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Non-Linear Models and the Forward Discount Anomaly: An Empirical Investigation

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Abstract

In this paper, we propose a non-linear approach to explain the forward discount anomaly. We use two classes of non-linear models: models with changes in mean and long memory process. Our empirical results show that the non-stationarity of the forward discount series is the causes of the rejection of the Forward Rate Unbiased Hypothesis (FRUH). By investigating the forward discount series, we show that are characterized by a stationary long memory behavior which is amplified by the presence of breaks.

Keywords: Structural breaks models, Long range dependence, Exchange rates

1. Introduction

One of the most persistent debates on the international finance concerns the forward discount anomaly. If the forward rate unbiasedness hypothesis holds, then under risk neutrality and rational expectations the current forward exchange rate is an unbiased predictor of the future spot rate. This hypothesis is generally rejected in empirical literature. Engel's (1996) sums up the empirical results:" First, empirical tests routinely reject the null hypothesis that the forward rate is a conditionally unbiased predictor of future spot rate. Second, models of the risk premium have unsuccessful at explaining the magnitude of this failure of unbiasedness".

In empirical literature, to test if the forward exchange rate provides an unbiased forecast of the future spot rate, we regress the forward discount (defined as the difference between the forward and the future spot exchange rates) on the spot exchange rate return. If the FRU hypothesis holds, then the coefficient β associated to the forward discount variable must be equal to the unity, the intercept to zero and the innovations must follow a white noise (iid). Most empirical works find a negative value of β . For example, Froot (1990) notes that the average value of β is equal to -0.88 over 75 published papers. This means that there is not only a problem of rejection of the unbiasedness hypothesis but also a problem in forecasting of the direction of changes.

Until now, there is no consensus about the true reasons of this rejection of the Forward Rate Unbiased Hypothesis (FRUH). Many economics arguments have been proposed to explain this anomaly. Bilson (1986). Fama (1984). and Sweeney (1986) suggest that central bank interventions can be in the origin of the forward discount anomaly. Barnhart and Szakmary (1991) have tried to explain the rejection of FRUH and the instability of the β coefficient by including in the estimated regression a variable that represents the intervention of central Banks. The authors have tried many Banks intervention variables but their results are not significant. Engel (1996) advances some other explanations:"Some progress has been made toward understanding the empirical findings when one allows for peso problems, learning, and possibly a group of agents whose irrational expectations lead to speculative bubbles through a bandwagon effect...".

Several others works have tried to explain the discount anomaly by the wrong econometric specification of the estimated model or by the statistical properties of the data. In the 80 decades researchers have been concentrated their attentions on the non-stationarity of the spot and forward exchange rates series. Recent works favor statistical artifact leads by analyzing the true nature of the non stationarity of the forward discount series, see for instance Baillie and Bollerslev (1994, 2000). Maynard and Phillips (2001). Sakoulis and Zivot (2001) and Choi and Zivot (2007).

In this paper, we privilege this last leads. We follow the empirical strategy proposed by Charfeddine and Guégan (2007). see also Choï and Zivot (2007). First, we analyze the statistical properties (in term of non-stationarity) of the spot, forward and forward discount exchanges rates. Then, by using the Markov switching model of Hamilton (1989) and long memory processes, we show that all the forward discount series are simultaneously characterized by the presence of long memory and breaks. We show also that the non stationarity of the forward discount series is the principal reason behind the rejection of the hypothesis that the current forward rate is an unbiased predictor of the future spot rate.

In this paper, our empirical results differ from previous works in three ways. First, we use a recent and different data sets. Second, we use for the first one the EURO/USD series to test the FRUH. Finally, we use the Markov switching model of Hamilton (1989) to detect the presence of breaks, contrary to all previous works in the literature which have used the Bai and Perron (1998) procedures.

The remaining of the paper is organized as follows. Section 2 presents the forward unbiased hypothesis and reports an exhaustive literature review concerning the forward discount anomaly. Section 3 reports the empirical results concerning the FRU Hypothesis and unit root tests results. Section 4 investigates the presence of breaks and long memory inside the forward discount series. Section 5 concludes.

2. Forward discount anomaly and empirical literature

In this section after introducing the forward unbiased hypothesis, we give an exhaustive review of recent empirical literature concerning the forward discount anomaly.

2.1 Forward Rate Unbiased Hypothesis (FRUH)

We say that a foreign exchange market is efficient when the two hypothesis of risk neutrality and rational expectations hold. Moreover, if these two hypothesis are verified the future anticipated rate of depreciation must offset the interest rate fluctuations. This hypothesis is known, in the literature, as the uncovered interest rate parity which is given by,

$$\frac{S_t^{k,e}}{S_t} = \frac{1 + i_t}{1 + i_t^*} \quad (1)$$

where $S_t^{k,e}$ represents the spot exchange rate at time t+k and S_t the spot exchange rate at time t. i_t and i_t^* denote respectively the domestic and foreign interest rate (for k periods maturity). The symbol *e* means that the variable is expected. Under logarithmic notations this relationship is approximately, (Note 1)

$$s_t^{k,e} - s_t = i_t - i_t^* \quad (2)$$

where $s_t^{k,e}$ =Log($S_t^{k,e}$) and s_t =Log(S_t). Equation (2) is obtained by considering $Log\ (1 + x) \approx x$, for small values of x that approach zero, we set here $x = i_t$, i_t^* . The second version of interest rate relationship is the covered version which implies that the current forward rate is an unbiased predictor of the future spot rate. We suppose here that there are no transactions costs and no tax, etc. The covered interest parity implies that,

$$\frac{F_t}{S_t} = \frac{1 + i_t}{1 + i_t^*}, \quad (3)$$

where F_t denotes the forward exchange rate for k periods, S_t the spot exchange rate. i_t and i_t^* denote respectively the nominal domestic and foreign interest rate (for k periods maturity). The logarithmic form of (3) is given by,

$$f_t - s_t = i_t - i_t^* \quad (4)$$

If the covered (2) and the uncovered (4) relationships hold, then the forward discount $f_t - s_t$ must be equal to the expected return on the spot exchange rate,

$$f_t - s_t = s_t^{k,e} - s_t, \quad (5)$$

which can be re-writen as follows,

$$f_t = s_t, \quad (6)$$

This later relationship implies that the forward exchange rate is an unbiased predictor of the future exchange rate. To test foreign exchange market efficiency researchers use, generally, a two particular equations. the first is a "level specification" which comes from (6).

$$s_t^k = \alpha + \beta f_t + \eta_{t+k} \quad (7)$$

Here η_{t+k} is an (iid) white noise.

The second is the well used "differences equation", where we regress the forward discount on the spot exchange return. This version follows from (5).

$$s_t^k - s_t = \alpha + \beta (f_t - s_t) + u_{t+k} \quad (8)$$

Under these two specifications if the FRUH holds, we get β =1, α =0 and/or the term error is a white noise. Using the first equation researchers have been largely accepted the hypothesis that the current forward exchange rate is an unbiased predictor of the future spot exchange rate, see for instance Levich (1979). Frenkel (1981). Edwards (1983). and Chiang (1986). (1988). This "level" specification do not have a sense unless the dependent and independent variables are stationary or if we will test the co-integration hypothesis between s_t^k and f_t , if these two variables are I(q) with q an integer. In the 80's century researchers have used the "level" specification to test the forward unbiasedness hypothesis and not the co-integration hypothesis, see for instance Meese and Singleton (1982) and Meese (1989) and Isard (1995). In that case, no sense can be done to that method because we have a fallacious estimation. This holds because the use of standard limiting distribution of usual statistics tests cannot be allowed and the obtained results are biased. However, by using the second specification the empirical finding show, contrary to all expectations, that the FRU Hypothesis is rejected for the majority of time series, see for instance Baillie and Bollerslev (1994, 2000). Maynard and Phillips (2001). Sakoulis and Zivot (2001) and Choi and Zivot (2007). Furthermore, not only the FRUH is rejected but we get a significantly negative value of β . Note here that in the rest of the paper, we will use only the differences specification given by equation (8) to investigate the FRU Hypothesis.

2.2 Reviews of the empirical literature

In recent literature, several econometrics paths have been followed to test the FRU Hypothesis. Unit root tests and cointegration hypothesis take the wide part of the 80-90 empirical literature. Using equation (7). a contrasting results have been obtained. Results depend on periods and money of study. The majority of works have showed that the spot and forward exchanges rates are non stationary and follow an I(1) process. In that case, a possible solution is to test the cointegration hypothesis. Cointegration implies that Granger causality (1969) must runs in at least one direction, that is, at least one of the exchange rates is predictable using current available information. In that sense, the FRUH suggests that s_t^k and f_t are cointegrated with a cointegrating vector [1,-1]. This result has been interpreted as foreign exchange market inefficiency. The empirical literature concerning this hypothesis is also controversial. Some works have accepted this hypothesis, see for example Mark et al. (1993) whose provide evidence for cointegartion between s_t^k and f_t with a vector [1,-1] and Hakkio and Roch (1989). While some other works have obtained opposite results, see for example Evans and Lewis (1995) and Zivot (2000).

Despite this empirical disparity, there exist a some consensus between researchers concerning the stationarity of the spot exchange return, $s_t^k - s_t$. So, if this series is stationary, the forward discount series, $f_t - s_t$, must also be stationary so that β do not deviates from it's expected unity value. Despite this unanimous consensus about the stationarity of the spot exchange rate return Δs_t series, the true nature of the forward discount series remains ambiguous. Mark et al. (1993) show that the $f_t - s_t$ series is stationary (an I(0) process). Crowder (1994) show that the forward discount is an I(1) process. This non-stationarity of the forward discount series is considered as the principal cause of the rejection of the forward unbiasedness hypothesis. This comes from the fact that, if we regress an I(1) process on I(0) process, then the value of β will deviate from it's expected value of unity.

Recently, some studies have investigated the hypothesis of presence of non-linearity inside the forward discount series. Baillie and Bollerslev (1994, 2000). and Maynard and Phillips (2001) have showed that the $f_t - s_t$ series follows an I(d) process where d is a fractional parameter. For the majority of investigated series, we get an estimated value of the long memory parameter d which is higher than 0.5. This means that these series are characterized by a non-stationary long memory behavior. In all cases, there are a mean-reverting behavior in the forward discount. This lead to conclude that regression (8) is not well specified because the left-hand-side variable ($s_t^k - s_t$) and the right-hand-side variable ($f_t - s_t$) have a different degrees of integration.

More recently, researchers have used models with changes in regimes to explain the non stationarity of the forward discount series, see for instance Sakoulis and Zivot (2001). Choi and Zivot (2007) and Baillie and Kiliç (2007). As shown in the literature concerning this kind of models, the presence of breaks inside time series can creates a spurious long memory behavior. In that case, the observed long-range dependence will be a spurious behavior, see for instance Diebold and Inoue (2001). Granger and Hyung (2004) and Charfeddine and Guégan (2007, 2009b) .

3. The FRU Hypothesis and Unit Root Tests

This section has two mains objectives. First, we confirm the rejection of the FRU Hypothesis in the four currencies (Euro, French Swiss, pound sterling and Canadian dollar) in terms of US dollar. Then, we examine the hypothesis of non stationarity of the forward discount series.

In the following subsection, we present the data and their properties. Then, in the second subsection, we confirm the rejection of the FRU Hypothesis. Finally, in the third subsection, we applied unit root tests on the forward discount series.

3.1 The Data

Four times series will be used to investigate the forward discount anomaly. We use a weekly data exchange rates for four countries against the US dollar. This data span the period 06-01-1999 to 16-08-2006 for the Euro/Dollar series and 29-10-1997 to 16-08-2006 for the three others series. All time series are obtained from the Datastream base. The s_t and f_t are the logarithm of the level rates of the spot and forward exchange rates multiplied by 100.

Figures 1, 2, 3, and 4 report the trajectories and the autocorrelation functions of the spot. spot return and forward discount series of each exchange rate series. These figures show that the s_t and $f_t - s_t$ series seem to be non stationary. The ACF of each forward discount series show the presence of long range dependence behavior. Tables 1, 2 and 3 report the corresponding descriptive statistics for each series. Following these tables, we observe that the s_t ,

Δs_t and the $f_t - s_t$ series have a Skewness and Kurtosis statistics that differ significantly from those of the normal distribution. Jarque Bera test confirms the later results and shows that for all series the null hypothesis of normal distribution is rejected. This first analysis suggests that the right and left variables in equation (8) seem to have a different degree of integration.

3.2 Rejection of the FRUH

The results of estimations of the FRUH, equation (8). are reported in Table 4. The results show that the hypothesis of β =1 is rejected three times out of four. Same results are obtained for the jointly hypothesis of β =1 and α =0 which is also rejected three times out of four for the following series (EURO/USD, CHF/USD and CAN/USD). In conformity with the empirical literature, the β coefficient has a negative value which is higher than one in absolute value. This means that not only the FRUH is rejected but also the forward discount do not predicts the true direction of exchange rates fluctuations. Table 1 reports also the Durbin Watson (DW). the Q-stats of Ljung-Box Q(12). Q(24) and the R^2 coefficient. This table shows also that the four series possess a small R^2 coefficients and a DW statistic close to 2.

<Insert table 1)

To assess the possibility that the left and right variables in equation (8) have different degrees of integration, we propose, in the following subsection, to analyze the statistical properties, in term of stationary, of the spot exchange rates return and the forward discount for each series.

3.3 Unit root tests

The unit roots statistics used in this paper to investigate the unit root hypothesis are the ADF, KPSS and the ADF-GLS statistics of Elliot, Rothenberg and Stock (1996). The use of ADF test allows us to compare our results with the 80-90 decades existing empirical literature. The use of KPSS and ADF-GLS tests is motivated by the fact that these tests are known to have good powers against the alternative of long-range dependence, see for instance Lee and Schmidt (1996). Following Ng and Perron (2001) and Choï and Zivot (2007), the lag length of the ADF-GLS test was selected using the modified AIC with a maximum lag of 15.

Tables 5, 6 and 7 report the results of these unit root tests. Following these tables, the spot (s_t) and forward (f_t) exchange rates series are non stationary and follow an I(1) process.

<Insert table 5>

These Tables show also that the forward discount series ($f_t - s_t$) are non stationary. For the spot exchange return series, unit root tests show that are stationary and follow in I(0) process. No significant difference exists between the three unit root tests. At 5% level significance, the ADF and KPSS tests find a same result except for the Δs_{t+1} CAN/USD time series. At 10% level significance, the results are more mixed. For example, using the KPSS test, three of the forward discount series are stationary. In contrast, using the ADF test, only one series is stationary.

<Insert table 6>

The ADF-GLS unit root test results reported in Table 7 provide evidences for non stationarity of the spot and forward series, these series follow an I(1) process. Moreover, Table 7 shows that the forward discount is characterized by the presence of a unit root. For all the forward discount series, we fail to reject the hypothesis of a unit root in the forward discount. Results concerning the spot exchange return are similar to those of the ADF and KPSS tests.

<Insert table 7>

From the results reported in tables 5, 6 and 7, the dependent and independent variable in equation (8) have a different order of integration. This means that the non-stationarity of the forward discount series is an interesting path in order to investigate the causes of the rejection of the FRU Hypothesis. (Note 2)

4. Long memory process versus switching models

The rejection of the hypothesis of stationarity of the forward discount series in the previous section can be explained by the low power of unit root tests against the alternative of long range dependence. If it is the case, the forward discount series will be characterized by a long memory behavior. Moreover, this long range dependence behavior can be a spurious behavior that is created by the presence of breaks, see for instance Diebold and Inoue (2001). Granger and Hyung (2004) and Charfeddine and Guégan (2009b). Thus, it is very important to investigate the true nature of this non stationarity and to determine if the forward discount series are characterized by a long range dependence behavior, or by the presence of breaks, or simultaneously by the two behaviors.

To do that, we use the following strategy. First, we start by estimating the fractional long memory parameter from each forward discount series by using the well known GPH technique of Geweke and Porter-Hudak (1983) and the Exact Local Whittle (ELW) method of Shimotsu et Phillips (2005). Then, we use the Markov switching model of Hamilton (1989) to investigate the possibility of the presence of breaks. After that, we describe briefly the empirical strategy proposed in Charfeddine and Guégan (2007) in order to determine if the long memory behavior observed on the forward discount series is a true behavior or spurious one.

4.1 Long memory methods

In the last three decades, several long memory estimation methods have been developed. The first semi-parametric method proposed in the literature is the GPH technique of Geweke and Porter-Hudak (1983). Recently, Shimotsu and Phillips (2005) propose an alternative method, the Exact Local Whittle (ELW), based on the Local Whittle semi-parametric methods of Künsch (1987) and Yajima (1989).

The GPH technique is based on the log-periodogram. For frequency near zero, the fractional long memory parameter d can be estimated from the following least squares regression,

$$Log\left\{I(w_j)\right\} = a - dLog\left\{4\sin^2(w_j/2)\right\} + \varepsilon_t, \qquad j = 1, \ldots\ldots, m$$

where w_j is the periodogram of the process $(y_t)_t$ at frequency $w_j = 2\pi j/T$ Consistency requires that m grows slowly with respect to the sample size. It is suggested to take $m = T^r$ with $r = 0.5$. In recent literature, many researchers have suggested to use a frequency of order $O(T^{4/5})$, see for instance Hurvich et al. (1998). Maynard and Phillips (2001). Kim and Phillips (2000) and Choi and Zivot (2007). The ordinate least-square estimator of d is asymptotically normal with standard error equal to $\pi(6m)^{1/2}$, see for instance Geweke and Porter-Hudak (1983) and Robinson (1995).

The second semi-parametric method used in this paper is the Exact Local Whittle (ELW). see Shimotsu and Phillips (2005). This method avoids some approximation in the derivation of the Local Whittle estimator proposed by Künsch (1987) and Yajima (1989). The method is more attractive than the Local whittle (LW) method because of it's more interesting asymptotic properties. The estimated value $\hat{d}_{ELW}$ is obtained as follows:

$$\hat{d}_{ELW} = Arg\,min_{d\in[d_1,d_2]}\ R(d),$$

where d_1 and d_2 are the lower and upper bounds of the admissible values of d such that $-\infty < d_1 < d_2 < \infty$ and,

$$R(d) = LogG(d) - 2d\frac{1}{m}\sum_{j=1}^{m} Log(\omega_j),$$

where m is the truncation parameter, and $G(d) = \sum_{j=1}^{m} I_{\Delta^d_y}(\omega_j)$ where $I_{\Delta^d_y}(\omega) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T}\Delta^d_{y_t} e^{iwt}\right|^2$ is the periodogram of $\Delta^d_{y_t} = (1-L)^d y_t$.

Under certain consistency and asymptotic normality assumptions given in Shimotsu and Phillips (2005). the ELW estimator $\hat{d}_{ELW}$ satisfies,

$$\sqrt{m}(\hat{d}_{ELW} - d) \rightarrow_d N(0, 1/4) \quad \text{when } T \rightarrow \infty$$

The results of the estimated fractional long memory parameter of the forward discount series (EURO/USD, CHF/USD, UK/USD and CAND/USD series) are reported in tables 8 and 9. For both methods, we use different values of the frequency m $=T^{0.6}$, $T^{0.7}$ and $T^{0.8}$.

<Insert table 8>

Tables 8 and 9 show that the discount forward series ($f_t - s_t$) are non stationary. This confirms the slowly decaying behavior observed in the ACF on Figure 1, 2, 3 and 4. Moreover, this result is in pair with some previous empirical works which have also found a fractional long memory behavior inside the discount forward series, see for instance Baillie et Bollerslerv (1994, 2000). Phillips and Maynard (2000) and Choi and Zivot (2007) among others. These authors have suggested that the rejection of the FRU Hypothesis is due to the presence of long memory components in the forward discount. Thus, if the independent variable in (8) is stationary and the dependent variable is integrated with a fractional order d, then regression (8) is a misspecified specification and the estimated value of the β parameter do not be consistent, see Engel (1996).

<Insert table 9>

These results concerning the estimated values of the long memory parameter d are slightly higher than those obtained by Baillie and Bollerslev (1994) and Choï and Zivot (2007). For example, when we use the ELW method, the estimated order of integration lies inside (0.75, 0.97) contrary to those reported by Choï and Zivot (2007) where d lies in the range (0.536, 0.866). These disparities can be explained by the higher frequency of our data a weekly data than the monthly data frequency in their works. These disparities can also be due to the difference on the periods of study.

Economically, this hypothesis of long range dependence, in the forward discount series, is very difficult to justify. So, the complexity and the heterogeneity of agents on exchange markets make difficult to suppose that today's exchange rate changes can have a long lasting effects. Thus, we suggest that only strong changes can influence the exchange rate at long horizon. Moreover, the fact that exchange rates fluctuate from one minute to another is not consistent with the long memory behavior hypothesis. In recent econometric empirical literature, many works have showed that long range dependence can be created by the presence of breaks inside time series, see for instance Granger and Hyung (2004) and Charfeddine and Guégan (2009a and 2009b). Therefore, it's very important to investigate the true nature of the long memory behavior observed from the ACF and confirmed by the GPH and ELW methods. The following section analyze this hypothesis of the presence of breaks inside the forward discount series by using the Markov switching model of Hamilton (1989). This hypothesis is economically less-difficult to justify than the long range dependence hypothesis. For example, Banks interventions, heterogeneity of agents inside the exchange market or the peso problem are the reasons that explain the presence of breaks inside the forward discount series.

Thus, we suppose that the long memory behavior detected using the two semi-parametric methods (ELW and GPH). is a spurious behavior. Then, we use the empirical strategy proposed by Charfeddine and Guégan (2007) to investigate the true nature of the long range dependence detected in the forward discount series.

The empirical strategy proposed in Charfeddine and Guégan (2007) is as follow: First one start by estimating the dates of breaks using the Markov switching model. Then, we adjust the data sets from the obtained breaks. After that, we compare the fractional long memory parameter *d* before and after adjusting the original series. Finally, we conclude.

4.2 Models with changes in regimes

This subsection introduces briefly the Markov switching model of Hamilton (1989). We say that a process $(y_t)_t$ follows a MS-5MV-AR(0) Markov Switching process with Five States in Mean and Variance and without autoregressive order if it takes the following form,

$$y_t = \mu_{s_t} + u_{s_t}$$

where $u_t \rightarrow N(0, \sigma^2_{s_t}), et$

$$\mu_{s_t} = \mu_1 s_{1t} + \mu_2 s_{2t} + \mu_3 s_{3t} + \mu_4 s_{4t} + \mu_5 s_{5t}$$

$$\sigma^2_{s_t} = \sigma^2_1 s_{1t} + \sigma^2_2 s_{2t} + \sigma^2_3 s_{3t} + \sigma^2_4 s_{4t} + \sigma^2_5 s_{5t}$$

with $s_{jt} = 1$ if $s_t = j$, et $s_{jt} = 0$, otherwise, j=1,2,3,4, 5 et $p_{ij} = \Pr[s_t = j / s_{t-1} = i]$ et $\sum_{j=1}^{5} p_{ij} = 1$ .To select the appropriate MS-NMV-AR(0) model (N is the number of regimes N=1,2,3,4 or 5). we use the Garcia's test (1998). the residual analysis, the AIC and the HQ criteria's (not reported here). The results concerning the selected Markov switching MS-NMV-AR(0) models are reported in table 10.

<Insert table 10>

From this table, it appears that the hypothesis of simultaneously changes in mean and variance is more supported by the four forward discount series. Moreover, it appears that the data supports the presence of a large number of breaks. Table 11 lists the dates of breaks.

<Inert table 11>

These dates of breaks are selected using the filtered and smoothed probabilities. Table 11 shows also a number of breaks respectively equal to 13, 11, 10 and 14 for the Euro, Franc Swiss, British Pound and the Canadian dollar currencies against the U.S dollar. Presence of breaks inside these series is supported by some economics events. The Peso Problem, the heterogeneity of agents and intervention of central Banks inside exchanges rates markets are generally the major reasons that cause the presence of breaks.

5. Long range dependence or structural changes models?

This section tries to give an answer for the following question: which process describes the true Data Generated Process (DGP) of the forward discount series: long range dependence or models with changes in regime. Moreover, one can also check the possibility that these series are characterized by the presence of these two behaviors. If this later case occurs, then the forward discount series will be characterized by the presence of long memory behavior which is amplified by the presence of breaks.

To analyze the true nature of the long memory behavior observed on the forward discount series. We use the empirical strategies proposed by Charfeddine and Guégan (2007). Once we have selected the breaks dates, the following step consists on filtering out these breaks. Then, we re-estimate another one the fractional long memory parameter d from the adjusted date. Finally, we compare the two estimated parameter d (before and after filtering out the breaks).

<Insert table 12>

The results of the estimated long memory parameter after adjusting the data are reported in table 12 and 13. From these tables, it appears that there are a significantly difference between these values and those obtained after adjusting the data from breaks. Using the GPH method and for a frequencies m= $T^{0.6}$, the estimated value of d is not significantly different from zero. Contrary, for higher values of m, we detect the presence of a stationary long memory behavior, see table 12.

<Insert table 13>

Using the ELW method, see table 13, the results support also the alternative hypothesis of a stationary long memory behavior. Results seem also to depend on the values of the frequencies m. In all cases, the new fractional estimated long memory parameter d lies on (0.3, 0.7), contrary to initial data (before adjusting from breaks) where the values of d lies on (0.55, 1). This means that the long memory behavior observed in the ACF of the forward discount series is a true behavior which is amplified by the presence of breaks inside these time series.

6. Conclusion

This paper advances some new economics and econometrics explanations for the rejection of the FRU hypothesis. Our empirical analysis shows that the dependent and independent variables of the equation used to test the FRU Hypothesis are not integrated with a same order. This is the principal cause of the rejection of the FRU Hypothesis. Also, we have showed that the forward discount series are subject to many breaks. In that case, changes in regimes are caused by changes in anticipations, by the presence of risk premium and the peso problem. Moreover, we have showed that the forward discount series are characterized by the presence of a long range dependence behavior which amplified by the presence of breaks.

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Notes

1- The use of the logarithmic form allows to avoid the Siegel paradox. Siegel (1972) suggests that the level relationship must be verified from the two exchanges sides which contradicts the Jensen inequality, E(1/x)> 1/E(x).

2- In the rest of the paper, we concentrate our analysis only in equation (8). Moreover, our analysis is also concentrated on the forward discount series which is considered as the origin of the forward discount anomaly.

Table 1. Descriptive statistics of s_t

	EURO/USD	CHF/USD	UK/USD	CAN/USD
Mean	0.066	0.491	0.364	0.341
Median	0.072	0.490	0.374	0.383
Std. Dev	0.138	0.087	0.122	0.104
Skew.	-0.134	-0.009	0.064	-0.791
Kurt.	1.632	2.014	1.832	2.371
J-B	32.216	18.614	26.436	55.535
Prob	0.000	0.000	0.000	0.000
Q(12)	4407.8	4941.3	5029.1	4993.8
Q(24)	8201.8	9201.8	9270.1	9003.4

Table 2. Descriptive statistics of $\Delta s_{t+1} = s_{t+1} - s_t$

	EURO/USD	CHF/USD	UK/USD	CAN/USD
Mean	0.0002	-0.0003	0.0002	-0.0003
Median	0.000	4.1E-05	0.0003	0.0002
Std. Dev	0.014	0.014	0.010	0.009
Skew.	0.024	-0.204	-0.128	0.038
Kurt.	2.943	3.135	3.295	3.613
J-B	0.093	3.074	2.529	6.329
Prob	0.954	0.215	0.282	0.042
Q(12)	8.547	23.478	4.7668	9.2949
Q(24)	18.960	33.650	13.495	22.414

Table 3. Descriptive statistics of $f_t - s_t$

	EURO/USD	CHF/USD	UK/USD	CAN/USD
Mean	6.91E-05	-0.0003	-0.0004	4.11E-05
Median	5.8E-05	-0.0004	-0.0003	3.85E-05
Std. Dev	0.0003	0.0002	0.0003	0.0002
Skew.	0.101	0.549	-0.306	0.1665
Kurt.	1.521	2.568	1.752	2.328
J-B	36.941	23.081	32.048	9.319
Prob	0.000	0.000	0.000	0.009
Q(12)	4416.8	4649.6	4794.7	4695.5
Q(24)	8145.4	8302.5	8808.3	8308.8

Table 4. Estimation of the percentage change specification

$$s_t^k - s_t = \alpha + \beta(f_t - s_t) + u_{t+k}$$

			t-stat	F-stat			Q-stats	
Currencies	α	β	$\beta=1$	$\alpha=0,\ \beta=1$	R^2	DW	Q(12)	Q(24)
EURO/USD	0.058	-5.175	-2.807	3.969	0.014	2.045	9.08	21.397
	(0.071)	(2.20)						
CHF/USD	-0.247	-4.561	-2.560	3.302	0.010	1.964	5.581	15.628
	(0.121)	(2.171)						
UK/USD	-0.014	-1.702	-1.29	1.406	0.001	1.829	24.428	34.098
	(0.073)	(2.091)						
CAN/USD	-0.045	-3.377	-1.951	2.57	0.005	2.086	9.977	22.952
	(0.044)	(2.243)						

t-stat are in parentheses.

Table 5. Results of the ADF Unit Root Test.

Currencies	s_t	f_t	$f_t - s_t$	Δs_{t+1}
EURO/USD	-0.457 (0)	-0.456 (0)	-0.458 (0)	-20.10*** (3)
CHF/USD	-0.755 (0)	-0.755 (0)	-0.659 (0)	-20.92*** (3)
UK/USD	0.374 (0)	0.376 (0)	-1.116 (0)	-13.10*** (3)
CAN/USD	-0.984 (0)	-0.982 (0)	-1.909* (0)	-22.12*** (3)

Note: (.) is the number of lag length selected by the (AIC) and the Schwartz criteria.

Critical values of the ADF test at significance level of 1%, 5% and 10% are respectively -2.57, -1.94, -1.61.

*,*** indicate that the corresponding statistics are respectively significant at the 10% and 1% levels.

Table 6. Results of the KPSS Unit Root Test.

Currencies	s_t	f_t	$f_t - s_t$	Δs_{t+1}
EURO/USD	.742 (17)	1.610 (16)	0.631*(7)	0.404**(16)
CHF/USD	.52 (17)	1.518 (17)	1.019 (5)	0.185***(17)
UK/USD	.201 (17)	1.200 (17)	0.517*(7)	0.244***(17)
CAN/USD	.612 (16)	1.739 (17)	0.674*(5)	0.519*(17)

Note: (.)lag length.

Critical values of the KPSS test at significance level of 1%, 5% and 10% are respectively 0.739, 0.463, 0347.

*,**,*** indicate that the corresponding statistics are respectively significant at the 10%, 5% and 1% levels.

Table 7. Results of the ADF-GLS Unit Root Test.

Currencies	s_t	f_t	$f_t - s_t$	Δs_{t+1}
EURO/USD	.051 (0)	0.051 (0)	-0.509(4)	-5.216***(10)
CHF/USD	-0.931 (0)	-0.932 (0)	-0.750(6)	-6.669***(15)
UK/USD	0.850 (3)	-0.849 (3)	-0.812(5)	-9.815***(0)
CAN/USD	.230 (0)	0.229 (0)	-0.465(3)	-7.252***(15)

Note: (.) Bandwidth

Critical values of the ADF-GLS test at significance level of 1%, 5% and 10% are respectively -2.58, -1.98, -1.62.

*,**,*** indicate that the corresponding statistics are respectively significant at the 10%, 5% and 1% levels.

Table 8. Estimation of the fractional long memory parameter d using the GPH method.

Currencies	EURO/USD	CHF/USD	UK/USD	CAN/USD
$T^{0.6}$	0.964 (6.025)	0.899 (8.289)	1.055 (8.953)	1.102 (10.094)
$T^{0.7}$	0.761 (8.014)	0.735 (8.670)	0.912 (10.742)	1.006 (12.679)
$T^{0.8}$	0.583 (8.273)	0.535 (8.920)	0.740 (11.586)	0.817 (13.515)

(.) t-stats in parenthesis.

Table 9. Estimation of the fractional long memory parameter using the ELW method.

Currencies	EURO/USD	CHF/USD	UK/USD	CAN/USD
$T^{0.6}$	0.868 (17.316)	0.843 (18.040)	0.816 (17.463)	0.895 (19.153)
$T^{0.7}$	0.821 (18.912)	0.790 (19.522)	0.784 (19.373)	0.883 (21.820)
$T^{0.8}$	0.802 (22.627)	0.792 (23.973)	0.754 (22.82)	0.882 (26.724)

(.) t-stats in parenthesis.

Table 10. Estimation of the Markov switching model MS-NVM-AR(0) on the forward discount series.

	Euro/USD	**CHF/USD**	**UK/USD**	**CAN/USD**
Par.	MS-5VM-AR(0)	MS-4VM-AR(0)	MS-5VM-AR(0)	MS-5VM-AR(0)
$\mu 1$	-0.030 (0.000)	-0.081 (0.000)	-0.059 (0.000)	-0.023 (0.000)
$\mu 2$	-0.020 (0.000)	-0.068 (0.000)	-0.046 (0.000)	-0.015 (0.000)
$\mu 3$	0.001 (0.000)	-0.043 (0.000)	-0.036 (0.000)	-0.002 (0.000)
$\mu 4$	0.030 (0.000)	-0.015 (0.000)	-0.016 (0.000)	0.012 (0.000)
$\mu 5$	0.045 (0.000)	-	0.007 (0.000)	0.033 (0.000)
σ_1^2	2.8E-5 (0.000)	1.2E-4 (0.000)	4.2E-5 (0.000)	5.1 E-5 (0.000)
σ_2^2	4.7E-7 (0.000)	1 E-5 (0.000)	1.1 E-5 (0.000)	1.3E-5 (0.000)
σ_3^2	1.5E-4 (0.000)	9.6E-5 (0.000)	1.7E-5 (0.000)	3.0 E-5 (0.000)
σ_4^2	1.9E-5 (0.000)	5.5E-5 (0.000)	4.7E-5 (0.000)	2.8E-5 (0.000)
σ_5^2	5.8E-5 (0.000)	-	6.8 E-5 (0.000)	3.2E-5 (0.000)
$p11$	0.956	0.864	0.887	0.922
$p12$	0.043	0.035	0.055	0.068
$p13$	0.001	0.001	0.057	0.009
$p14$	0.001	-	0.001	0.001
$p21$	0.069	0.074	0.062	0.058
$p22$	0.906	0.114	0.938	0.933
$p23$	0.024	0.011	0.001	0.088
$p24$	0.001	-	0.001	0.001
$p31$	0.001	0.018	0.014	0.001
$p32$	0.010	0.001	0.007	0.025
$p33$	0.970	0.967	0.951	0.965
$p34$	0.018	-	0.026	0.009
$p41$	0.001	0.001	0.001	0.001
$p42$	0.001	0.001	0.001	0.001
$p43$	0.015	0.006	0.033	0.009
$p44$	0.951	-	0.943	0.980
$p51$	0.001	-	0.001	0.001
$p52$	0.001	-	0.001	0.001
$p53$	0.001	-	0.001	0.001
$p54$	0.001	-	0.001	0.013
L(.)	**1477.95**	**1541.787**	**1652.22**	**1831.97**

p-values in parenthesis.

Table 11. Breaks dates for the forward discount series

	EURO/USD	CHF/USD	UK/USD	CAN/USD
1	17:03:1999	20:05:1998	09:12:1998	14:01:1998
2	23:08:2000	15:07:1998	02:06:1999	19:08:1998
3	27:12:2000	20:01:1999	07:03:2001	11:08:1999
4	04:07:2001	10:03:1999	05:09:2001	06:12:2000
5	01:08:2001	31:05:2000	23:10:2002	18:04:2001
6	15:08:2001	06:12:2000	19:11:2003	25:10:2002
7	05:09:2001	18:04:2004	03:11:2004	25:12:2002
8	18:06:2003	05:10:2005	02:03:2005	04:02:2004
9	03:12:2003	26:10:2005	22:06:2005	24:03:2004
10	17:12:2003	15:03:2006	07:12:2005	15:12:2004
11	16:06:2004	26:04:2006	-	01:06:2005
12	15:06:2005	-	-	13:07:2005
13	12:10:2005	-	-	19:04:2006
14	-	-	-	31:05:2006

Table 12. Estimation of the fractional long memory parameter using the GPH method after filtering the Breaks.

Currencies	EURO/USD	CHF/USD	UK/USD	CAN/USD
$T^{0.6}$	0.460 (4.365)	0.484 (6.057)	0.280 (2.232)	0.293 (2.346)
$T^{0.7}$	0.591 (7.709)	0.470 (6.180)	0.349 (4.268)	0.542 (6.446)
$T^{0.8}$	0.542 (9.125)	0.447 (7.036)	0.369 (6.717)	0.586 (8.514)

(.) t-stats in parenthesis.

Table 13. Estimation of the fractional long memory parameter using the ELW method after filtering the Breaks.

Currencies	EURO/USD	CHF/USD	UK/USD	CAN/USD
$T^{0.6}$	0.579 (6.977)	0.437 (5.499)	0.196 (2.466)	0.436 (5.487)
$T^{0.7}$	0.667 (10.842)	0.468 (8.003)	0.352 (6.019)	0.545 (12.739)
$T^{0.8}$	0.585 (12.827)	0.436 (10.130)	0.454 (10.548)	0.648 (15.056)

(.) t-stats in parenthesis.

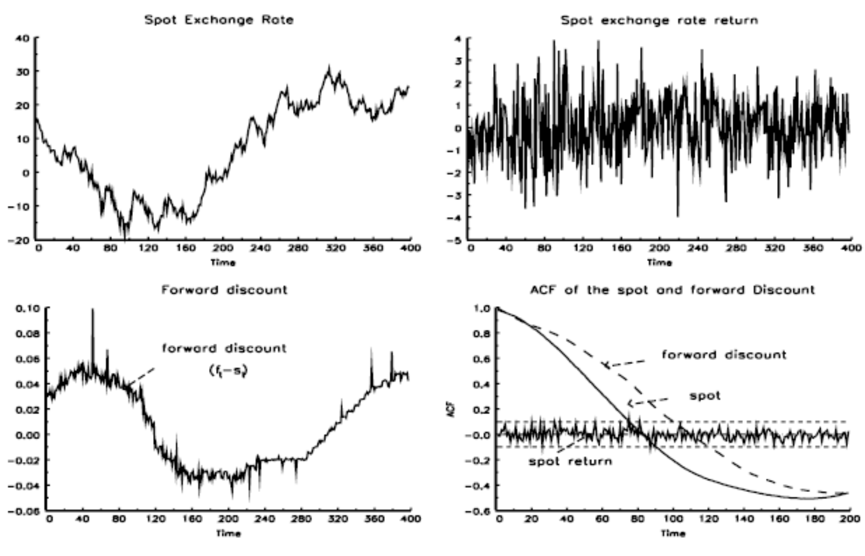

Figure 1. Trajectories and ACF functions of the spot, the forward and the forward discount of the EURO/USD exchange rate series

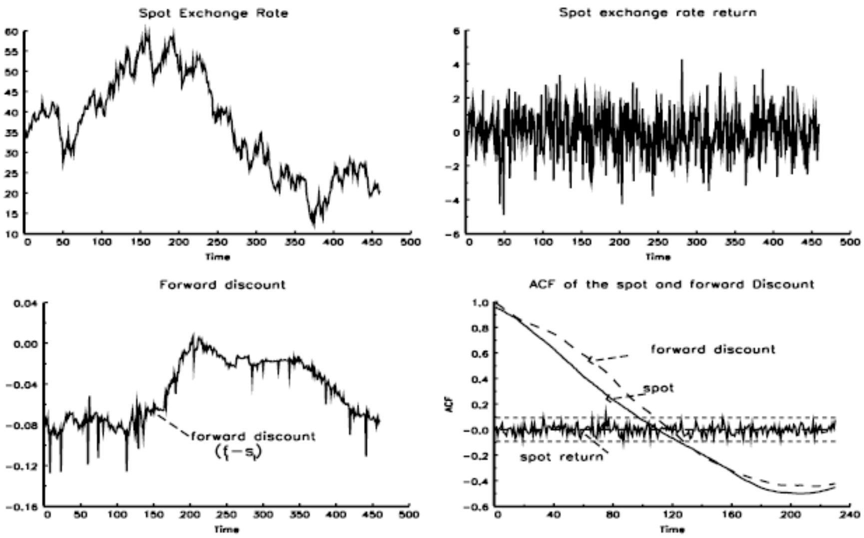

Figure 2. Trajectories and ACF functions of the spot, the forward and the forward discount of the CHF/USD exchange rate series

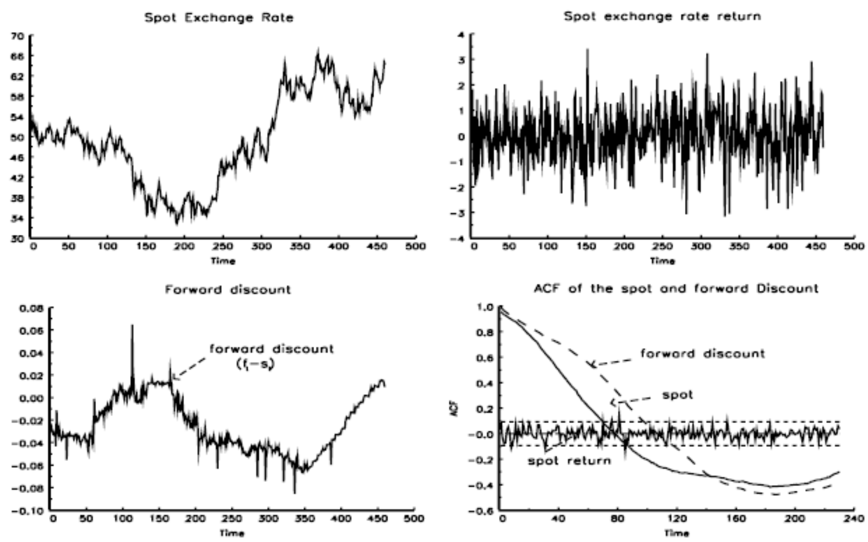

Figure 3. Trajectories and ACF functions of the spot, the forward and the forward discount of the UK/USD exchange rate series

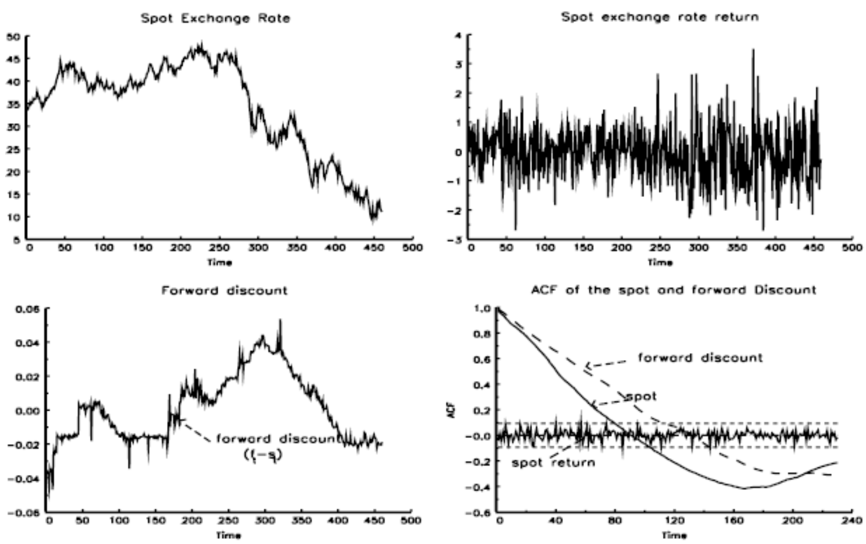

Figure 4. Trajectories and ACF functions of the spot, the forward and the forward discount of the CAN/USD exchange rate series

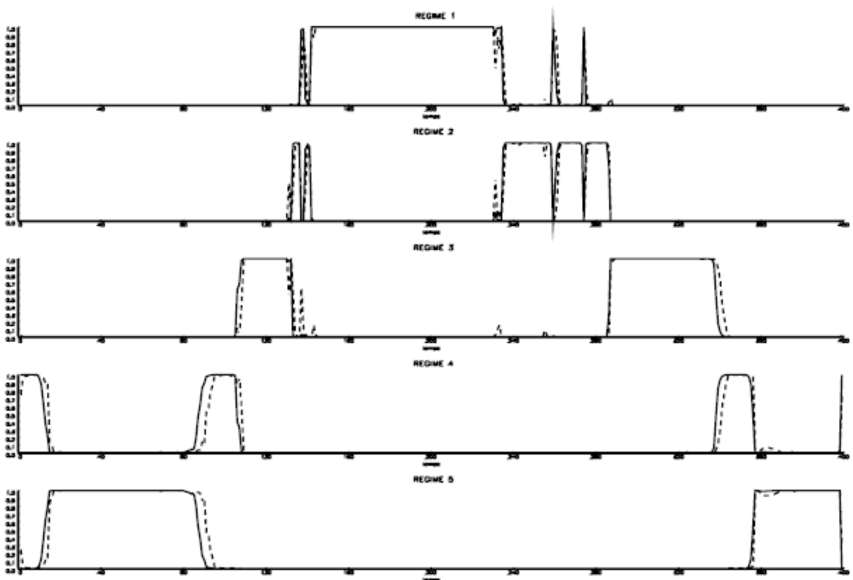

Figure 5. Filtered (dash line) and Smoothed probabilities (solid line) for the EURO/USD forward discount series

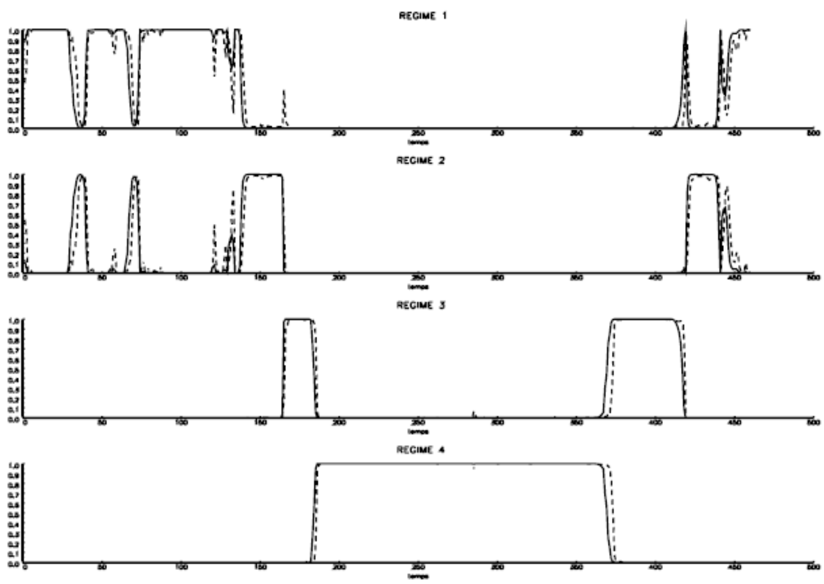

Figure 6. Filtered (dash line) and Smoothed probabilities (solid line) for the CHF/USD forward discount series

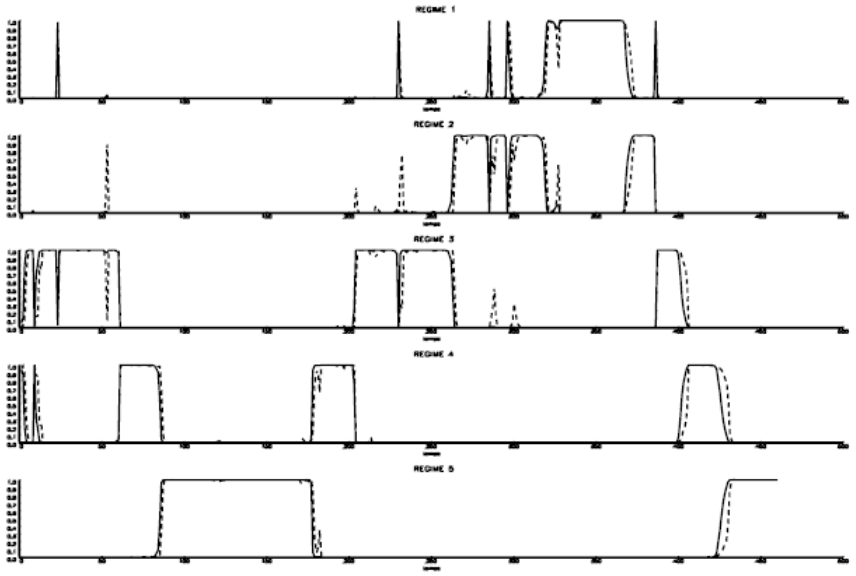

Figure 7. Filtered (dash line) and Smoothed probabilities (solid line) for the UK/USD forward discount series

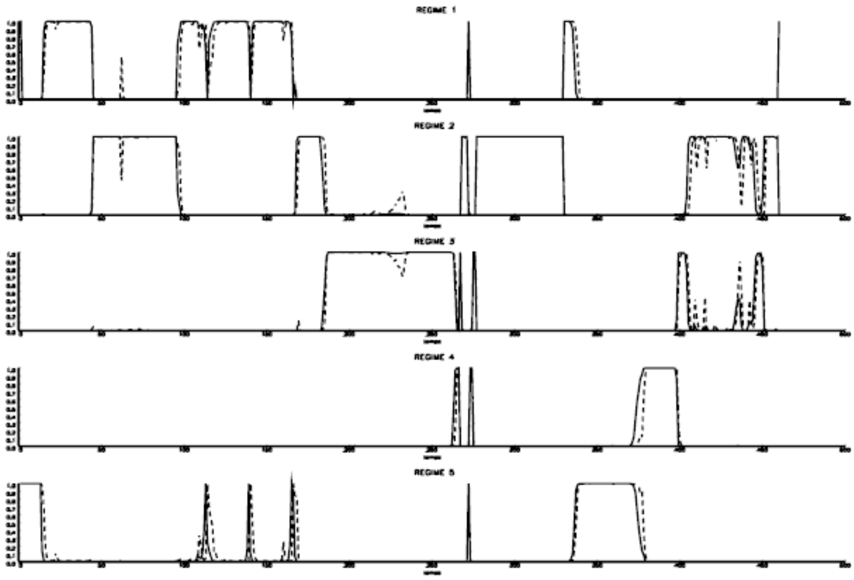

Figure 8. Filtered (dash line) and Smoothed probabilities (solid line) for the CAN/USD forward discount series.

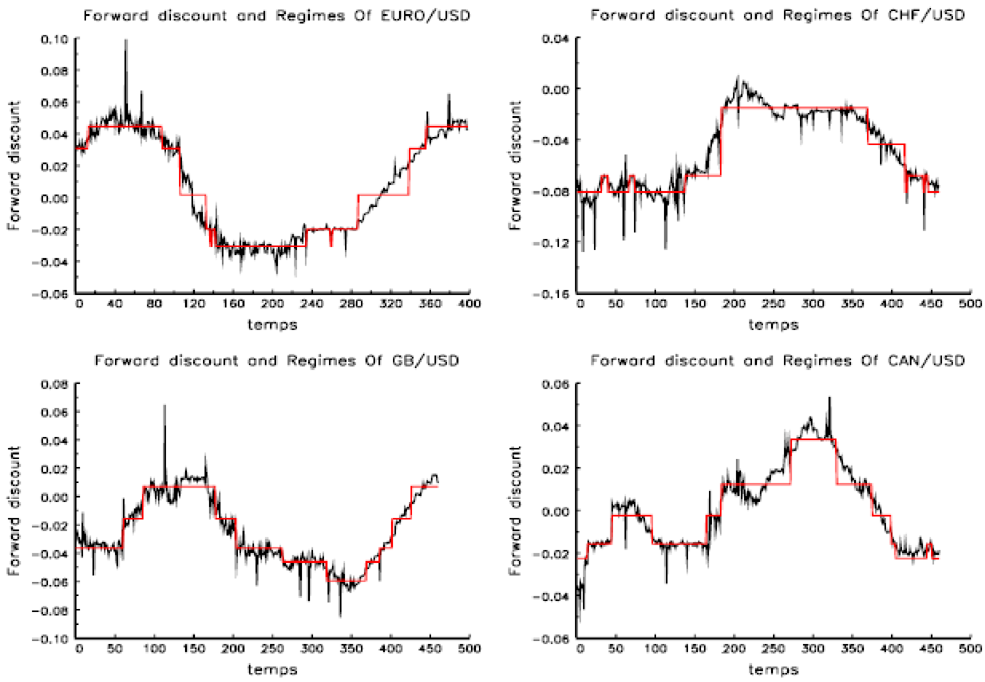

Figure 9. Trajectories and regimes detected using the Markov switching model.