Manipulation of the Large Shareholder with the Existence of a Potential Raider

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Received: November 20, 2013 Accepted: December 7, 2013 Online Published: January 23, 2014
doi:10.5539/ijef.v6n2p40 URL: http://dx.doi.org/10.5539/ijef.v6n2p40

Abstract

This paper theoretically discusses why and how a large shareholder adopts a manipulation strategy. In the spirit of Glosten and Milgrom (1985) and Easley and O’Hara (1987), our model considers a market with a potential raider who is able to alter (say increase) the fundamental value of a firm by paying some costs. The results suggest that when the large shareholder privately knows the existence of the raider, he will probably manipulate the stock price lower in order to attract the raider buying the shares of the firm and to intervene. In particular, the large shareholder is going to manipulate the stock price in the situation when the benefit of the raider’s intervention is moderate and the shareholder possesses sufficiently many shares of the stocks. Although the manipulative strategy may result in trading losses, the intervention of the raider increases the shareholder’s total wealth.

Keywords: informed trading, market manipulation, market microstructure

1. Introduction

In the traditional market microstructure model, such as Kyle (1985), and Glosten and Milgrom (1985) among others, the informed trader buys (sells) when he knows the price is undervalued (overvalued). In their setting, the manipulative strategy does not exist because of some specific assumptions (Note 1).

Most of these papers do not take the endowments of market participants into account. In the real world, however, an investor’s trading strategy is usually influenced by his endowment holding. For example, Allen, Litov, and Mei (2004) provide evidences that large investors and corporate insiders are able to manipulate the stock price and benefit themselves. Therefore, this paper considers a theoretical model to show how and why an informed shareholder manipulates the stock price to attract a potential raider who is able to improve the operations of the firm and increase the firm value by paying some costs.

A raider who has power to alter (say increase) the true value of a firm usually holds a long position of the stocks before intervention. However, if the stock price is too high to cover the cost of intervention, it prevents the raider from intervening. Vila (1989), Kyle and Vila (1991), and Bagnoli and Lipman (1996) show that the raider will manipulate the price downward and buy stocks back with the reduced price before beginning his action to intervene. One way to manipulate the price downward is to short sell the stocks. However, large traders or institutional investors usually face short selling constraints in many exchanges. Nevertheless, we argue that even if the raider does not have the power to manipulate the price, the large shareholder, even if without the power to intervene, will manipulate the price downward in order to encourage the raider entering the market. Our model shows that as long as the large shareholder holds enough shares, he has motivation to adopt the manipulative strategy because the intervention of raider improves his wealth.

In our setting, the intervention is costly for the raider. Without any extra benefits, the raider will intervene only if he holds sufficiently many shares of the stock. For simplicity, we assume that the raider does not take any position of the stock in the beginning (Note 2). Therefore, his strategy can be seen as two: (1) buy shares and does intervene, and (2) does not enter the market.

On the other hand, the large shareholder hopes the raider to intervene because this makes his endowment holding more worthy. Thus, the shareholder will try to do some actions to encourage the raider intervening. He can do this from two ways: first, to lower the cost of intervention, and second, to lower the stock price. In our setting, the large shareholder is not able to change the cost, but it is possible for him to influence the stock price by changing
his trading strategy. Sometimes, to do this makes him lose in the short run, but benefits him more in the long run. This motivates him to temporarily trade irrationally even though he possesses superior information. We define this kind of trading behavior as “price manipulation.” The main goal of this study attempts to discuss the situations under which the equilibrium does or does not contain the manipulative strategy.

Before beginning of our model establishing, the following surveys the definitions of market manipulation and the development of the related study in the prior research. We also show the focus of this paper by comparing with the previous related literature.

Various definitions of manipulation are claimed in the past study. The most general definition is identified in the U. S. Securities and Exchange Commission: “Manipulation is intentional conduct designed to deceive investors by controlling or artificially affecting the market for a security. Manipulation can involve a number of techniques to affect the supply of, or demand for, a stock. They include: spreading false or misleading information about a company; improperly limiting the number of publicly-available shares; or rigging quotes, prices or trades to create a false or deceptive picture of the demand for a security.” (Note 3) In addition, Cherian and Jarrow (1995) say that “market manipulation occurs when an individual (or a group of individuals) trades a firm’s shares in a manner such that the share price is influenced to his advantage.” According to these definitions, in order to benefit some specific people, any actions attempting to influence the price to abnormally moves can be defined as manipulation.

There is a large number of authors study various kinds of market manipulation (Note 4). Allen and Gale (1991) divide them into three categories: Information-based manipulators manipulate the asset price by insider trading or release false information, and the examples are in Vila (1989), Benabou and Laroque (1992), and Van Bommel (2003). Action-based manipulators do this by taking actions to alter the true or perceived value of the assets, and the examples are in Kyle and Vila (1991), Vila (1989), Bagnoli and Lipman (1996), Goldstein and Gumabel (2008). Even if not possessing any private information and the power to alter the asset value, trade-based manipulators can make profits by just buying and selling (Note 5).

In this paper, we focus on the situation where a trade-based manipulator will get loss in certain round trade but this benefits him more in total wealth. We define the manipulation strategy in this paper as a trading strategy that makes a trader lose from trading but improves his total wealth.

The usage of “market manipulation” in our model is more likely to that in Chakraborty and Yilmaz (2004a; 2004b) and Brunnermeier (2000). In their concepts, a manipulator sometimes may take a “wrong” position and get short-run losses for long run profits (Note 6). Both papers built environments to show why the informed traders adopt manipulative strategies. When informed traders receive private information in different timing, Brunnermeier (2000) suggests that an early informed trader can exploit his information twice: before and after it becomes public information. In this specific information structure, the early informed trader is possible to take a manipulative strategy to disturb information revealing and enhance the informational advantage after the public announcement. Chakraborty and Yilmaz (2004a; 2004b) show that when the existence of the informed trader is uncertainty, the equilibrium contains market manipulation as long as the rounds of trade is sufficiently large.

Our model is in the spirit of Easley and O’Hara’s (1992) model which considers the uncertainty of the existence of private information. In their model, however, the informed traders need not manipulate the stock price because he knows the realized value of the asset and no exogenous power is able to alter the asset value. Consequently, the informed trader buys when receiving good signal, sells when receiving bad signal, and quits the market when not receiving any private information. To build an economy with market manipulation, our model, instead, assumes that there is a potential raider who possesses the power to improve the operations of the firm and increase the stock value. We also assume that this potential raider may or may not exist. Because the informed trader usually possesses private information regarding a company’s operations and management, we also assume that only the informed trader knows whether the raider exists or not. This assumption is important to build an asymmetric information environment between the inside and the outside shareholders and is helpful to outline that how the insider’s trading decision can influence the further development of a firm. We show that the informed trader, who possesses some shares of the firm and knows the existence of the raider, will manipulate the stock price in several specific situations in order to encourage the raider enter the market and intervene the operation of the firm. Even if the informed trader will get loss from the trade, the intervention of the raider benefits him more from increasing the value of his endowments.

Kyle and Vila (1991) provide a model that investigates the interaction between trading strategy and takeover activity (Note 7). In their model, because to takeover is costly, a bidder will declare a takeover to increase the firm (or stock) value only if he holds enough shares of the firm. In order to buy the shares in a cheaper price, he shorts
the stocks to manipulate the stock price downward before takeover activity. To extend Kyle and Vila’s (1991) framework, Bagnoli and Lipman (1996) show that there are another manipulators who can pool with the bidder and earn positive profits. The meanings of “manipulation” between the two papers are different. Kyle and Vila emphasize an agent’s ability to alter the fundamental firm value whereas Bagnoli and Lipman focus on how to alter the perceived value or price of the asset.

The action-based manipulation, such as Kyle and Vila (1991) and Bagnoli and Lipman (1996), are usually illegal and prohibit in the most security exchange around the world. Our model, however, discuss the manipulation of a large shareholder who does not possess the power to alter the firm value. Although the meaning of manipulation in our model is a little similar to Bagnoli and Lipman, we focus on the motivation of manipulation rather than the possibility of speculation. We study why and how the manipulator encourages an outside investor to intervene the operation of the firm.

The remainder of this article is organized as follows. Section 2 describes the basic model which includes of the market participants and the market structure. Section 3 analyzes the equilibrium and the related properties. We also provide the analysis and explanations for these results. The conclusion and the implications for further research are provided in Section 4.

2. Model

We begin our model building by introducing the four types of market participants: the potential raider, the large shareholder, the noise traders, and the competitive market maker. For tractability, we assume that all of them are with risk neutral preferences.

There may or may not exist a potential raider who is with the power to increase the stock value from $V_d$ to $V_u$ by any exogenous reason (Note 8). The intervention costs the raider $c$. For simplicity, we assume that the raider does not hold any share of the stocks and we do not consider the raider’s manipulative strategy. In reality, the raider may be a potential merger who is considering whether to buy the firm. Therefore, he will do intervention if and only if he decides to buy the stock of the firm. In other words, he has only two alternative strategies: first, to buy the stock and to intervene, and second, to quit the market.

Only the large shareholder knows whether the raider exists or not before the trading. We assume the large shareholder has $h$ shares as his endowment holding.

The behaviors of the noise traders and the market maker are similar to the setting in the traditional microstructure models. Each one of the noise traders buys and sells with equal probability, 0.5, for any exogenous motivations. The competitive market maker does not know about the existence of the raider. He sets fair ask and bid prices equal to the expectation of the stock value conditional on all the available information.

For convenience, some notations and assumptions are selected. The prior probability that there is a raider is denoted by $q$. If the raider does not enter the market or there is no raider, the partitions of the informed and the noise traders are $\alpha$ and $1-\alpha$, respectively. If there exists a raider and he enter the market, the market maker face the trading order of the raiders, the informed traders, and the noise traders with probabilities $\beta$, $\alpha(1-\beta)$, and $(1-\alpha)(1-\beta)$, respectively.

The trading mechanism is similar to the sequential trade models a la Gloseten and Milgrom (1985) and Easley and O’Hara (1987). In the beginning, the market maker rationally expects the market participants’ trading strategy and set fair ask and bid prices. After observing the ask and the bid prices, the market participants are randomly drawn to enter the market and to execute their order. The amount that the market participants can trade is limited to be one unit in each round of trade. After the trade, the market is closed (Note 9).

3. The Equilibrium and Main Results

We first give some additional notations before beginning the analysis of the equilibrium. For the informed trader, he has two decision nodes. First, when he knows that there is no raider, he buys with probability $q_2$ and sells with probability $1-q_2$. Second, when he knows that the raider exists, he buys with probability $q_1$ instead of $q_2$ (Note 10). For the raider, he decided whether to enter (buy the stock and intervene) or to quit the market. The probability that he enters the market is denoted by $p_1$. For simplicity, we assume that if the payoff of to enter and to quit is the same, the raider will quit the market.

By rationally expecting the traders’ strategies, the market maker set a fair ask and bid prices that equal to the expected stock value. Therefore, the ask prices ($P_a$) and bid price ($P_b$) are functions of the traders’ strategy and we describe them mathematically as following:

$$P_a = E[\hat{V} \mid buy] = f_s(p_1, q_1, q_2)$$ (1)


and
\[ P_e = E[\bar{V} \mid \text{sell}] = f_e(p_1, q_1, q_2) \]  
(2)
where \( f_a \) and \( f_b \) are the ask and bid prices functions, respectively.

**Lemma 1.** The large shareholder will sell the stocks if there is no potential raider; that is, \( q_1^* \) is equal to zero.

The intuition for Lemma 1 is obvious. If there is no raider, the value of the stock must equal to its worst possible outcomes, \( V_d \). Without any additional information for the stock value or the existence of the raider, the stock prices, ask and bid, must be overvalued. Therefore, the large trader will sell the stocks to benefit himself.

By the consideration of Lemma 1, we continue to deduce the price formation and Lemma 2 concludes the rule for the raider’s decision.

**Lemma 2.** In the equilibrium, the raider’s strategy will satisfy the following rules:

\[
p_1^* = \begin{cases} 1 & \text{if and only if } \frac{(V_u - V_d)}{c} \geq \delta^* \\ 0 & \text{otherwise} \end{cases}
\]

where \( \delta^* = \frac{1 - \alpha + (1 + \alpha)\beta q + 2\alpha q(1 - \beta)q_1^*}{(1 - \alpha)(1 - q)} \).

Lemma 2 shows that the criterion for the raider’s strategy is whether the benefits can offset the cost of intervention. \( V_u - V_d / c \) measures the ratio of the benefits to the cost. We show that \( \delta^* \) is the threshold of the decision rule and the raider will enter the market only if the benefit-cost ratio is larger than \( \delta^* \). In other words, the raider will intervene only if the benefits to intervene are at least \( \delta^* \) times as much as the costs.

An interesting finding is that the value of the threshold, \( \delta^* \), depends on the large shareholder’s strategy, \( q_1^* \). (Note 11) Specifically, \( \delta^* \) is positively related to \( q_1^* \). This also implies that the raider is more likely to enter the markets if the large shareholder is with less willing to buy the stock. This is because the strategy of the large shareholder influences the pricing of the market maker. If the large shareholder is with higher willing to buy the stock, the ask price will be higher. This results to fewer profits for the raider to intervene and the raider will less likely to enter the market.

Consequently, the large shareholder faces the dilemma of making decision. If the raider is expected to enter the market, the true value of the stock is expected to increase. The large shareholder should buy the stock to make more expected profits. However, the buying of the large shareholder results to a higher ask price and reduces the willing of the raider to intervene. Furthermore, because of the endowment holding of the large shareholder, the intervention of the raider can increase the large shareholder’s wealth. As a result, the large shareholder will attempt to take some actions to attract the raider entering the market as long as he holds sufficiently many shares of the stock. In the following, we discuss the large shareholder’s strategy and investigate whether he will manipulate the stock price to attract the raider entering.

We describe the market equilibrium in the following. For convenience to discuss, we separate the market situations into three cases.

**Proposition 1.** In the case with \( \frac{V_u - V_d}{c} < \frac{1 - \alpha + (1 + \alpha)\beta q + 2\alpha q(1 - \beta)q_1^*}{(1 - \alpha)(1 - q)} \), the equilibrium exists with (1) \( p_1^* = p_2^* = V_d \), (2) \( p_1^* = 0 \) and (3) \( q_1^* = t, q_2^* = 0 \) for \( t \in [0,1] \).

The result of Proposition 1 shows the equilibrium when the benefit of intervention, \( V_u - V_d \), is not great enough (relative to \( c \)). In this case, no matter how the informed trader manipulate the price, the trading gains for the raider are never enough to offset the cost of intervention. So the raider will never enter the market. The rational market maker also expects the result, and knows that the value of the stock is always \( V_d \) whether the raider exists or not. Therefore, the market maker will set both of the ask and the bid prices to be \( V_d \) even if the existence of the raider is uncertain. For the large shareholder, buying and selling provide the same payoff, so he can choose to adopt each one as his strategy or to adopt a mixed strategy.

**Proposition 2.** In the case with \( \frac{V_u - V_d}{c} > \frac{1 - \alpha + (1 + \alpha)\beta q + 2\alpha q(1 - \beta)q_1^*}{(1 - \alpha)(1 - q)} \), the equilibrium exists with

\[
i \quad P_d^* = (1 - l_d)V_d + l_dV_u, \quad P_b^* = (1 - l_b)V_d + l_bV_u.
\]
Proposition 2 shows that when the benefit of intervention \((V_u - V_d)\) is relatively much greater than the cost, the raider will always enter the market. When the benefit of intervention is so great that it always offset the cost, the raider will always buys the stock and intervene no matter how high the ask price is. Moreover, it is not necessary for the large shareholder to manipulate the price because the stock value will always be \(V_u\). The large shareholder will always choose buying as his best strategy. Finally, because the market maker does not know the existence of the potential raider, he can only set the ask and the bid prices based on expectation.

The previous two propositions show that when the benefit of intervention is sufficiently great or little, the raider’s strategy is independent to the large shareholder’s. In the remaining case, however, the equilibrium is more complex because both of the raider’s and the large shareholder’s strategy are influenced by the other’s strategy. In other words, each of them has to forecast the other’s action when making decision.

Reminding the result of Lemma 2, the raider’s strategy depends on the large shareholder’s strategy and the pricing of the market maker. In addition, the large shareholder’s final wealth and optimal strategy are influenced by his endowment holding. Therefore, the endowment holding plays an important determination of the large shareholder’s strategy and the market equilibrium. Proposition 3 shows that the equilibrium may contain market manipulation under some certain situations.

**Proposition 3** In the case with

\[
\frac{1 - \alpha + (1 + \alpha)\beta q}{(1 - \alpha)(1 - q)} \leq \frac{V_u - V_d}{c} \leq \frac{1 - \alpha + (1 + \alpha)\beta q + 2\alpha(1 - \beta)q}{(1 - \alpha)(1 - q)},
\]

the equilibrium exists with the large shareholder’s endowment holding, \(h\). Specifically, under the situation with

\[
h \leq \frac{1 - q}{1 - \beta q},
\]

the equilibrium exists with

1. \(P_A^* = P_B^* = V_d\),
2. \(p_1^* = 0\),
3. \(q_1^* = 1, q_2^* = 0\).

On the other hand, under the situation with

\[
h > \frac{1 - q}{1 - \beta q},
\]

the equilibrium exists with

1. \(P_A^* = (1 - \kappa_A)V_d + \kappa_A V_u\), \(P_B^* = (1 - \kappa_B)V_d + \kappa_B V_u\),
2. \(p_1^* = 1\),
3. \(q_1^* = 0, q_2^* = 0\)

where

\[
\kappa_A = \frac{(1 - \alpha)(1 - \beta)q}{1 - \alpha + (1 + \alpha)\beta q} \quad \text{and} \quad \kappa_B = \frac{(1 - \beta)q}{1 - \beta q}.
\]

Proposition 3 emphasizes the role of the large shareholder’s endowment holding when the benefit of intervention is neither extreme great nor few. We find that the endowment of the large shareholder determines his trading strategy. If the large shareholder holds enough many shares of the stock, he will sell the stock. Intuitively, the raider’s intervention benefits the large shareholder because his endowment can be more worthy. Consequently, the large shareholder hopes the raider to intervene, and will attempt to influence the stock price by his trading strategy. The selling strategy lower the ask price, and lower the raider’s cost to buy the stock. This also attracts the raider entering the market and intervene the operation of the firm. On the other hand, it is similarly explained that if the shareholder does not hold sufficiently many shares of the firm, it is not worthy to manipulate the price. Furthermore, it is proven that the threshold of the endowment is \((1-q)(1-\beta q)\), which is related positively to \(\beta\) and negatively to \(q\). This result implies that the endowment needed by a shareholder to adopt a manipulate strategy is negatively to the probability that the raider exists, \(q\). Intuitively, when the probability that the raider exists is higher, the market maker expects higher probability for the stock price to be \(V_u\) and select higher ask and bid prices.
Because the higher bid price lowers the informed shareholder’s cost to take the manipulative strategy, it is more likely for the informed trader to adopt the manipulative strategy even with less shares endowment.

The finding of interest in this study is the existence of manipulative strategy. We show that when the large shareholder holds sufficiently many shares of the firm, he will sell the stock in order to manipulate the stock price lower and attracts the raider entering. Moreover, the large shareholder will get a trading loss if he taking this strategy. Mathematically, his profits in this trading can be described as the following equation.

\[
\text{net trading gain} = P_B - V_u = (1 - \kappa_B)(V_d - V_u) = \frac{1-q}{1-\beta q}(V_d - V_u) < 0
\] (4)

Although the large shareholder loses from the trade, the intervention of the raider increases the fundamental value of the shares, and improves the large shareholder’s wealth. Actually, as long as the large shareholder holds enough endowment shares, the increase on the wealth will cover the trading losses. Consequently, this manipulative strategy makes the large shareholder get short-term losses in the trade and long-term profits form the increase of the wealth.

Specifically, equation (4) also implies that the cost of manipulation for the large shareholder is \((1-q)(1-\beta q)\) times the benefit of intervention for each share of his endowment, \(V_u-V_d \). Consequently, the large shareholder will not adopt manipulative strategy unless he has more than \((1-q)(1-\beta q)\) shares of the stock. This intuitively explains why the threshold of the large shareholder’s endowment in Proposition 3 is \((1-q)(1-\beta q)\).

4. Conclusion

Market manipulation is illegal in many countries and the regulators also devote to enforce laws against manipulative trades. However, as Allen and Gale (1992) mentioned, although lots of the action-based and information-based manipulations were successfully eliminated, trade-based manipulation is very difficult to eradicate. This interests a large number of authors to study the motivation of market manipulation.

In this study, we establish an environment in which the shareholder of a firm may manipulate the stock price to encourage the potential raider, who has power to alter the fundamental value of the firm, to buy the stocks and to intervene. In the equilibrium, the raider will never take a costly intervention unless he has enough many shares of the firm and the benefit of intervention is great enough. The raider, with no share in the beginning, will buy some shares in the low price before intervention. Consequently, if the benefit is not great enough or the ask price is high, the raider will not enter the market.

We find three different kinds of equilibrium in the different situations. First, when the benefit of intervention is very little, the raider will neither enter the market nor intervene because the gains of trade are never able to offset the cost of intervention. Furthermore, the large shareholder will sell the stocks. Second, when the benefit of intervention is very great, the raider will buy the stocks and intervene. In this case, the large shareholder also expects that the stock value will increase after the intervention. Therefore, he will buy the stock for making more profits. The final case discusses the equilibrium with the moderate benefit of intervention. The large shareholder with the endowment holding of the shares hopes the raider to intervene because this increases the value of his shares. The benefit, however, is not great enough for the raider to adopt a costly intervention strategy. Therefore, the large shareholder needs to manipulate the ask price lower to attract the raider entering even if this makes the large shareholder lose in the trade. We show that the equilibrium contains manipulative strategy as long as the shareholder holds sufficiently many shares of the firm. In other words, if the shareholder holds sufficiently many shares of the firm, he will adopt a selling strategy to lower the stock price and make the raider enter the market.

Our model provides an environment in which the large shareholder adversely affects the stock price to attract the raider’s intervention. The large shareholder gets losses in the trade but gains more from the increase of his aggregate wealth. Although some specific settings make the results concise, this study also identifies the existence of market manipulation. We show that the traders sometimes do not seek to maximize the gain through the trading because additional benefits may influence the optimal strategy of the traders even if they may suffer trading losses. Consequently, to investigate the motivations of manipulation more deeply, the sources of the benefit can be the important and interesting issue for the further research.

Acknowledgements

The author thanks the editor and an anonymous referee for their helpful comments and suggestions. Remaining errors and weakness are the author’s responsibility.

References


**Notes**

Note 1. Cherian and Jarrow (1995) provided five specific assumptions and showed that there is no manipulation opportunity if these assumptions are given.

Note 2. We can also assume that the raider has few stock shares, but it will make the model difficult to solve. We believe that this assumption will not change the results but make the model tractable.


Note 4. For the survey, see Putnins (2012).

Note 5. There is a spate of literature study trade-based manipulation, such as Kyle (1984), Allen and Gorton (1992), Allen and Gale (1992), Fishman and Hargerty (1992), Kumar and Seppi (1992), John and Narayanan (1997), Huddart et al. (2001), Chakraborty and Yilmaz (2004a,b; 2008) and Aggarwal and Wu (2006).

Note 6. Chakraborty and Yilmaz (2004a,b) define a manipulator as an investor who “trade in a wrong direction for short-term losses and long-term profits”; while Brunnermeier (2005) describes that “Manipulative trading is intended to move the price in order to enhance the informational advantage in the next period.”

Note 7. Vila (1989) also takes an example to explain a similar action-based manipulation.

Note 8. There are some reasons why the raider can increase the firm value. For example, the raider may have good reputation so that the firm will become more creditable. Or, the raider may have the power to improve or to monitor the operations of the firm, and this can benefit the firm’s earning.

Note 9. For simplicity, we do not consider the learning ability and the prices adjustment of the market maker in our model. Different to those traditional sequential trade models which pay attention to investigate the prices process, our focus is on the market participants’ behaviors, especially on the large shareholder’s, trading strategy. This assumption can make our model more tractable without changing the main results.

Note 10. We ignore the strategy that the large trader do neither buy nor sell. Without the consideration of the transaction costs, the large trader only takes no action only if the ask price is higher and the bid price is lower than the fundamental value. In our model, however, because the large shareholder knows the fundamental value of the asset, the probability that he neither buying nor selling is always zero.

Note 11. In fact, the threshold, $\delta^*$, will also be related to many exogenous variables, including the proportions of the market participants and the prior probability of the existence of the raider. However, equation (2a) does not directly imply the impact of these variables on the threshold because the value of $q_1$, which is also influences by the exogenous variables, is not solved.

**Appendix**

Appendix A. Proof of lemma 1

If there is no raider, the value of the stock must be $V_d$. Therefore, the optimization problem can be written as following.

$$ q_2^* = \arg \max_{q_2 \in [0, 1]} [q_2 (V_d - P_d) + (1 - q_2) (P_B - V_d) + h V_d ] $$

(A1)

Simple calculation shows that

$$ q_2^* = \arg \max_{q_2 \in [0, 1]} [q_2 (2V_d - P_A - P_B) + P_B - V_d + h V_d ] $$

It is clear that the objective function is linearly related to $q_2$ with a slope $2V_d P_A P_B$. Because the ask and bid price is the expected value of the possible stock value, $V_u$ and $V_d$, the prices must lie between $V_u$ and $V_d$. That is, $V_d \leq P_B \leq P_A \leq V_u$. As a result, it is clear that the slope must be negative. Consequently, it is obvious that $q_2^* = 0$ solve the maximization problem.

Appendix B. Proof of lemma 2

By instead $q_2 = 0$ into equations (1) and (2), they can be rewritten as (A2) and (A3) as follows:

$$ P_d = \frac{a_1 V_u + a_2 V_d}{a_1 + a_2} $$

(A2)
where

\[ a_1 = q \left\{ p_1 \left[ \beta + (1 - \beta)\alpha q_1 + \frac{1}{2}(1 - \beta)(1 - \alpha) \right] \right\}, \]

and

\[ a_2 = q(1 - p_1) \left[ \alpha q_1 + \frac{1}{2}(1 - \alpha) \right] + \frac{1}{2}(1 - q)(1 - \alpha) \cdot \]

\[ P_B = \frac{b_1 V_u + b_2 V_d}{b_1 + b_2} \quad \text{(A3)} \]

where

\[ b_1 = q p_1 \left[ (1 - \beta)\alpha(1 - q_1) + \frac{1}{2}(1 - \beta)(1 - \alpha) \right], \]

and

\[ b_2 = q(1 - p_1) \left[ \alpha(1 - q_1) + \frac{1}{2}(1 - \alpha) \right] + \frac{1}{2}(1 - q)(1 - \alpha) \cdot \]

We first solve the profit maximization problem of the raider. Given the ask price, if the raider buys and shares of the firm and then intervene, his final wealth, denoted by \( W_M \), is as equation (A4).

\[ W_M = V_u - P_A - c \quad \text{(A4)} \]

On the other hand, if the raider does not enter the markets, his final wealth will be zero. Thus, the raider will enter the market if and only if the (A4) is positive. According to the rule, we can conclude that \( p_1^* = 1 \) if and only if \( V_u - P_A - c > 0 \).

By instead (A2) into (A4), and instead \( p_1 \) by one, we have

\[ E[W_M \mid p_1 = 1] = \frac{(1 - \alpha)(1 - q)}{1 - \alpha + (1 + \alpha)\beta q + 2\alpha(1 - \beta)q_1}(V_u - V_d) - c \quad \text{(A5)} \]

By simple calculation, we can solve the condition that makes (A5) be positive as the main result of Lemma 2.

Appendix C. Proof of proposition 1

Because \( \frac{V_u - V_d}{c} < \frac{1 - \alpha + (1 + \alpha)\beta q}{(1 - \alpha)(1 - q)} \) implies that \( \frac{V_u - V_d}{c} < \delta^* \) for all \( q_1^* \in [0,1] \), we can shows \( p_1^* \) must equal to zero. This also means that even if the raider exists, he will never enter the market. In this case, the stock value is always equal to \( V_d \). Thus, it is obvious to show that \( P_A^* = P_B^* = V_d \).

Then we go to solve the large shareholder’s optimal strategy. The optimization problem for the large shareholder can be described mathematically as

\[ q_1^* = \arg \max_{q_1 \in [0,1]} E[q_1(V - P_A) + (1 - q_1)(P_B - V) + hV] \quad \text{(A6)} \]

where \( V = p_1 V_u + (1 - p_1)V_d \).

Because \( p_1^* \) must equal to zero, \( V \) always equals to \( V_d \). Also, by instead both of the ask and the bid prices by \( V_d \), the equation (A6) can be deduced to be

\[ q_1^* = \arg \max_{q_1 \in [0,1]} \{q_1 \cdot 0 + (1 - q_1) \cdot 0 + hV_d\} \cdot \]

As a result, \( q_1^* \in [0,1] \) solve the optimization problem.

Appendix D. Proof of proposition 2

In this case, \( \frac{V_u - V_d}{c} > \delta^* \) for all \( q_1^* \in [0,1] \), we can shows \( p_1^* \) must equal to one. This also means that no matter what strategy the large shareholder adopts, the raider will always choose to enter the market.

By instead \( p_1^* = 1 \) into the price formations, (A2) and (A3) can be rewritten as
\[ P_A = V_d + \frac{q(1 - \alpha + (1 + \alpha) \beta + 2 \alpha (1 - \beta) q_1^*)}{1 - \alpha + (1 + \alpha) \beta q + 2 \alpha (1 - \beta) q q_1^*} (V_u - V_d) \]  
(A7)

and

\[ P_B = V_d + \frac{q(1 - \beta)[1 + \alpha - 2 \alpha q_1^*]}{1 + \alpha - (1 + \alpha) \beta q - 2 \alpha (1 - \beta) q q_1^*} (V_u - V_d) \]  
(A8)

By simple calculation, it is obvious to show that \( V_d < P_A < P_B < V_u \). We then go to discuss the large shareholder’s strategy by reminding equation (A6). Because \( p_1^* = 1 \), we can instead \( \overline{P} = 1 \) into (A6) and rewrite it as

\[ q_1^* = \arg \max_{q_1 \in [0, 1]} E[q_1(V_u - P_A) + (1 - q_1)(P_B - V_u) + h \overline{P}] . \]  
(A9)

Because \( V_u - P_A \) is positive and \( P_B - V_u \) is negative, it is trivial that \( q_1^* = 1 \) is the solution of this maximization problem.

Finally, the ask and the bid prices can be shown by directly instead \( q_1^* = 1 \) into (A7) and (A8). The proof is complete.

Appendix E. Proof of proposition 3

According to the result of Lemma 2, it is obviously to show that \( q_1^* = 1 \) implies \( p_1^* = 0 \) under the situation with

\[ \frac{1 - \alpha + (1 + \alpha) \beta q}{(1 - \alpha)(1 - q)} \leq V_u - V_d \leq \frac{1 - \alpha + (1 + \alpha) \beta q + 2 \alpha (1 - \beta) q}{(1 - \alpha)(1 - q)} . \]

Also, \( q_1^* = 0 \) implies \( p_1^* = 1 \). The large shareholder is trying to choose a strategy (from \( q_1^* = 1 \) and \( q_1^* = 0 \)) to make his expected final wealth maximized.

If the large shareholder adopts \( q_1^* = 1 \) as his strategy, this make the intervention of the raider. Then the market maker will set the fair ask and bid prices to be \( V_u \). Equation (A6) shows the expected wealth, denoted by \( \pi_L^1 \), will be \( hV_u \). On the contrast, if the large shareholder adopt the selling strategy, \( q_1^* = 0 \), the raider will buy the stock and intervene. Moreover, according to the pricing rule of (A2) and (A3) with \( q_1^* = 0 \) and \( p_1^* = 1 \), the market maker with rational expectation will set the ask and the bid price to be \( P_A^* = (1 - \kappa_A) V_u + \kappa_A V_u \), and \( P_B^* = (1 - \kappa_B) V_d + \kappa_B V_u \), respectively. Then the expected wealth of the large shareholder can be simplified to be \( \pi_L^0 = hV_u + \frac{(q - 1)}{1 - \beta q} (V_u - V_d) \). The optimization problem can be deduced to be

\[ q_1^* = \arg \max_{q_1 \in [0, 1]} \{ \pi_L^0(q_1) \} . \]  
(A10)

Simple calculation shows that \( \pi_L^1 > \pi_L^0 \) if and only if \( h > (1 - q)/(1 - \beta q) \). Then we can conclude that the large shareholder adopts buying strategy if \( h > (1 - q)/(1 - \beta q) \), and adopts selling strategy in the other case.

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