the accuracy of the model is equivalent to testing the hypothesis $H_0: \alpha = \hat{\alpha}$ versus $H_1: \alpha \neq \hat{\alpha}$. Under the null hypothesis, the likelihood-ratio statistic, denoted by $LR_{uc}$, follows the chi-square distribution with one degree of freedom. That is:

$$LR_{uc} = -2Log\left[\frac{\alpha^k(1-\alpha)^{T-k}}{\hat{\alpha}^k(1-\hat{\alpha})^{T-k}}\right] \sim \chi^2(1) \tag{27}$$

**6. Empirical analysis**

*6.1 Data description and preliminary analysis*

In this paper, we consider daily NASDAQ stock market index data. The choice of this index is attributable to the subprime crisis that has started in the US since 2007. The return series cover the period going from 01/01/2003 to 10/07/2008. In order to investigate the effect of this crisis on the VaR estimation, we break up our sample into two periods: the first covers the *stability period* (calm period) with a number of 1140 observations (from 01/01/2003 to 16/07/2007) and the second period covers the crisis period having 247 observations (from 17/07/2007 to 10/07/2008). We introduce and characterize our dataset at this stage to set the statistical properties of the series during the two periods. More specifically, we consider return series $r_t$ of closing prices expressed in (3) (Note 8). Table 1provides the descriptive characteristics for the return series, while descriptive graphs (price, daily returns) are included in Figure 1.

Summary statistics clearly indicate that the index is more risky and less profitable in the crisis period than in the normal (stable) one. Indeed, means values are positive in the stability period and negative in crisis one, and standard deviations are higher for this turbulent period. During this period, the return series are extremely volatile which lead to a succession of extremely large positive and negative returns within a very short time span.

The application of some unit root tests indicates that the series are stationary for the two sub-samples series. In addition, it is clear that the normality hypothesis is rejected for the first sub-sample, and accepted for the second one (crisis period). Indeed, we notice that in the stability period, skewness is significantly negative. Moreover, excess kurtosis is significantly different from zero. This situation indicates that the empirical distribution of returns displays fatter tails than the Gaussian distribution in this period. That is, large changes are more often to occur than a normal distribution would imply. For the crisis period, the skewness, kurtosis and Jarque-Berra statistics indicate that the normality hypothesis could not be rejected.

*6.2 GARCH family models fitting*

In our empirical study, we use first the maximum likelihood method to give parameters estimates for each model defined presciently (Note 9). These estimates could be used to assess the in-sample performance of various GARCH models in forecasting VaR for long and short positions. In this subsection, we estimate three GARCH family models (GARCH(1,1), IGARCH(1,1) and GJR-GARCH(1,1)) under the normal, Student-t, and skewed Student-t distributions innovations return series. Table 2 compares the estimation results of these models estimations. According to this table, we verify that stationarity condition is obtained, i.e. $a + b < 1$, and $(a + b)$ is close to unity for all GARCH specifications during the two periods. In addition, estimated parameters $a$ and $b$ are all positive and significantly different from zero almost in all considered $GARCH$ specifications. We find that $(a + b)$ increases when a student error model is fitted instead of normal error. In the case of Student error fitting, we observe that the coefficient $\eta$ is statistically significant at 5% confidence level for the first sub-sample. Consequently, this result indicates that NASDAQ return series is fat tailed and exhibits a leptokurtic characteristic especially in the first period, due to the rejection of the normality hypothesis for return series. However, we observe a large degree of freedom which isn't statistically significant when the data cover the crisis period. This is explained by the presence of normality detected in this period.

For the skewed student-t distribution, the asymmetric parameters are negative and statistically significant in the stability period confirming the fact that the density distribution of NASDAQ return series is skewed to the left side. However, the asymmetric parameters are also negative in the crisis period but insignificantly different from zero. Besides, the empirical results showed that fat-tail phenomenon is strong in the calm period than in the crisis one because the student parameters are significantly different from zero under the three models. As a result, we may conclude that the skewed-student-t-GARCH family models considered outperformed the other models with the normal and student-t distributions innovations in capturing the asymmetry and fat tails of the NASDAQ return distribution.

In order to compare the quality of fitness among the three $GARCH$ specifications, we report the ranking of these models based on AIC (Akaike's information criterion), SH (Schwartz), and HQ (Hannan-Quinn) information criterions (Table 3). According to these criterions, we rank the best models. We use the mean ranks to indicate the average rank for each specification. The model selection procedure is based on two steps. First, we choose the best innovation distribution for each model and we select the best one based on the mean ranks. Then, we choose the best GARCH specification for the three best distributions. Generally, student-t error models perform better than the normal and Skewed Student-t error models in the stability period. However, in the crisis period, results show the superiority of the normal distribution (table 3). From the empirical analysis of these models, the GJR-GARCH model performs the best in the two periods. Thus, we select this model in order to compute in-sample $\mathrm{VaR}$ estimations during these two periods. This result could be explained by the asymmetry in volatility which could be detected by the GJR-GARCH specification in the two considered periods.

By taking into account for the volatility asymmetry, we notice that the parameter describing the asymmetric feature in the volatility is positive and statistically significant in the two considered periods. This evidence indicates that unexpected negative returns resulted in more volatility than unexpected positive returns of the same magnitude. In addition, it is clear that volatility clustering is observed in the daily returns graphic, especially in crisis period (graph 1). The GARCH coefficient is statistically significant for all models considered and is found to be around 0.87, 0.9 and 0.8 for GARCH, IGARCH and GJR-GARCH models respectively, under the three distributions for our daily financial time series relative to turbulent period. Given the values of this coefficient, it is obvious that large values of $\sigma_{t-1}^2$ will be followed by large values of $\sigma_t^2$, and small values of $\sigma_{t-1}^2$ will be followed by small values of $\sigma_t^2$.

*6.3 Assessing the VaR model performance*

In order, to assess the performance of based Value-at-Risk model (GJR-GARCH), we present a range of summary statistics that address a number of these different aspects of VaR models to risk managers. Two types of performance criterion are employed: *sample coverage* and *Kupiec LR test for unconditional coverage*.

By adopting VaR as a quantitative measurement of downside risk, risk managers desire to achieve a failure rate equal to the fixed level confidence $\alpha$.

6.3.1 VaR model performance in stability period

During the first period (stability period), the VaR results computed by the normal, Student-t, and skewed Student-t via the best GJR-GARCH(1,1) model for the long and short trading positions are given in Table 4. The first remark is that the LR Kupiec test accepts the null hypothesis for all confidence levels, for the three distributions in long and short position VaR (except of the 0.5% confidence level). Then, the failure rate is significantly equal to the prescribed confidence levels. This result clearly indicates that the GJR-GARCH(1,1) is very successful in VaR estimation for our data set.

If comparing between distributions, we observe that these later perform better at low confidence levels. Indeed each innovation distribution has the lowest value of sample coverage in the 0.25% confidence level for long and short trading positions. On other hands, the comparison between the three distributions shows that Student-t and skewed Student-t perform better than the normal distribution according to the lowest value of sample coverage (0.0747% versus 1.006%).

6.3.2 VaR model performance in crisis period

As mentioned in Table 5, we obtain the same conclusion for the Kupiec's LR test. Indeed, the null hypothesis is rejected only for 0.5% confidence level and for long trading position. In addition, some values of LR test are not available, due to the absence of exceptions especially for skewed Student-t distribution for long position VaR. Moreover, we note that the Kupiec's LR test accept its null hypothesis for all confidence levels under the three distributions. This result shows that the selected GJR-GARCH model is also successful in VaR calculation in the crisis period. It is clear that the normal distribution performs better in this period. This result could be explained by the return series normality in the turbulent period (Table1).

*6.4 Effects of subprime crisis on value at Risk estimation*

To investigate the effects of subprime crisis on the VaR's amounts computed by the selected GJR- GARCH(1,1) model, we analyze the descriptive statistics of VaR values in the two sub-periods considered. According to tables 6 and 7, we deduce that the means and standard deviations VaR values, in crisis period, are larger, in absolute value, than those in stable period for the case of long and short trading positions. These findings could be explained by the speculative behavior of investors in the US stock market and in particular to moves of taking buy and sell positions in order to realize short term gains. These speculative actions may lead to generalized increases in almost of all the of financial assets prices to levels that do not reflect market reality but rather to the formation of a speculative bubble. These bubbles lead to a situation where asset prices are significantly different from their fundamental values.

The subprime crisis has led to the deterioration of the American mortgage market. It has been initiated due mainly to an excess liquidity and quickly degenerated into a credit crisis followed by stock market crisis. The stock market prices have rapidly and greatly decreased while the trade volumes ought to increase. Consequently, the volatility of Nasdaq100 index return series, induced by the stock market speculation, has increased during the subprime crisis generating the rise in VaR values.

Finally, we conclude that the subprime crisis has significant effects on Value-at-Risk estimates as it pushes up the amount of maximum losses supported by speculative investors.

## 7. Conclusion

GARCH type models are widely used to model financial market volatilities in risk management applications. They may be used to model risk attributes such as volatility clustering and the long-range dependence structure that exists in financial indices. In this research work, the GARCH(1,1), IGARCH(1,1), and GJR-GARCH(1,1) models with the normal, Student-t and skewed Student-t error distributions are investigated to estimate one-day-ahead VaR for daily NASDAQ index returns. We focus on the investigation of the effects of American subprime crisis on Value-at-Risk estimation using some GARCH family models listed above. Also, we have assessed the performance of VaR models using the sample coverage and Kupiec LR test for unconditional coverage. The first empirical result confirms that the GJR-GARCH(1,1) specification is chosen in terms of information criterion in the two considered periods. This choice led us to a very successful one-day-ahead VaR calculation. Indeed, the LR Kupiec test shows that this model accept the null hypothesis of equality between the failure rate and the specified VaR confidence levels for in the case of long and short trading positions. If we choose between distributions innovations, we conclude that Student and Skewed Student are preferred in the calm period and that the normal distribution performs the best for the turbulent period due to the observed normality in the return series. Finally, we conclude that the subprime crisis has significant effects on Value-at-Risk estimates as it pushes up the amount of maximum losses supported by speculative investors.

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**Notes**

Note 1. For comprehensive overview of VaR, See Duffie and Pan (1997), Jorion (2001).

Note 2. In this study, we will consider normal, Student and skewed Student-t distribution for index return series.

Note 3. Hansen (1994) and Paolella (1997) have proposed other asymmetric Student-t densities.

Note 4. AR-APARCH model with a skewed Student density forecasts correctly (both in- and out-of-sample) the 1-day-ahead VaR for three international stock indexes and three US stocks of the Dow Jones index (Giot and Laurent, 2003).

Note 5. The density $f(z_t/1/\xi, v)$ is the symmetric of $f(z_t/\xi, v)$ with respect to the mean, hence $\ln(\xi)$ is better solution to indicate sign of the skewness._

Note 6. The asymmetry coefficient $\xi > 0$ is defined such that the ratio of probability masses above and below the mean is $\frac{P(z \geq 0/\xi)}{P(z < 0/\xi)} = \xi^2$

Note 7. For details see Hung et al(2007).

Note 8. We chose to multiply return series by 100 to express the returns on percentage

Note 9. We use the G@RCH 2.3 Ox package for the model estimation.

Table 1. Summary statistics on daily returns during two periods

| | Stability period | Crisis period |
|---|---|---|
| *Summary statistics* | | |
| Obs | 1139 | 247 |
| Min | -4.301 | -4.496 |
| Max | 5.925 | 4.284 |
| Mean | 0.059 | -0.047 |
| St Dev | 1.155 | 1.574 |
| *Unit root tests* | | |
| ADF | -35.308 | -17.834 |
| P-P | -35.399 | -18.009 |
| KPSS | 0.094 | 0.113 |
| Z-A | -13.561 | -8.629 |
| *Normality* | | |
| Skewness | 0.039 | -0.009 |
| (p-value) | (0.594) | (0.949) |
| Excess Kurtosis | 1.34 | 0.048 |
| (p-value) | (0.000) | (0.879) |
| JB stat | 85.551 | 0.028 |
| (p-value) | (0.000) | (0.986) |

Note: this table reports summary statistics on daily stock market returns before and on crisis period. ADF, P-P, KPSS and Z-A denote the augmented Dickey Fuller, Phillips-Perron, Kwiatkowski-Phillips-Schmidt-Shin and Zivot and Andrews unit root tests respectively

Table 2. GARCH family parameter estimation in the two period

| Period | Stability period | | | Crisis period | | |
|---|---|---|---|---|---|---|
| Distribution | Normal | Student | Skewed t | Normal | Student | Skewed t |
| GARCH model | | | | | | |
| $\mu$ | 0.0565 | 0.0632* | 0.0493* | 0.0176 | 0.0216 | 0.0140 |
| | (1.838) | (2.080) | (2.440) | (0.204) | (0.251) | (0.164) |
| $w$ | 0.0062 | 0.0043 | 0.0075* | 0.1161 | 0.1181 | 0.0807* |
| | (1.413) | (1.029) | (1.729) | (1.182) | (1.192) | (2.270) |
| $a$ | 0.0249* | 0.0235* | 0.0471* | 0.0752* | 0.0758* | 0.0807* |
| | (3.154) | (2.789) | (3.726) | 2.084 | 2.060 | (2.270) |
| $b$ | 0.9691* | 0.9722* | 0.9392* | 0.8796* | 0.8782* | 0.8825* |
| | (97.58) | (96.18) | (55.27) | (15.95) | (15.84) | (17.39) |
| Student DF, $\eta$ | --- | 13.763* | --- | --- | 59.022 | --- |
| | | (2.817) | | | NaN | |
| Assymetry, $\ln(\xi)$ | --- | --- | 10.854* | --- | --- | 53.909 |
| | | | (3.433) | | | (0.393) |
| Tail, $\upsilon$ | --- | --- | -0.0761* | --- | --- | -0.1241 |
| | | | (-2.007) | | | (-1.485) |
| IGARCH model | | | | | | |
| $\mu$ | 0.0556 | 0.0636* | 0.0471* | 0.0154 | 0.0210 | 0.0085 |
| | (1.807) | (2.091) | (2.319) | (0.179) | (0.255) | (0.101) |
| $w$ | 0.0007 | 0.0003 | 0.0020 | 0.0302 | 0.0317 | 0.0288 |
| | (0.484) | (0.238) | (1.263) | (1.231) | (1.300) | (1.247) |
| $a$ | 0.0266* | 0.0248* | 0.0503* | 0.0909* | 0.0932* | 0.0927* |
| | (3.338) | (2.948) | (3.793) | (2.239) | (2.328) | (2.461) |
| Student DF, $\eta$ | --- | 12.539* | | --- | 53.843 | --- |
| | | (2.995) | | | NaN | |
| Assymetry, $\ln(\xi)$ | --- | --- | 9.176* | --- | --- | 57.948 |
| | | | (3.904) | | | NaN |
| Tail, $\upsilon$ | --- | --- | -0.0814* | --- | --- | -0.1401 |
| | | | (-2.106) | | | (-1.658) |
| GJR –GARCH model | | | | | | |
| $\mu$ | 0.0439 | 0.0531 | 0.0284 | -0.0227 | -0.0172 | -0.0552 |
| | (1.408) | (1.738) | (1.407) | (-0.266) | (-0.197) | (-0.629) |
| $w$ | 0.0047 | 0.002 | 0.0053 | 0.1917 | 0.1939 | 0.2763* |
| | (1.239) | (0.609) | (1.444) | (1.544) | (1.609) | (2.370) |
| $a$ | 0.0085 | 0.0036 | -0.0115 | -0.0465 | -0.0516 | -0.1074 |
| | (1.014) | (0.424) | (-1.114) | (-0.967) | (-1.006) | (-1.820) |
| $b$ | 0.9701* | 0.9743* | 0.9547* | 0.8473* | 0.8464* | 0.7968* |
| | (112.7) | (117.3) | (53.52) | (12.69) | (13.04) | (12.35) |
| $\varphi$ | 0.0314* | 0.0386* | 0.0949* | 0.244 | 0.2546 | 0.4523 |
| | (2.093) | (2.379) | (3.761) | (1.905) | (1.898) | (1.923) |
| Student DF, $\eta$ | | 13.145* | --- | --- | 38.604 | --- |
| | | (2.995) | | | (0.317) | |
| Assymetry, $\ln(\xi)$ | | --- | -0.1008* | --- | --- | -0.1955 |
| | | | (-2.616) | | | (-1.840) |
| Tail, $\upsilon$ | | --- | 12.971* | --- | --- | 14.808 |
| | | | (2.970) | | | (0.983) |

Notes: The numbers in the parentheses represent the t-Student statistic of corresponding tests significance. The log-likelihood is the maximized value of the log likelihood function. (*) indicates that the parameter is statiscally significant at 5% level.

Table 3. Model selection based on the information criteria and ranking

| | Stabilty period | | | | Crisis period | | | |
|---|---|---|---|---|---|---|---|---|
| | AIC | SC | HQ | *Mean rank* | AIC | SC | HQ | *Mean rank* |
| GARCH-Normal | 3.0328 (3) | 3.0505 (2) | 3.0395 (3) | *2.66* | 3.6676 (1) | 3.7201 (1) | 3.6887 (1) | ***1 (3)*** |
| GARCH- Student | 3.0245 (2) | 3.0466 (1) | 3.0328 (1) | ***1.33 (3)*** | 3.6746 (3) | 3.7401 (2) | 3.7009 (2) | *2.33* |
| GARCH-Skewed | 3.0243 (1) | 3.0509 (3) | 3.0343 (2) | *2* | 3.6738 (2) | 3.7526 (3) | 3.7054 (3) | *2.66* |
| IGARCH- Normal | 3.0341 (3) | 3.0474 (3) | 3.0391 (3) | *3* | 3.6669 (1) | 3.7063 (1) | 3.6827 (1) | ***1 (2)*** |
| IGARCH- Student | 3.0242 (2) | 3.0419 (1) | 3.0309 (1) | ***1.33 (2)*** | 3.6739 (3) | 3.7264 (2) | 3.6949 (2) | *2.33* |
| IGARCH- Skewed | 3.0239 (1) | 3.0460 (2) | 3.0323 (2) | *1.66* | 3.6714 (2) | 3.7370 (3) | 3.6977 (3) | *2.66* |
| GJR- Normal | 3.0304 (3) | 3.0525 (3) | 3.0387 (3) | *3* | 3.6419 (1) | 3.7075 (1) | 3.6683 (1) | ***1 (1)*** |
| GJR- Student | 3.0206 (2) | 3.0471 (1) | 3.0306 (1) | ***1.33 (1)*** | 3.6486 (3) | 3.7273 (2) | 3.6802 (3) | *2.66* |
| GJR-Skewed | 3.0192 (1) | 3.0501 (2) | 3.0309 (2) | 1.66 | 3.6428 (2) | 3.7346 (3) | 3.6796 (2) | *2.33* |

Table 4. Long and Short position VaR calculated by GJR-GARCH(1,1) before crisis period

| Long position VaR | | | Short position VaR | | |
|---|---|---|---|---|---|
| Quantile | Sample coverage | LR statistic | Quantile | Sample coverage | LR statistic |
| *Normal distribution* | | | | | |
| *5%* | 0.5215% | 0.6328 | *95%* | 0.0833% | 0.0166 |
| *2.50%* | 0.3922% | 0.686 | *97.50%* | 0.5675% | 1.4075 |
| *1%* | 0.1394% | 0.2142 | *99%* | 0.3146% | 1.0382 |
| *0.50%* | 4.2989% | 68.8550* | *99.50%* | 0.1494% | 0.5722 |
| *0.25%* | **0.1006%** | 0.4109 | *99.75%* | **0.0747%** | 0.2854 |
| *Student distribution* | | | | | |
| *5%* | 0.5302% | 0.6993 | *95%* | 1.0561% | 2.8796 |
| *2.50%* | 0.5719% | 1.6597 | *97.50%* | 0.8348% | 3.6903 |
| *1%* | 0.3865% | 1.9971 | *99%* | 0.7371% | 8.8672* |
| *0.50%* | 4.7371% | 93.0795* | *99.50%* | 0.4124% | 5.9468 |
| *0.25%* | **0.0747% (1)** | 0.2854 | *99.75%* | **0.1624%** | 1.6116 |
| *Skewed Student distribution* | | | | | |
| *5%* | 1.8449% | 9.3567* | *95%* | 0.0833% | 0.0166 |
| *2.50%* | 1.3606% | 10.8340* | *97.50%* | 0.1293% | 0.0769 |
| *1%* | 0.5618% | 4.6058* | *99%* | 0.3865% | 1.9971 |
| *0.50%* | 4.7371% | 93.0795* | *99.50%* | 0.3247% | 3.2293 |
| *0.25%* | **0.0747% (1)** | 0.2854 | *99.75%* | **0.1624%** | 1.6116 |

Table 5. Long and Short position VaR calculated by GJR-GARCH(1,1) in crisis period

| Long position VaR | | | Short position VaR | | |
|---|---|---|---|---|---|
| Quantile | Sample coverage | LR statistic | Quantile | Sample coverage | LR statistic |
| *Normal distribution* | | | | | |
| *5%* | 1.8841% | 1.8551 | *95%* | 1.01% | 0.6408 |
| *2.50%* | **0.0362%** | 0.0015 | *97.50%* | 0.33% | 0.1259 |
| *1%* | 0.4493% | 0.4941 | *99%* | **0.09%** | 0.0205 |
| *0.50%* | 4.2754% | 16.3972 | *99.50%* | 0.22% | 0.2457 |
| *0.25%* | 0.1123% | 0.1225 | *99.75%* | 0.25% | NaN |
| *Student distribution* | | | | | |
| *5%* | 0.7971% | 0.352 | *95%* | 1.38% | 1.2131 |
| *2.50%* | 0.3261% | 0.1259 | *97.50%* | 0.33% | 0.1259 |
| *1%* | 0.4493% | 0.4941 | *99%* | **0.09%** | 0.0205 |
| *0.50%* | 4.6377% | 20.9656 | *99.50%* | 0.14% | 0.1164 |
| *0.25%* | **0.2500%** | NaN | *99.75%* | 0.25% | NaN |
| *Skewed t distribution* | | | | | |
| *5%* | 1.3768% | 1.2131 | *95%* | 1.52% | 1.233 |
| *2.50%* | 1.4130% | 2.8588 | *97.50%* | 1.49% | 2.123 |
| *1%* | 1.0000% | NaN | *99%* | 1.17% | 2.8769 |
| *0.50%* | 5.0000% | NaN | *99.50%* | 0.59% | 1.4287 |
| *0.25%* | **0.2500%** | NaN | *99.75%* | **0.47%** | 1.6431 |

Table 6. Descriptive statistics calculated by GJR-GARCH-Student in the stability period and by GJR-GARCH- normal in the crisis period

| Quantile | Long position VaR | | | | Short position VaR | | | |
|---|---|---|---|---|---|---|---|---|
| | mean | | Standard deviation | | mean | | Standard deviation | |
| | Stability | Crisis | Stability | Crisis | Stability | Crisis | Stability | Crisis |
| 5% | -1.9132 | -2.5138 | 0.4323 | 0.5480 | 2.0471 | 2.4684 | 0.4383 | 0.5480 |
| 2.50% | -2.3456 | -2.9911 | 0.5273 | 0.6530 | 2.4969 | 2.9456 | 0.5372 | 0.6530 |
| 1% | -2.8896 | -3.5459 | 0.6469 | 0.7751 | 3.0710 | 3.5005 | 0.6634 | 0.7751 |
| 0.50% | -3.2915 | -3.9238 | 0.7352 | 0.8582 | 3.5015 | 3.8783 | 0.7581 | 0.8582 |
| 0.25% | -3.6915 | -4.2739 | 0.8232 | 0.9352 | 3.9355 | 4.2285 | 0.8535 | 0.9352 |

Table 7. Descriptive statistics calculated by GJR-GARCH-Student in the stability period and by GJR-GARCH- normal in the crisis period

| Quantile | Long position VaR | | | | Short position VaR | | | |
|---|---|---|---|---|---|---|---|---|
| | mean | | Standard deviation | | mean | | Standard deviation | |
| | Stability | Crisis | Stability | Crisis | Stability | Crisis | Stability | Crisis |
| 5% | -2.0664 | -2.5138 | 0.4707 | 0.5480 | 1.8718 | 2.4684 | 0.4082 | 0.5480 |
| 2.50% | -2.5357 | -2.9911 | 0.5755 | 0.6530 | 2.2779 | 2.9456 | 0.4989 | 0.6530 |
| 1% | -3.1318 | -3.5459 | 0.7085 | 0.7751 | 2.7905 | 3.5005 | 0.6133 | 0.7751 |
| 0.50% | -3.5761 | -3.9238 | 0.8077 | 0.8582 | 3.1708 | 3.8783 | 0.6982 | 0.8582 |
| 0.25% | -4.0214 | -4.2739 | 0.9071 | 0.9352 | 3.5510 | 4.2285 | 0.7830 | 0.9352 |

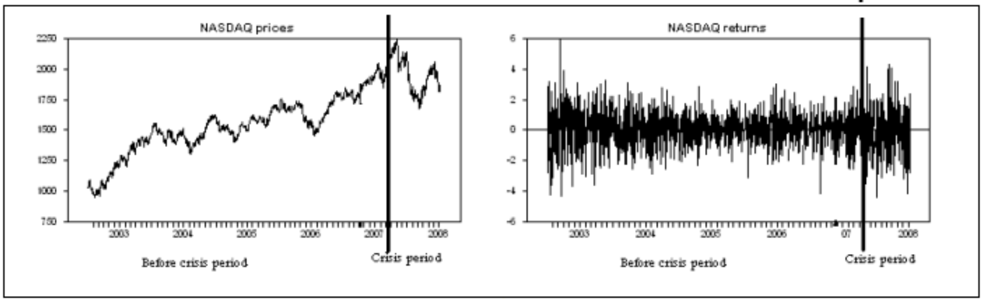


Figure 1. Graph of prices and return series in the two periods

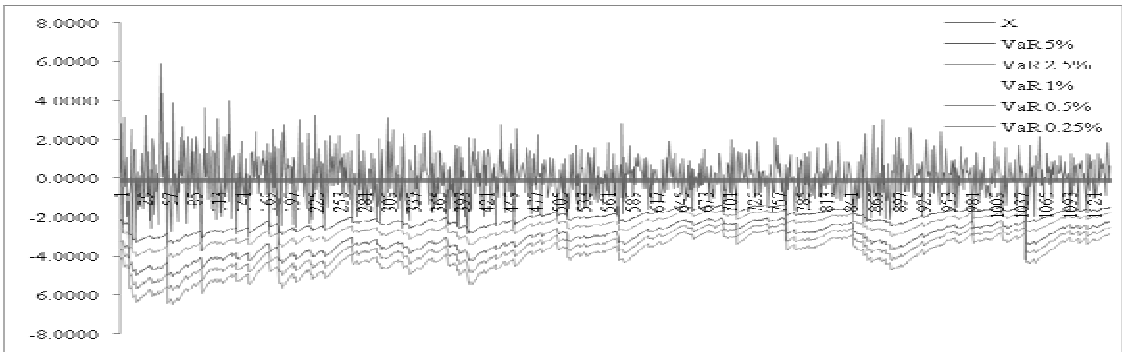


Figure 2. Returns (X) and long position VaR calculated by GJR-GARCH(1,1) with skewed Student t in the stability period

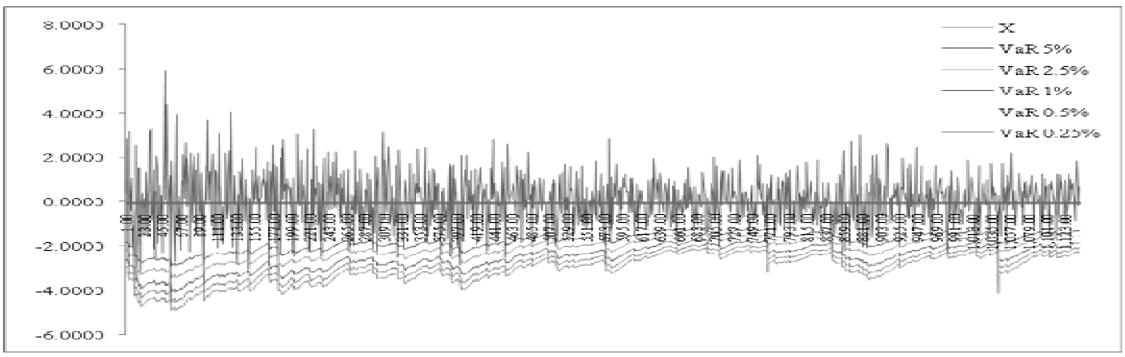


Figure 3. Returns (X) and long position VaR calculated by GJR-GARCH(1,1) with Student in the stability perio

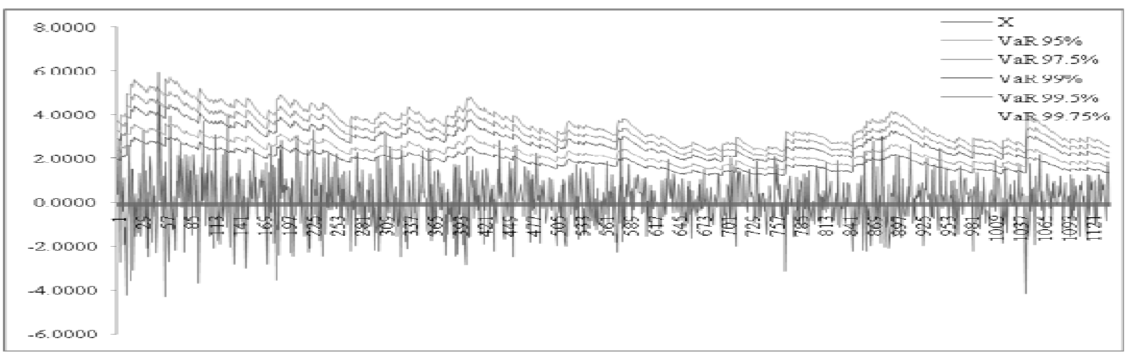


Figure 4. Returns (X) and short position VaR calculated by GJR-GARCH(1,1) with skewed Student t in the stability period

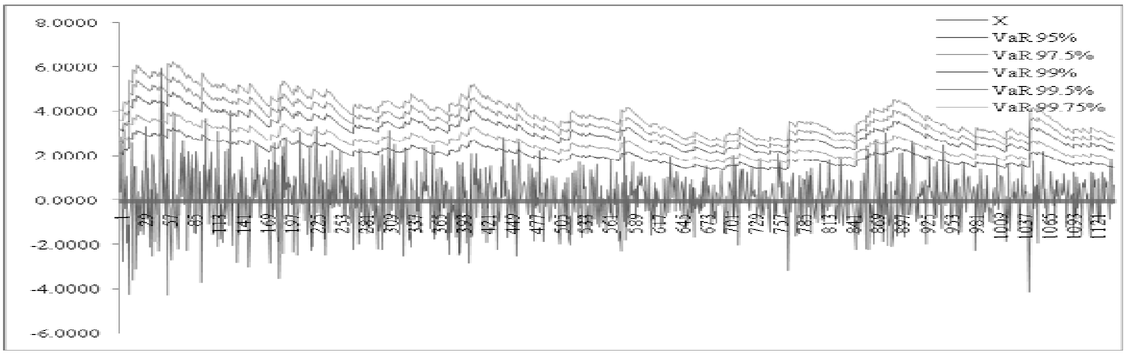


Figure 5. Returns (X) and short position VaR calculated by GJR-GARCH(1,1) with Student t in the stability period

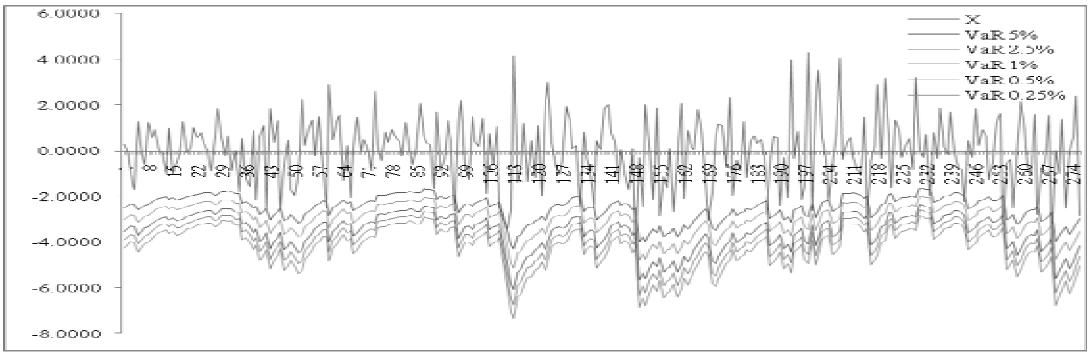


Figure 6. Returns (X) and long position VaR calculated by GJR-GARCH(1,1) with normal in the crisis period

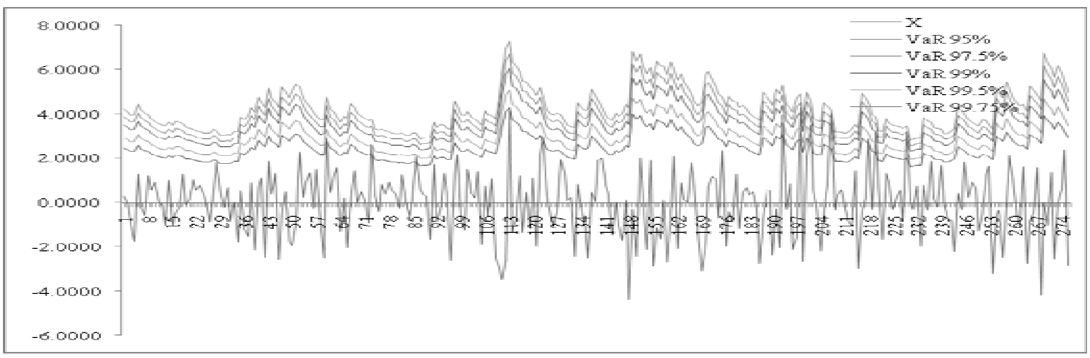


Figure 6. Returns (X) and short position VaR calculated by GJR-GARCH(1,1) with normal distribution in the crisis period