Nontraded Goods and Deflationary Bias in a New Keynesian Open Economy Model

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Abstract
This paper develops a New Keynesian open economy model that embeds nontraded goods and produces multiple discretionary equilibria arising in the form of expectations traps. In contrast to the results in previous studies, we find the presence of nontraded goods narrows the region for a deflationary monetary surprise, suggesting that it is critically important for central banks to control inflation expectations in open economies.

Keywords: discretionary monetary policy, time inconsistency problem, nontraded goods, deflationary bias

1. Introduction
Following the seminal work of Obstfeld and Rogoff (1995; 1996), the New Keynesian open economy model has become the standard framework for analyzing monetary policy in an open economy setting. Monopolistic competition and nominal price rigidities are the two core assumptions in this model. However, in the absence of a commitment device or reputational considerations, these assumptions generate a time-consistency problem for monetary policy (Note 1).

The intuition underlying the optimal policy choice given discretion in a standard New Keynesian open economy setting is set out below. The central bank policy maker confronts a trade-off between the benefits of unanticipated expansionary monetary surprise and the costs of realized inflation. The combination of nominal rigidities and monopolistic competition then generates an inflationary bias through the conventional aggregate demand channel, given that the unanticipated expansionary monetary surprise can reduce the markup distortion, eventually increasing production and household utility. However, in contrast to closed economy settings, this expansionary monetary effect depreciates the value of the local currency, thereby raising the domestic currency price of imports relative to exports. The optimal policy decision thus requires the monetary authority to weigh the welfare gain from exploiting the aggregate demand channel against the loss in welfare arising through the terms of trade channel.

In earlier studies, Corsetti and Pesenti (2001; 2005) and Tille (2001) have discussed the relative strengths of the aggregate demand and terms of trade channels in open economy models. Both found that the terms of trade channel tends to dominate the aggregate demand channel, except in very large countries, implying that the deflationary bias prevails in the circumstances of an open economy. However, several recent studies have challenged this finding. For example, Arseneau (2012) introduced a monetary demand distortion in a New Keynesian open economy model, suggesting that it is critically important for central banks to control inflation expectations in an open economy. Likewise, Cooke (2010) developed a two-country dynamic general equilibrium model to analyze the optimal rate of inflation under discretion, finding that the relative monopoly markup affects the terms of trade to the extent that inflation can be higher in a more open economy, despite the presence of a less favorable short-run Phillips curve. Lastly, Evans (2012) formulated a two-country perfect foresight overlapping generations model, arguing that increased openness to international trade can increase a country’s long-run incentive to create inflation (Note 2).

However, all of the abovementioned studies ignore the presence of nontraded goods. Consequently, the objective of this paper is to examine which bias is more common in open economic circumstances with embedded nontraded goods. For this purpose, we develop a two-country monetary general equilibrium model to analyze the inflationary (or deflationary) bias in open economies by considering the presence of nontraded goods. We
therefore extend the work of Arseneau (2012) by explicitly introducing nontraded goods, as in Obstfeld and Rogoff (2000). Our main result is that multiple discretionary equilibria continue to arise, as in Arseneau (2012), in the form of expectations traps (Note 3). However, the presence of nontraded goods also narrows the region of a deflationary monetary surprise, thereby strengthening the result first illustrated by Arseneau (2012). The reason for this is that the presence of nontraded goods serves to reinforce the positive effect of the aggregate demand channel given the inflationary monetary policy, while the negative effect of the terms of trade channel remains unchanged.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 discusses discretionary monetary policy. Section 4 presents a simple numerical example. Finally, Section 5 concludes.

2. The Model

Our model draws on Arseneau (2012). There is perfect information in the two countries, denoted Home and Foreign (Note 4). Money enters the model via a cash-in-advance constraint and the households in each country provide labor to a continuum of firms in their own country. We assume firms set prices one period in advance every period, and that they have monopoly power in the goods market. Finally, there are two kinds of producers in each country: one that produces traded goods sold both domestically and abroad, another that produces nontraded goods sold only domestically. We assume free exchange rates.

2.1 Central Bank

The central bank in the Home country controls the money supply by making a lump-sum transfer, \( \chi_t - 1 \), of Home monetary supply to the Home households in each period \( t \). Here \( \chi_t \) is defined as the gross growth rate of the Home country monetary supply, \( \chi_t \). The objective of the central bank is to select the discretionary rate of growth in domestic money that maximizes the discounted lifetime utility of the domestic representative agent. The representative household’s money stock follows \( M_{t+1} = \chi_t M_t \). Without loss of generality, we normalize the period 0 money stock, so that \( M_0^* = 1 \). The Foreign country proceeds in a similar way.

2.2 Firms

The technology is a linear production function of labor where one unit of labor produces one unit of output in the global economy. Firms set prices one period in advance and maximize profits each period. For the traded goods, let \( Y_{H,t}(h) = l_{H,t}(h), \forall h \in [0,1] \), and for the nontraded goods, let \( Y_{N,t}(h) = l_{N,t}(h), \forall h \in [0,1] \). There are parallel production functions for Foreign-produced traded goods, denoted \( Y_{F,t}(f) = l_{F,t}(f), \forall f \in [1,2] \), and for Foreign-produced nontraded goods, denoted \( Y_{N,F}(f) = l_{N,F}(f), \forall f \in [1,2] \). The normalized profit function for a traded goods producer in the Home country is as follows.

\[
\max_{p_{H,t+1}(h)} \left( p_{H,t+1}(h) - w_{t+1} \right) c_{t+1}^w(h), \quad \forall h \in [0,1],
\]

where \( c_{t+1}^w(h) \) is the expected demand function for the traded goods produced in the Home country for the consumers in both countries. After deriving the firm’s optimization problem, we obtain the price level \( p_{H,t+1}(h) = \Phi w_{t+1} \), where \( \Phi = \theta(\theta - 1) \) is the markup and \( w_{t+1} \) is the labor wage. The nontraded goods producers solve the same problem. As usual, Foreign firms solve the problem in a similar way. We state these problems explicitly below.

\[
\max_{p_{N,t+1}(h)} \left( p_{N,t+1}(h) - w_{t+1} \right) c_{t+1}^N(h), \quad \forall h \in [0,1]
\]

\[
\max_{p_{N,F}(f)} \left( p_{N,F}(f) - w_{t+1} \right) c_{t+1}^N(f), \quad \forall f \in [1,2]
\]

\[
\max_{p_{N,F}(f)} \left( p_{N,F}(f) - w_{t+1} \right) c_{t+1}^N(f), \quad \forall f \in [1,2]
\]
nontraded goods, respectively. Finally, \( w^*_{t+1} \) is the Foreign labor wage.

2.3 Households

We assume households trade domestic bonds as well as domestic money. Bonds are available in zero net supply, so \( B_t = B_{t+1} = 0 \) must hold in equilibrium for all \( t = 0, 1, 2, \ldots \) in the global economy. We define the preferences of the representative Home household over the consumption of an aggregate consumption basket and labor,

\[
U_t = \sum_{n=0}^{\infty} \beta^n \left( \frac{1}{1 - \sigma} c_t^{\sigma} - l_t \right)
\]

where \( 0 < \beta < 1 \) is the subjective discount rate, \( 0 < \sigma < 1 \) is the reciprocal of the elasticity of intertemporal substitution, and \( l_t = l_{tH} + l_{tF} \) is the total labor supply for the traded and nontraded goods firms.

Following Obstfeld and Rogoff (2000), we define a basket of the representative household’s consumption of the Home and Foreign traded goods and the Home nontraded goods bundles as a Cobb–Douglas form as follows.

\[
c_t = \frac{1}{\eta^\gamma} c_{tH} \eta^\gamma c_{tN}, \quad \text{where} \quad \eta^\gamma = \eta^\gamma (1 - \eta)^{1-\eta}, \quad 0 < \eta < 1,
\]

where \( \eta \) is the share of traded goods. The preferences over Home and Foreign traded products have an Armington form,

\[
c_{tH} = \frac{1}{\gamma^\eta} c_{tH}^{\gamma} c_{tF}^{1-\gamma}, \quad \text{where} \quad \gamma^\eta = \gamma^\eta (1 - \gamma)^{1-\gamma}, \quad 0 < \gamma < 1,
\]

where \( \gamma \) is the share of traded goods produced in the Home country. The consumption of the traded goods produced in the Home country bundles, \( c_{tH} \), the consumption of the traded goods produced in the Foreign country bundles, \( c_{tF} \), and the consumption of the nontraded goods produced in the Home country bundles, \( c_{tN} \), are defined as follows.

\[
c_{tH} = \left( \int_0^1 c_{tH}(h) \frac{\sigma-1}{\sigma} dh \right)^{\frac{\sigma}{\sigma-1}}, \quad \text{where} \quad \sigma > 1, \quad h \in [0,1],
\]

\[
c_{tF} = \left( \int_0^1 c_{tF}(f) \frac{\sigma-1}{\sigma} df \right)^{\frac{\sigma}{\sigma-1}}, \quad \text{where} \quad \sigma > 1, \quad f \in [1,2]
\]

\[
c_{tN} = \left( \int_0^1 c_{tN}(h) \frac{\sigma-1}{\sigma} dh \right)^{\frac{\sigma}{\sigma-1}}, \quad \text{where} \quad \sigma > 1, \quad h \in [0,1]
\]

Where \( \sigma \) is the elasticity of substitution between differentiated goods producing market power for the firms. The domestic currency price index for overall real consumption is

\[
P_t = P_{tH}^{\eta} P_{tN}^{1-\eta}
\]

and the price index for traded consumption is

\[
P_{tH} = P_{tH}^{\eta} P_{tF}^{1-\eta}
\]

where

\[
P_{tH} = \left( \int_0^1 P_{tH}(h)^{1-\sigma} dh \right)^{\frac{1}{1-\sigma}}, \quad \text{where} \quad \sigma > 1, \quad h \in [0,1]
\]

\[
P_{tN} = \left( \int_0^1 P_{tN}(h)^{1-\sigma} dh \right)^{\frac{1}{1-\sigma}}, \quad \text{where} \quad \sigma > 1, \quad h \in [0,1]
\]

\[
P_{tF} = \left( \int_0^1 P_{tF}(f)^{1-\sigma} df \right)^{\frac{1}{1-\sigma}}, \quad \text{where} \quad \sigma > 1, \quad f \in [1,2]
\]

where \( P_{tH}, P_{tN} \) and \( P_{tF} \) are the price indexes for traded goods produced in the Home country, the price index for nontraded goods produced in the Home country, and the price index for traded goods produced in the Foreign country, respectively. These price indexes are defined as the minimum amount of expenditure needed to buy one
unit of the composite consumption indexes, \( c_{H,t} \), \( c_{N,t} \), and \( c_{F,t} \). The Home commodity demand functions for the representative product resulting from cost minimization are given by the following,

\[
c_{T,t}(h) = \left[ \frac{P_{T,t}(h)}{P_{H,t}} \right]^{-\theta} c_{H,t}, \quad \text{where } \theta > 1, \ h \in [0,1]
\]

(16)

\[
c_{S,t}(h) = \left[ \frac{P_{S,t}(h)}{P_{H,t}} \right]^{-\theta} c_{S,t}, \quad \text{where } \theta > 1, \ h \in [0,1]
\]

(17)

\[
c_{T,f}(f) = \left[ \frac{P_{T,f}(f)}{P_{F,f}} \right]^{-\theta} c_{F,f}, \quad \text{where } \theta > 1, \ f \in [1,2]
\]

(18)

where \( c_{T,t}(h) \), \( c_{N,t}(h) \) and \( c_{T,f}(f) \) are the demands for typical Home traded goods, typical Home nontraded goods, and typical Foreign traded goods, respectively. Also,

\[
c_{H,t} = \gamma \left[ \frac{P_{H,t}}{P_{T,t}} \right]^{-1} c_{T,t}, \quad c_{F,t} = (1 - \gamma) \left[ \frac{P_{F,t}}{P_{T,t}} \right]^{-1} c_{T,t},
\]

(19)

and

\[
c_{T,f} = \eta \left[ \frac{P_{T,f}}{P_{F,f}} \right]^{-1} c_{F,f}, \quad c_{N,f} = (1 - \eta) \left[ \frac{P_{N,f}}{P_{F,f}} \right]^{-1} c_{T,f}.
\]

(20)

In the producer’s currency price setting, the law of one price holds for the traded goods. Then we have

\[
P_{H,t}(h) = S_t P_{H,t}^*(h), \forall h \in [0,1]
\]

(21)

\[
P_{F,f}(f) = S_t P_{F,f}^*(f), \forall f \in [1,2]
\]

(22)

Where \( S_t \) is the nominal exchange rate. Next, we define the Home country’s terms of trade, \( \text{ToT} = \frac{P_{H,t}}{P_{F,t}} \), as the relative price of exports to imports, denominated in the local currency. We use the per household money stock to scale the nominal variables in the cash-in-advance and budget constraints. This allows the Home country’s normalized cash-in-advance and budget constraints for the representative household to be expressed as

\[
p_f c_t \leq m_t + (x_t - 1) + b_t - \frac{b_{t+1} x_t}{R_t}
\]

(23)

\[
p_f c_t + \frac{b_{t+1}}{R_t} + m_{t+1} x_t \leq m_{t+1} + (x_{t+1} - 1) + b_{t+1} + \int_{0}^{1} d_{H,t}(h) dh + \int_{0}^{1} d_{N,t}(h) dh + w_{t+1}
\]

(24)

where \( d_{H,t}(h) \) and \( d_{N,t}(h) \) respectively denote the domestic currency dividends for the traded- and nontraded-goods-producing firms, and \( R_t \) is the gross nominal interest rate between time \( t \) and \( t+1 \).

The representative household in the Home country takes as given the sequences \( p_{H,t}(h) \), \( p_{F,f}(f) \), \( d_t(h) \), \( w_t \), \( R_t \), \( x_t \), \( x_{t+1} \), and chooses quantities \( c_{H,t}(h) \), \( c_{N,t}(h) \), \( c_{F,t}(f) \), \( m_{t+1} \), \( b_{t+1} \) to maximize (5) subject to (6), (7), (8), (9), (10), (23), and (24). The Foreign households solve the problem in a similar way.

2.4 Private Sector Equilibrium

In the initial period, the central banks in both countries announce a monetary policy plan. Let \( x = \{ x_t | t = 0,1,2,... \} \) denote the Home monetary policy plan and \( x^* = \{ x_t^* | t = 0,1,2,... \} \) the Foreign monetary policy plan. Taken together, let \( \Pi(x,x^*) = (x,x^*) \) be world monetary policy. We assume that both central banks do not have a commitment technology, so they must set policy sequentially, choosing \( x_t \) (or \( x_t^* \)), at the beginning of each period \( t \). Once private agents reveal deviation from the initial plan, they will resolve their optimal problems. This puts the central banks at risk of falling into an expectations trap. Here, we assume a simple trigger strategy for forming private sector expectations. Given the monetary policy announcement, private agents fully trust the policy plans in both countries in the initial period. If a central bank deviates from the initial policy plan to surprise private agents, then the global economy will immediately fall into the discretionary
equilibrium, and persist there forever.

Conditional on global monetary policy plans, we obtain a monopolistically competitive equilibrium. We define the monopolistically competitive equilibrium as a set of restrictions on prices and quantities in line with the following definition.

**Definition 1** Taking \( \Pi(x, x^*) \) as given, in the monopolistically competitive equilibrium, (i) firms set prices to maximize profit; (ii) households maximize utility; and (iii) all markets clear in every period in the Home and Foreign countries.

Set the optimization problem for households in the Home country as follows.

\[
L = \sum_{t=0}^{\infty} \left( \frac{1}{1-\alpha} c_{it}^{1-\alpha} - l_t \right) + \beta' \lambda_t \left[ m_t + (x_t - 1) + b_t - \frac{h_{t+1} x_t}{R_t} - p_t c_t \right] \\
+ \beta' \mu_t \left[ m_t + (x_t - 1) + b_t + \int_0^1 d_{H,t}(h) dh + \int_0^1 d_{F,t}(h) dh + w_t l_t - p_t c_t - \frac{h_{t+1} x_t}{R_t} - m_{t+1} x_t \right]
\]

where \( \lambda_t \) and \( \mu_t \) are the Lagrange multipliers. Differentiating with respect to \( c_t, l_t, m_t, \) and \( b_t \), respectively, we have

\[
c_t^{-\sigma} = \lambda_t p_t + \mu_t p_t \\
1 = \mu_t w_t \\
-\mu_t x_t + \beta \mu_{t+1} + \beta \lambda_{t+1} = 0 \\
-\mu_t \frac{x_t}{R_t} + \beta \mu_{t+1} + \beta \lambda_{t+1} - \lambda_t \frac{x_t}{R_t} = 0
\]

and the transversality condition is

\[
\lim_{t \to \infty} \beta' (\lambda_t + \mu_t) m_t = 0
\]

Using \( m_t = 1, b_t = h_{t+1} = 0 \) and the market-clearing conditions in the equilibrium, we obtain the following restrictions in the short run for a monopolistically competitive equilibrium in both countries.

\[
c_t = (\eta \gamma + 1 - \eta) \Phi_H \frac{x_t^{\eta \gamma + 1 - \eta} x_t^{(1-\gamma)}}{p_{H,t}^{\eta \gamma + 1 - \eta} p_{F,t}^{(1-\gamma)}} , \quad c_t^* = (1 - \eta \gamma) \Phi_F \frac{x_t^{1-\eta \gamma} x_t^{1-\eta \gamma}}{p_{H,t}^{1-\eta \gamma} p_{F,t}^{1-\eta \gamma}}
\]

where

\[
\Phi_H = \frac{1}{(\eta \gamma + 1 - \eta)^{\eta \gamma + 1 - \eta} (1 - \eta \gamma)^{(1-\gamma)}} , \quad \Phi_F = \frac{1}{(1 - \eta \gamma)^{1-\eta \gamma} \eta \gamma^{1-\eta \gamma}}
\]

and

\[
l_t = \frac{x_t}{p_{H,t}} , \quad l_t^* = \frac{x_t^*}{p_{F,t}}
\]

3. Discretionary Monetary Policy

We assume there is no commitment device in either country. In addition, the assumption of setting prices one period in advance generates nominal price rigidities, while monopolistic competition implies that output falls below the efficient level. Therefore, the Home central bank is tempted in each period to set money growth unexpectedly high to stimulate the own real economy. Moreover, these monetary policy changes also influence behavior in the Foreign country.

**Definition 2** Define \( \check{x}_t - x_t \) as the optimal monetary surprise in the Home country, where \( \check{x}_t \) is the gross growth rate of the Home country’s monetary supply in period \( t \) that maximizes the instantaneous utility of the Home households subject to restrictions implied by (31) and (32) in the short run, taken as given \( \Pi(x, x^*) \). The Foreign central bank solves the problem in a similar way.
3.1 Optimal Monetary Surprise

Now we turn to derive the optimal monetary surprise for the Home country’s central bank. (Note 5)

\[
\max_{\{c_t\}} \frac{1}{1-\sigma} c_{t}^{1-\sigma} - l_t, \quad \text{subject to}
\]

\[
c_t = (\eta \gamma + 1 - \eta) \Phi H \frac{x^{\gamma+1-\eta} x^{*}(1-\gamma)}{p_{H,t}^{\gamma+1-\eta} p_{F,j}^*}, \quad l_t = \tilde{x}_t, \quad \tilde{x}_t \in [\beta, \bar{x}].
\]

where \( \Phi_H = \frac{1}{(\eta \gamma + 1 - \eta)^{\gamma+1-\eta} \eta (1-\gamma)^{0(1-\gamma)}} \), \( \tilde{x}_t \in [\beta, \bar{x}] \).

After a few calculations, we obtain

\[
\tilde{x}_t = (\eta \gamma + 1 - \eta) \Phi_H \frac{1}{\beta} x^2
\]

Let \( \Omega = [\gamma, \Phi_H, \beta, \sigma, \eta] \) be the structural parameters vector of the model, and \( g(\Omega, x) \) denote the solution to this problem for the Home central bank; therefore we obtain

\[
g(\Omega, x) = \left( (\eta \gamma + 1 - \eta) \Phi_H \frac{1}{\beta} x - 1 \right) x
\]

**Proposition 1** The optimal solutions for the central banks in the global economy are given by

\[
g(\Omega, x) = \left( (\eta \gamma + 1 - \eta) \Phi_H \frac{1}{\beta} x - 1 \right) x, \quad \text{and}
\]

\[
g^*(\Omega^*, x^*) = \left( (1 - \eta \gamma) \Phi_F \frac{1}{\beta} x^* - 1 \right) x^*. \quad \text{(Note 6)}
\]

According to Proposition 1, we find that the optimal money supply surprise can be either positive or negative. In addition, the optimal monetary policy generated by the Home central bank is independent of the monetary policy of the Foreign central bank. Furthermore, because of the introduction of nontraded goods in this model, we obtain a new parameter \( \eta \) in the optimal solutions for the central banks, \( g(\Omega, x) \) and \( g^*(\Omega^*, x^*) \), respectively.

We consider the effect of the share of nontraded goods, \( 1-\eta \), in Section 4.

3.2 The Policy Trade-Off under Discretion

As discussed in the New Keynesian open economy literature, there are two separate channels for generating unanticipated monetary surprises, which influence the welfare of households. The first channel is the aggregate demand channel. Monopolistic competition implies that output falls below the efficient level and prices are set one period in advance. Therefore, in each period, central banks are tempted to set money growth unexpectedly higher as a means of stimulating production. However, this channel only exists in a closed economy, as noted by Ireland (1997), where the optimal money supply surprise is always positive.

In open economy settings, there is a second channel known as the strategic terms of trade channel. If the Home central bank implements a surprise monetary expansion at this time, it deteriorates the value of the local currency, which worsens the short-run terms of trade. Because traded goods trade throughout the world, Home households need to work harder to meet the demands for the relatively cheaper Home-produced goods, driving up labor inputs in the Home country. We can see this more easily by transforming (32) as

\[
\frac{l_t}{l_t^*} = \frac{\tilde{x}_t}{\bar{x}} \frac{p_{F,J}^*}{p_{H,J}}
\]

This equation is identical to that in Arsenneau (2012). In other words, the negative effect of the terms of trade channel remains unchanged after considering the presence of nontraded goods. However, the increment in Home export incomes also leads Home import demand to increase. As in Arsenneau (2012), the surprise monetary expansion raises household consumption in the Home country, at the same time as increasing household consumption in the Foreign country by the same quantity, eventually maintaining the ratio of consumption in both countries. However, given the introduction of nontraded goods, by transforming (31) we obtain
\[ c_t = \frac{\left( \eta \gamma + 1 - \eta \right) \Phi_x}{(1 - \eta \gamma) \Phi_x} \left[ \frac{\bar{x}_t}{x_t} \right]^{-\gamma} \left[ \frac{p_{t,x}}{p_{t,\hat{c}}} \right]^{1 - \gamma} \]  

(36)

implying that an inflationary surprise \( \bar{x}_t \) can increase consumption by the Home country’s households. Further, the smaller is \( \eta \), the larger the share of nontraded goods, and the stronger this effect. Put differently, the presence of nontraded goods serves to reinforce the positive effect of the aggregate demand channel for the Home country’s households given an inflationary monetary policy. This is the key finding of our analysis.

3.3 Multiple Time-inconsistent Discretionary Nash Equilibria

We now analyze the optimal monetary surprises as a discretionary Nash equilibrium in the current model.

**Definition 3** Taking \( \Pi (x^*, \tau^*) \) as given, in a Nash equilibrium, (i) the Home central bank chooses \( x^d \), (ii) the Foreign central bank chooses \( x^d^* \), and (iii) global households and firms respond optimally, and thus can never be surprised.

We find a threshold level of inflation expectations formed by the private sectors in the initial period such that there is no incentive for the central banks in either country to introduce a monetary surprise under any parameterization of the model. Letting \( g(\Omega, x) = 0 \) and \( g^\prime(\Omega^*, x^*) = 0 \), the following expressions yield the respective threshold levels for both countries.

\[ \hat{x} = \frac{\beta}{(\eta \gamma + 1 - \eta) \Phi_x} \]  

(37)

\[ \hat{x}^* = \frac{\beta}{(1 - \eta \gamma) \Phi_x} \]  

(38)

Denoting \( \Pi^d = (x^d, x^d^*) \) as the global monetary policy in the time-inconsistent discretionary Nash equilibrium, we have the following proposition as noted by Arseneau (2012).

**Proposition 2.** The multiple time-inconsistent discretionary Nash equilibria are defined as follows.

<table>
<thead>
<tr>
<th></th>
<th>( x^* &lt; \hat{x}^* )</th>
<th>( x^* = \hat{x}^* )</th>
<th>( x^* &gt; \hat{x}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x &lt; \hat{x} )</td>
<td>( \hat{x}, \beta )</td>
<td>( \beta, x^* )</td>
</tr>
<tr>
<td></td>
<td>( x = \hat{x} )</td>
<td>( \hat{x}^*, \beta )</td>
<td>( \hat{x}^<em>, x^</em> )</td>
</tr>
<tr>
<td></td>
<td>( x &gt; \hat{x} )</td>
<td>( \tau, \beta )</td>
<td>( \tau, \hat{x}^* )</td>
</tr>
</tbody>
</table>

In the Home country, when the initial inflation expectation is equal to the threshold level, such that \( x = \hat{x} \), the gains in welfare from exploiting the aggregate demand channel precisely counterbalance the welfare loss that occurs through the terms of trade channel, implying that there are no gains from introducing a monetary surprise in the Home country. Therefore, for the discretionary Nash equilibrium, \( x \) is an interior solution. Moreover, this is independent of the behavior of the Foreign central bank. When \( x > \hat{x} \), inflation expectations formed by the private sector in the initial period are higher than the threshold level, the welfare loss from the strategic terms of trade channel is dominated by welfare gains from the aggregate demand channel, and the Home central bank prefers an expansionary monetary surprise, resulting in an inflationary bias. Conversely, when \( x < \hat{x} \), initial inflation expectations formed by the private sector in the Home country are lower than the threshold level, the welfare loss that occurs through the strategic terms of trade channel dominates the welfare gains from the aggregate demand channel, and the Home central bank prefers a conservative monetary surprise, resulting in a deflationary bias.

4. A Simple Numerical Example

In this section, we provide a simple numerical example. The purpose is to illustrate that the region of the deflationary monetary surprise will be narrower when we include nontraded goods in the analysis by Arseneau (2012). For this exercise, we use 45-degree analysis. Figure 1 examines the case of a small Home country, \( \gamma = 0.1 \); the substitution elasticity between differentiated goods, \( \theta = 2 \), connoting a 100% markup; \( \eta = 1 \), \( \eta = 0.8 \), and \( \eta = 0.6 \) for the three different proportions of the share of tradable goods; and \( \beta = 0.9 \) (Note 7).
The 45-degree line illustrates expected household inflation, with the remaining three lines depicting the optimal monetary surprise for different shares of traded goods. The cross points indicate the threshold level of inflation expectations formed by the private sector. At these points, the gains in welfare from exploiting the aggregate demand channel precisely counterbalance the welfare loss that occurs through the terms of trade channel, implying that there are no gains from introducing a monetary surprise in the Home country. The region to the left of the cross point indicates the region of a deflationary monetary surprise. Conversely, the region to the right of the cross point identifies the region of an inflationary monetary surprise. Where all goods are tradable, \( \eta = 1 \), the region of a deflationary monetary surprise is quite large, but when \( \eta \) becomes smaller and the share of nontraded goods increases, the region for a deflationary monetary surprise quickly contracts. The intuitive interpretation is that the presence of nontraded goods serves to reinforce the positive effect of the aggregate demand channel given the inflationary monetary policy, while the negative effect of the terms of trade channel remains unchanged.

Table 1 provides a summary of the threshold country sizes, for the Home country, \( \hat{\gamma} \), when \( \eta = 0.5 \), for various levels of the markup and the initial inflation expectations formed by the private sector, \( x \), and setting the constant discount rate at \( \beta = 0.9 \) in the Home country. Compared with Arseneau (2012), the threshold country size \( \hat{\gamma} \) is smaller. Therefore, the deflationary bias that occurs in New Keynesian open economy models is very susceptible to the share of nontraded goods.

![45-degree diagram](image)

**Figure 1. The 45-degree diagram**

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.9</th>
<th>1</th>
<th>1.025</th>
<th>1.05</th>
<th>1.1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta ) = 2</td>
<td>0.38</td>
<td>0.31</td>
<td>0.30</td>
<td>0.29</td>
<td>0.26</td>
<td>0.13</td>
<td>0.03</td>
</tr>
<tr>
<td>( \theta ) = 6</td>
<td>0.67</td>
<td>0.69</td>
<td>0.66</td>
<td>0.64</td>
<td>0.60</td>
<td>0.38</td>
<td>0.22</td>
</tr>
<tr>
<td>( \theta ) = 8</td>
<td>0.75</td>
<td>0.73</td>
<td>0.71</td>
<td>0.69</td>
<td>0.64</td>
<td>0.41</td>
<td>0.24</td>
</tr>
<tr>
<td>( \theta ) = 12</td>
<td>0.83</td>
<td>0.78</td>
<td>0.76</td>
<td>0.73</td>
<td>0.69</td>
<td>0.44</td>
<td>0.27</td>
</tr>
</tbody>
</table>

5. Conclusion

In this study, we developed a two-country monetary general equilibrium model to analyze the inflationary (or deflationary) bias in open economies after explicitly considering the presence of nontraded goods. We found that the presence of nontraded goods narrows the region of a deflationary monetary surprise, thereby strengthening the result first illustrated by Arseneau (2012). For simplicity, we applied the approach of Obstfeld and Rogoff (2000) to introduce nontraded goods. However, alternative modeling strategies may be valuable in future research. It would perhaps also be interesting to introduce Calvo-style sticky prices or imperfect information and learning when examining the inflation and output dynamics in New Keynesian open economy models.
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References


**Notes**

Note 1. The seminal contributions regarding the time-inconsistency problem are by Kydland and Prescott (1977) and Barro and Gordon (1983). Subsequently, many studies have considered the time-inconsistency problem in a closed economy framework, including Ireland (1997), Neiss (1999), Chari et al. (1998), Albanesi et al. (2003a, 2003b), and King and Wolman (2004), among others.

Note 2. Other studies discussing inflationary or deflationary bias in an open economy setting include those of Benigno (2002) and Benigno and Benigno (2003), among others.

Note 3. For the expectation traps hypothesis, see Albanesi et al. (2003b), Armenter (2008), and Christiano and Gust (2000).

Note 4. In the standard New Keynesian model with perfect information it is impossible to reproduce the delayed response of inflation to monetary policy shocks, requiring two modifications of the perfect information assumption. The first, as in Mankiw and Reis (2002), is to assume that information is sticky. The second is to introduce, as in Dellas (2006) and Dellas and Collard (2004), imperfect information and learning. These approaches are both successful in generating realistic inflation and output dynamics in the standard New Keynesian model. As we are most interested in inflationary (or deflationary) bias in open economies, the question of inflation dynamics is beyond the scope of this analysis, so we defer this theme to future research.

Note 5. We impose two bounds on money growth to ensure the existence of a monetary equilibrium. A lower bound on the net nominal interest rate $\left(1 - R_t^e\right)$ guarantees that the interest rate never falls below zero. The upper bound on $\bar{x}$ guarantees that private agents never abandon the use of money altogether.

Note 6. As our discussion closely parallels that of Arseneau (2012), we omit the details of the solution and the proof in the interests of brevity. However, the details are available upon request from the author.

Note 7. More precisely, $\gamma$ is the share of traded goods produced in the Home country. However, in this section we also define $\gamma$ to be the size of the Home country for the purpose of intelligible comparison.

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