Nonlinear Income Convergence and Structural Breaks: Further Empirical Evidence

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Received: August 2, 2012        Accepted: February 7, 2013        Online Published: March 18, 2013
doi:10.5539/ijef.v5n4p81        URL: http://dx.doi.org/10.5539/ijef.v5n4p81

Abstract

In this paper we reexamined the study done in King and Ramlogan-Dobson (2011) as well as Chong et al. (2008) by investigating the nonlinear convergence among the G16 countries using alternative methodology. We find that the results are sensitive to the method of analysis even after allowing for structural breaks. With semi-parametric model only six (6) cases of convergence were identified and eight (8) cases when we use nonlinear Fourier unit root test. With relative transition model all the sixteen (16) countries exhibit convergence with United States and Norway converging a little above group average.

Keywords: convergence, structural breaks, fractional integration, nonlinear Fourier transform

1. Introduction

In a recent paper King and Ramlogan-Dobson (2011) hereafter KR investigate the role of structural breaks in nonlinear income convergence applied to sixteen (16) OECD countries for the period 1950-2004. The study is an extension of a previous study by Chong et al. (2008) hereafter CHLL within a nonlinear framework but without incorporating structural breaks. Whereas CHLL (2008) find evidence of convergence for only four countries, KR using nonlinear LM test and in the presence of structural breaks find evidence of convergence for ten countries, which is more than double the number obtained under CHLL. The results seem to suggest that tests that ignore structural breaks are susceptible to misleading results. Our study revisits the income convergence hypothesis in the two studies by reexamining the issue within fractional integration framework, nonlinear Fourier transform and relative transition model to determine if the method materially affects the outcome. An attractive feature of fractional integration is that it allows for the existence of continuum of situation between I(0) and I(1) cases. In the case of nonlinear Fourier approximation it allows us to incorporate multiple structural breaks with unknown functional forms and hence to control for the effect of unknown forms of nonlinear deterministic terms in testing for a unit root. With respect to relative transition model, the approach provides a detailed characterization of the transition paths to equilibrium and also provide a simple test for convergence or divergence in the data generating process (DGP). Thus, the combination of these approaches enable us to determine the sensitivity of the results to different methods of analysis.

The rest of the study is organized in four sections. Section I has been the introduction. In section 2 a brief review of the literature is provided. Section 3 articulates the econometric tests and report the empirical results Section 4 concludes.

2. Literature Review

The literature on income convergence is vast and growing. At one end there is the neoclassical growth theory which states that over the long run there is the tendency of per capita income of different countries to converge to steady state (Solow 1956, Barro and Sala-i-Martin 1995, Bernard and Dulauf 1995). At the other end there is the ‘new’ endogenous growth theory that challenges the former insisting that social increasing returns to scale associated with human and physical capital cause divergence (Romer 1986, Lucas 1988). According to the new endogenous growth theory there is no automatic mechanism that prevent economies from divergent steady states. For example, the structure of incentives to invest which are different among countries is said to be one of the
Critical factors promoting divergence (North 1990, Barro and Sala-i-Martin 1997). It is therefore not surprising that the debate has attracted significant interests among scholars and development economists to determine which of the two competing schools of thought best describe convergence behavior.

Testing for convergence within time series framework had been based on unit root test of stochastic convergence indicated in Ben-David (1993), Bernard and Durlauf (1995). According to Carlino and Mills (1993) stochastic convergence implies that shocks to the income of a given country relative to the average income across a group of countries will be temporary. A common test for stochastic convergence involves testing for a unit root in the log of the ratio of per capita income relative to the group average or to the dominant economy. Failure to reject the unit root null hypothesis is indicative of divergence, while its rejection is supportive of stochastic convergence.

Whereas many studies on convergence had been carried out on the assumption of linearity, there is growing literature that it may be nonlinear and that linearity is too restrictive. Greasley and Oxley (1997), Kapetanios et al. (2003), Datta (2003), CHLL (2008), KR (2011) are indicative of this line of empirical enquiry. CHLL (2008), and KR (2011) adopt nonlinearity within a Smooth Transition Autoregressive process.

However, recent research claims that growth convergence cannot be appropriately investigated in a $I(0)/I(1)$ framework given the evidence in the empirical literature that aggregate output or its components are suitably modeled by fractionally integrated processes. By its design fractional integration accounts for the long-memory characteristic of the series through the differencing parameter $d$ that can take any values not necessarily integers.

The justification for fractional integration stems from the fact that it is a consequence of aggregation over heterogenous firms and multiple sectors (Lo and Haubrich (2001)).

One problem with KR (2010) study is that the use of conventional procedures for detecting and dating structural breaks tend to find spurious breaks, usually in the middle of the sample, when in fact there is only fractional integration in the data (Hsu 2001, Kramer and Sibbertsen 2002, Mayoral 2006). Indeed traditional test as indicated in KR and CHLL considers only integer integration versus short memory and structural breaks, even in cases where there is empirical evidence for the hypothesis of fractional integration. If the model is in fact fractionally integrated, contradictory results are likely to be found when different methods are adopted. This problem is also examined in our study by conducting a battery of tests.

We adopt three approaches. The first pertains to the semi-parametric long memory narrow band model of Geweke and Porter-Hudak (GPH, 1983). The second approach is the nonlinear Fourier transform which allows for structural breaks (Enders and Lee 2006, 2009; Becker et al. 2006). In this method the trigonometric terms is defined to capture unknown nonlinearities in the equilibrium level. The third approach is the relative transition model of Phillips and Sul (2007a, 2007b).

### 3. Econometric Method and Results

Let $Y_{it}$ be the per capita income level of country $i$ in time $t$ and $Y_{jt}$ the corresponding income of country $j$ in time $t$. Let $X_{it} = logY_{it} - logY_{jt}$ be the corresponding income ratio or per capital income differential. As in KR we use US as the dominant economy.

**Therefore**

\[ X_{it} = \alpha + \beta t + w_i - I(d) \quad i=1,2,\ldots,n, \quad i \neq j \]

or \((1-L)^d w_i = C(L) \varepsilon_i \sim iid(0,\delta^2)\)

where $L$ is the lag operator, $C(L) = \sum \xi_k L^k$, $C(0) = 1$, and $d$ is the fractional integration parameter, $k$=lag length, $w_i$ is a zero-mean fractionally integrated process. We assume that the process is invertible ($d > -0.5$). Consequently $w_i$ can be rewritten as an infinite AR($p$) process:

\[ \sum_{k=1}^{(d-1)} \pi_k (d) \xi_k = (d) \xi_k = \frac{k}{\Gamma(k+1)} \]

where \(\Gamma(.)\) is the gamma function.

The value of $d$ indicates the persistence of the shocks: the smaller $d$ the less persistent will be the shocks.

**Case 1:** When $-0.5 < d \leq 0$, $w_i$ is short memory, that is $I(0)$. The coefficient \(\pi_k\) in (3) reduces to \(1/k\) and decay rapidly towards zero. In the context of fractional integration we call this configuration rapid catching–up or short-memory catching-up or convergence ($\alpha \neq 0, \beta \neq 0$).

**Case 2:** When $0 < d \leq 0.5$, $w_i$ is a long memory stationary converging process. The autoregressive coefficients in (3)
decay smoothly. Any observed difference in the per capita income in the remote past still has an influence in the current year. We call this long memory catching-up. It occurs when a country spends a long time on the transition path towards the common equilibrium deterministic trend\((a \neq 0, \beta \neq 0)\).

Case 3: When \(0.5 < d < 1\), we have long memory non-stationary but mean-reverting converging process. The autocorrelation coefficients in (3) are characterized by a high persistence, meaning that any difference observed in per capita income in (the very far past has a long lasting influence. This transition dynamics is referred to as long memory mean-reverting catching-up \((a \neq 0, \beta \neq 0)\).

Case 4: When \(d \geq 1\), \(w_t\) is explosive. In this case, there is a magnification effect. Any initial difference is not expected to be reversed in future. We call this condition stochastic divergence \((a \neq 0, \beta \neq 0)\).

Similarly for conditional convergence \((a \neq 0, \beta = 0)\) three distinct cases may emerge:

- **Strict convergence** \(0 < d \leq 0\)
- **Long memory conditional convergence** \(0 < d < 0.5\)
- **Long memory mean-reverting convergence** \(0.5 < d < 1\)

Finally absolute convergence \((\alpha = 0, \beta = 0)\) occurs when \(d = 0\), long memory stochastic convergence when \(0 < d < 0.5\), long memory mean-reverting convergence when \(0.5 < d < 1\).

In general, \(d = 0\), the I(d) process is often called long memory process, because the autocovariance function is not reverting and when \(d\geq0.5\), the I(d) is nonstationary, but mean reverting and when \(d\geq1\) the I(d) is a purely non-stationary process.

Several authors have focused on a semi-parametric estimation of the memory parameter alone. An important property of stationary fractional series on which these semi-parametric methods are based is:

\[
\sum_{\lambda=\pm1}^{\pm\infty} \lambda^d = 0
\]

which is valid as \(\lambda \rightarrow 0^+\).

\[
\log(f(\lambda)) \approx k + d(-2\log \lambda)
\]

for small frequencies. From discrete Fourier transforms of equation (4) we obtain a regression of \(\log[I_{1}(\lambda_{i})]\) on a constant and \(-2\log \lambda_{j}\) for \(j=L, L+1, \ldots, m\), with \(L \geq 1\) and \(m<\infty\). This is the basis for the log periodogram (LP) regression of Geweke,Porter-Hudak (GPH, 1983) which uses narrow band, the broad band based Moulines and Soulier (1999) regression, and local whittle Gaussian maximum likelihood estimator of Robinson (1995a, 1995b). In this study we use GPH and test for bias using Davidson and Sibbertsen (2009) Hausman-type test and differentiate between spurious I(d) model and long memory in the fractional integration parameter results by splitting the sample into two, run the regression and test for parameter constancy of \(d\) using Shimotsu (2007) adjusted Wald (Wc) statistic distributed as \(\chi^2(b-1)\) where \(b\) is the number of samples.

### 3.1 Nonlinear Fourier Unit Root Test

The nonlinear Fourier unit root test relies on a Fourier function which captures structural change with a transition regime. It takes the form:

\[
\Delta X_t = \psi_0 + \theta X_{t-1} + \psi_1 \sin(2\pi k t/T) + \psi_2 \cos(2\pi k t/T) + \sum_{j=0}^{\phi} \Delta X_{t-j} + e_t
\]

where \(k (1 \leq k \leq 5)\) is the number of frequencies of the Fourier function, \(t\) is a trend term, \(T\) is sample size, and \([\psi_1 \sin(2\pi k t/T) + \psi_2 \cos(2\pi k t/T)\] captures structural change in the sequence \(\{X_t\}\). The unit root test allows for an unknown number of endogenous structural breaks with unknown functional forms.

There is nonlinearity and unknown breaks in the function if the hypothesis \(\psi_1 = \psi_2 = 0\) is rejected using F-statistics \(F(k)\) of Table 3 in Enders and Lee (2004). The \(K\) in \(F(k)\) is the \(k_{\text{min}}\) obtained from the regression (5) which gives the minimum residual sum of squares (RSS) for different frequencies. As earlier indicated rejection of the above hypothesis is indicative of the presence of structural breaks. If \(0 = 0\), (using the \(t_{10}\) statistics from Table 3, Enders and Lee 2004), there is unit root. However, if \(0\) is significantly different from zero, we reject the unit root through taking into account nonlinearity and possible structural breaks and therefore \(X_t\) is stationary (stochastic convergence).

### 3.2 Relative Transition Model

The relative transition paths to long-run equilibrium model was proposed by Phillips and Sul (2007a, 2007b). It is based on the reduced form of a Solow growth model allowing for heterogenous speeds of convergence and transition effects over time (see also Dufrenot et al. 2009). Let \(h_{i,t}\) be the relative transition path of country \(i\) at time \(t\) relative to the group of 16 countries with which she shares the same technology. We then have:

\[
h_{i,t} = Y_{i,t} / \left(N \sum Y_{j,t} - \delta_{i,t}(rT / T) \mu_{i,T} (rT / T) \rightarrow p \delta_{i,t} (rT / T) \mu_{i,T} (rT / T) \right) as T \rightarrow \infty
\]
where $N$ is the sample of countries, $T$ is the time span of the study, $r$ is the fraction of time corresponding to the observation $t$, $\mu_T(r)$ the common steady state growth curve, $\delta_i(r)$ the limiting transition curve for the economy, and $\Rightarrow p$ indicates convergence in probability.

The relative transition regression model is given by
\[
\log H_t = c - 2 \hat{p} \log(t) + \epsilon_t
\]
(6)

where $H_t = T^{-1} \sum (h_{i-1})^2$, $H_t \sim cT^{-2p}$ as $t \to \infty$, $i=1,2,\ldots,n$

where $\hat{p} > 0$ and statistically significant indicates convergence and $\hat{p}<0$ means divergence. The G16 countries was subdivided into three subgroups. Group A comprise five countries - USA, Canada, UK, and Germany; group B Australia, Austria, France, Italy, and Switzerland; group C Norway, Sweden, Denmark, Belgium, Finland and Netherlands. For each subgroup we fit the relative transition model for the period 1950-2006. We first removed the cyclical component in the log of per capita income using a Hodrick-Prescot Filter and then use the smooth component of the filter to estimate the relative transition coefficients.

The data were extracted from Angus Madison (2006) per capita GDP 1990 International Geary Khamis dollars table. The sample size is 1950-2006, two more data points than those used by KR. The fractional integration models were estimated using Time Series modeling (TSM) 4.34 (James Davidson 2002-2011) at http://www.timeseriesmodelling.com/ and OX 6.20 (J.A. Doonik, 1994-2010) was required as a complement to run the package. The nonlinear Fourier unit root was carried out using RATS 7.3 computer software. The relative transition curves were fitted using Eviews 7.0.

Table 1 presents the results based on GPH (1983) semiparametric long memory estimation method.

Table 1. Geweke-Porter-Hudak estimation and test results

<table>
<thead>
<tr>
<th>Country</th>
<th>$d$</th>
<th>$t$-value</th>
<th>Bias test</th>
<th>Sub Sample ds</th>
<th>$W_c$, $b=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.2495</td>
<td>0.714</td>
<td>1.332</td>
<td>0.835</td>
<td>30.33802*</td>
</tr>
<tr>
<td>Austria</td>
<td>0.8599</td>
<td>2.46</td>
<td>0.815</td>
<td>0.779</td>
<td>0.579203</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.934</td>
<td>2.673</td>
<td>0.223</td>
<td>0.913</td>
<td>0.039028</td>
</tr>
<tr>
<td>Denmark</td>
<td>1.283</td>
<td>3.671</td>
<td>-0.166</td>
<td>1.082</td>
<td>3.575408</td>
</tr>
<tr>
<td>Finland</td>
<td>0.8569</td>
<td>2.451</td>
<td>-0.161</td>
<td>0.961</td>
<td>0.959036</td>
</tr>
<tr>
<td>France</td>
<td>1.0507</td>
<td>3.006</td>
<td>0.609</td>
<td>1.051</td>
<td>7.96E-06</td>
</tr>
<tr>
<td>Germany</td>
<td>0.933</td>
<td>2.669</td>
<td>-0.216</td>
<td>0.964</td>
<td>0.085047</td>
</tr>
<tr>
<td>Italy</td>
<td>0.9988</td>
<td>2.857</td>
<td>-0.099</td>
<td>0.867</td>
<td>1.53732</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.811</td>
<td>2.321</td>
<td>-1.138</td>
<td>1.058</td>
<td>5.399175*</td>
</tr>
<tr>
<td>Norway</td>
<td>0.9916</td>
<td>2.837</td>
<td>-0.514</td>
<td>1.056</td>
<td>0.367033</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.0972</td>
<td>3.139</td>
<td>1.087</td>
<td>1.339</td>
<td>5.17423*</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.4005</td>
<td>4.006</td>
<td>-0.846</td>
<td>1.273</td>
<td>1.438646</td>
</tr>
<tr>
<td>UK</td>
<td>0.644</td>
<td>1.843</td>
<td>-0.5</td>
<td>0.833</td>
<td>3.161237</td>
</tr>
<tr>
<td>Japan</td>
<td>1.149</td>
<td>2.630</td>
<td>-0.288</td>
<td>1.147</td>
<td>2.212563</td>
</tr>
<tr>
<td>Canada</td>
<td>1.357</td>
<td>3.881</td>
<td>1.183</td>
<td>1.214</td>
<td>1.809696</td>
</tr>
</tbody>
</table>

Note:* Indicates rejection of the null at 5% level. $\chi^2_{0.95}(1) = 3.84$.

Our results, based on fractional integration, are inconclusive Eight (8) countries namely Denmark, France, Italy, Norway, Sweden, Switzerland, Japan and Canada indicate nonstationarity and hence divergence process. Six (6) countries (Austria, Belgium, Finland, Germany, Netherlands, UK) have estimated fractional integration parameter between 0.5 and 1.00 suggesting long memory but mean-reverting converging process. The transitional dynamics is in the tradition of long memory mean-reverting caching-up. Only Australia (d<0.5) shows evidence of stationary convergence or catching-up with USA. Of the seven (7) countries that show convergence, only four (4) namely Australia, Austria, Belgium and Germany were among those contained in KR converging countries. For the results based on fractional integration the adjusted Wald test do not support the view that structural breaks account for all the observed persistence.

Table 2 reports the results based on nonlinear Fourier unit root tests. First we note that the null hypothesis of linearity is rejected at 5 percent level for ten (10) countries based on the sample value of the F(k) statistic (those marked with *) in column three, where estimated k represent frequency with the minimum residual sum of squares (RSS). Eight (8) countries (Australia, Austria, Denmark, France, Italy, Sweden, Switzerland and UK) indicate evidence of convergence based on their estimated $\tau_{DF}$ values which exceed the critical value at $k_{min}$ frequency.
Thus, while KR obtain convergence for ten countries our results based on nonlinear Fourier approximation indicate convergence for eight countries, two less than the result in KR. Of the eight countries for which convergence is obtained six of them are contained in KR set of ten countries while the remaining two (Italy and United kingdom) were part of the five set of countries in KR for which divergence was the case.

Table 2. Results based on nonlinear Fourier Unit Root Test

<table>
<thead>
<tr>
<th>Θ</th>
<th>F(k)</th>
<th>Kmin</th>
<th>Lags</th>
<th>τDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.969*</td>
<td>5</td>
<td>3</td>
<td>-2.915*</td>
</tr>
<tr>
<td>2</td>
<td>7.701*</td>
<td>1</td>
<td>3</td>
<td>-5.272*</td>
</tr>
<tr>
<td>3</td>
<td>3.647</td>
<td>1</td>
<td>0</td>
<td>-2.979</td>
</tr>
<tr>
<td>4</td>
<td>6.237*</td>
<td>2</td>
<td>0</td>
<td>-2.949</td>
</tr>
<tr>
<td>5</td>
<td>8.770*</td>
<td>5</td>
<td>2</td>
<td>-3.703*</td>
</tr>
<tr>
<td>6</td>
<td>2.579</td>
<td>1</td>
<td>1</td>
<td>-2.753</td>
</tr>
<tr>
<td>7</td>
<td>4.018*</td>
<td>5</td>
<td>0</td>
<td>-3.132*</td>
</tr>
<tr>
<td>8</td>
<td>4.712</td>
<td>1</td>
<td>1</td>
<td>-2.845</td>
</tr>
<tr>
<td>9</td>
<td>12.311*</td>
<td>1</td>
<td>0</td>
<td>-5.256*</td>
</tr>
<tr>
<td>10</td>
<td>3.355</td>
<td>1</td>
<td>1</td>
<td>-2.353</td>
</tr>
<tr>
<td>11</td>
<td>3.492</td>
<td>3</td>
<td>0</td>
<td>-2.374</td>
</tr>
<tr>
<td>12</td>
<td>5.501*</td>
<td>3</td>
<td>3</td>
<td>-1.333</td>
</tr>
<tr>
<td>13</td>
<td>8.696*</td>
<td>2</td>
<td>4</td>
<td>-3.284*</td>
</tr>
<tr>
<td>14</td>
<td>20.960*</td>
<td>1</td>
<td>3</td>
<td>-5.498*</td>
</tr>
<tr>
<td>15</td>
<td>9.320*</td>
<td>1</td>
<td>4</td>
<td>-4.451*</td>
</tr>
</tbody>
</table>

Notes: τDF at 5% with kmin (1) -3.816, kmin (2) -3.270, kmin (3) =-3.059, kmin (5) -2.910, F(k): kmin (1) 7.137, kmin (2) 4.256 , kmin (3) 3.539, kmin (5) 3.139

Table 3. Log(t) test of transition convergence – Regression: Log Ht=c-2log(t) + et

<table>
<thead>
<tr>
<th>Countries</th>
<th>β</th>
<th>t-ratio</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. USA, Canada, Japan, UK, Germany</td>
<td>0.562</td>
<td>20.704</td>
<td>Convergence</td>
</tr>
<tr>
<td>B. Australia, Austria, France, Italy, Switzerland, France</td>
<td>0.608</td>
<td>16.239</td>
<td>Convergence</td>
</tr>
<tr>
<td>C. Norway, Sweden, Denmark, Finland, Belgium, Netherlands</td>
<td>0.322</td>
<td>10.318</td>
<td>Convergence</td>
</tr>
</tbody>
</table>

Table 3 and Figure 1 pertain to the results based on relative transition paths. The relative transition paths for each of the three subgroups reveal absolute convergence for all the subgroups (see Table 3). Figure 1 provides detailed characterization of the transition paths to long-run equilibrium or convergence of each of the G16 country. The relative transition curves show how the trajectories followed by these countries become closer over time. Some countries (Canada, USA, Switzerland, UK, Australia) start above average and follow a downward trend, while others (Japan, Italy, Germany) start below average and exhibit upward transition. As reported above in Table 3, the Log(t) test does not reject the null hypothesis of no convergence, thereby indicating that for the G16, there is a common factor driving their economies together in the long run. These factors include, but not limited to, technology, quality of their institutions, and labour productivity. The good news is that there is no evidence that the G16 countries are in general converging to a GDP per capita level below average, which is a sign of improvement in the standard of living over time. The USA and Norway show convergence slightly above the group average, while Japan’s performance is slightly below group average. It is remarkable to observe that KR also concluded that “Norway is the only country to catch up with the US over the entire period. Overall, the differences in the trajectories of the countries that are initially above and those initially below the average are reduced over time in all the countries.
4. Concluding Remarks

In this study we reexamined the results presented in CHLL and KR using alternative method to determine if the approach materially affected the results obtained. First we found that with respect to convergence with the United States only six countries (Austria, Australia, Belgium, Finland, Germany, Netherlands and United Kingdom) are convergent based on fractional integration. This is two more than in CHLL but four fewer than in KR. When we use nonlinear Fourier unit root test we obtain eight countries that are converging with the USA. This is two fewer than in KR. However, when we use relative transition path model we found that all the G16 countries exhibit convergence both in terms of the log(t) test and in terms of the trajectories of the logarithm of per capita income followed by these countries. The results show no evidence that the G18 countries are converging to a GDP per capital level below average. We conclude that findings of convergence studies are not robust to methodology adopted. It seems to us that relative transition model provides a more fruitful line of inquiry for future research.

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Foundation Discussion Paper, No. 1595.


