Default Risk under Different Colours of Noise

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Abstract

In this study we theoretically simulate default risk scenarios under various economic noises. We find that firms default more quickly with stronger economic shocks but simultaneously expose higher default probabilities during their deterioration, offering traders better visibility. When the macroeconomic environment exhibits positive autocorrelation, the volatility of assets' value increases with corporate creditworthiness, and vice versa. While positive autocorrelation forces liquidation more swiftly even for firms with higher risk tolerance, counter-cyclical economic movements reduce default risk for firms with greater sustainability. When a series of economic noises exhibits higher autocorrelation, although average default probabilities decline, firms tend to default more rapidly, making it tougher for traders to predict corporate failure.

Keywords: Colours of noise, Autocorrelation, Time series, Default risk, Merton Model

1. Introduction

It is debated by researchers whether a positively autocorrelated time-series of macroeconomic shocks damages corporate default risk more or less than a random sequence of economic noises. On the one hand, positive autocorrelation increases the probability of a series of contracting market conditions, implying increasing default risk. On the other hand, for any limited time period, the likelihood of at least one extreme harmful event is reduced compared with a chain of completely random noises, suggesting decreasing default risk. One should expect that positive autocorrelation increases default risk, since a firm may survive a single bad economic event but not a series of subsequent downturn conditions. However, positive time-series autocorrelation prevents a single major economic catastrophe and therefore decreases default odds. Furthermore, it is not entirely obvious how a negatively autocorrelated string of macroeconomic noises affects the creditworthiness of a firm, whether it contributes to or rather injures corporate survivability. A negative autocorrelation tends to compensate a destructive economic shock with a later constructive noise, thus reducing default risk. In contrast, it also minimizes the chances for successive favorable conditions and thus enhances default likelihoods. Moreover, it is still ambiguous whether a short memory autocorrelated time-series of economic noises hurts default likelihoods more or less than a longer memory process. These are the underlying riddles guiding our current study.

The financial literature has devoted considerable effort to exploring various issues concerning noise trading. Black (1986) describes noise as a phenomenon contrasted with information, but further explains that investors sometimes trade on noise, rather than fundamentals, as if it were information. Trueman (1988) provides a theory explaining why investors would rationally choose to engage in noise trading. De Long, Shleifer, Summers, and Waldman (1989, 1990, and 1991) investigate how stock prices respond to irrational noise trading, what the risks, expected gains and losses are from noise trading, and how noise traders can optimize their portfolio allocation and thus dominate the market. Palomino (1996) demonstrates how in imperfectly competitive markets with risk-averse investors, noise traders can expect higher incomes than rational investors. Bhushan, Brown, and Mello (1997) examine different noise-trading models and their consequential assets' prices. Dow and Gorton (1997) present a model for noise trading with delegated portfolio management, where managers cannot always identify profitable trading opportunities. Kelly (1997) discusses whether noise traders influence stock prices. Brown (1999) shows that unusual levels of sentiment among noise traders are correlated with excessive volatility of closed-end investment funds. Greene and Smart (1999) investigate whether noise traders provide further liquidity to the capital markets. Gemmill and Thomas (2002) show that noise-trader sentiment often leads to fluctuations in the discounted prices of closed-end funds, and Hughen and McDonald (2005) examine who the noise traders are and find no evidence for a systematic mispricing by these investors.

However, prior studies have given secondary attention to the types of noise affecting traders and in particular the colours of noise that could impact not only fixed-income investors but also shareholders. In this article we examine various colours of time-series economic noise and their long-lasting influence on default risk through theoretical computer simulations. Identifying the long-term implications of different colours of noise on corporate default risk can assist noise traders, momentum traders, and contrarians to achieve better performance in debt and equity related investments. These traders must assess and act upon the unique circumstances underlying the volatility of the market. Arnott (2005) quotes Benjamin Graham as saying that "in the short run the market is a voting machine, but in the long run it is a weighing machine."(Note 1) A single economic noise often has a negligible effect on the long-term default risk of a firm, yet a time-series of autocorrelated noises could substantially affect this matter.

We examine five different colours of noise: (1) a white noise, which represents a random macroeconomic shock, (2) a red noise, which denotes a positive autocorrelation in a sequence of shocks, (3) a pink noise, which depicts a combination of whitened and reddened noises, (4) a blue noise, which illustrates a negative autocorrelation in a string of economic shocks, and (5) a black noise, which classifies a longer term dependency and thus an extended but reduced shock over a wider time frame than the red noise, for instance. Noise traders often follow white noises, different levels of momentum traders are associated with the pink noise, the red noise, and the black noise spectra, and contrarians believe in the blue noise type of investment strategy.

We are motivated to investigate this topic to assist noise traders, momentum players, and contrarians to recognize the long-term impact of various macroeconomic shocks on default likelihoods. This link conveys further implications on yield spreads of outstanding debt issuances, as well as on expected equity returns. In essence, we explore how corporate creditworthiness fluctuates over time through different types of simulated macroeconomic shocks and whether a single random shock damages a firm's survivability more or less than a series of reduced noises. Furthermore, the Monte Carlo simulations hereafter allow us to conduct different intra-colour as well as cross-colour comparisons.

Our main findings confirm that in the presence of strong macroeconomic shocks, as opposed to a more stable economy, firms tend to default more quickly, but at the same time these firms also reveal higher default probabilities on the way to liquidation. Paradoxically, traders enjoy better predictive capabilities under these extreme conditions. In addition, we find that when a macroeconomic environment exhibits positive (negative) time-series autocorrelation, the expected variance of assets' value increases (decreases) with corporate creditworthiness. We further realize that while positive autocorrelation intensifies already existing trends and therefore forces liquidation more quickly even for firms with greater risk tolerance, counter-cyclical economic movements tend to reduce default risk for firms with higher sustainability. We also detect that when a series of economic noises exhibits a higher autocorrelation although average default probabilities decline, firms tend to default more quickly, thus making it more difficult for traders to predict corporate failure. Moreover, a longer memory series with reduced autocorrelation increases default likelihoods but simultaneously allows firms to survive over a longer period of time, as opposed to a shorter memory series having higher autocorrelation.

In the following sections we first present a brief theoretical background on the Merton (1974) model for assessing corporate default risk. We then describe five time-series colours of noise – the white, the red, the pink, the blue, and the black noises – along with their potential impact on default risk. Next, we construct five different computer simulations to measure the long-term influence of each colour of noise on corporate survival. Finally, we discuss the findings and their economic significance.

2. The Merton Model

Merton (1974) presents a model in which a firm's equity can be considered as a European call option on the firm's assets. The model can be simplified by assuming that the firm has one zero-coupon bond outstanding, which matures at time T. Because the model considers only one debt time to maturity, academicians and practitioners often employ the Macaulay duration to combine various debt components into a single maturity. The Merton model defines the following variables: V_0 as the market value of the firm's assets today, V_T as the stochastic market value of the firm's assets at time T, E_0 as the value of the firm's equity today, E_T as the value of the firm's equity at time T, D as the deterministic total amount of debt due to be repaid at time T, σ_V as the volatility of assets' return, σ_E as the volatility of the firm's equity, and r_f as the risk-free interest rate.

If $V_T < D$, the firm cannot service its debt and defaults at time T, and then the value of the equity becomes zero. However, if $V_T > D$, the firm pays back its outstanding debt at time T, and its equity value becomes $V_T - D$. Thus, the value of the firm's equity at time T can be written as

$$E_T = Max\{V_T - D, 0\} \tag{1}$$

The firm's equity is therefore a European call option on the market value of assets, with the total amount of debt due to be repaid at time T as the exercise price. The Black-Scholes (1973) model solves the value of the firm's equity today as follows:

$$E_0 = V_0\Phi(d_1) - De^{-r_f T}\Phi(d_2)$$
where
$$d_1 = \frac{\ln(V_0 / D) + (r_f + \sigma_V^2 / 2)T}{\sigma_V\sqrt{T}}; d_2 = \frac{\ln(V_0 / D) + (r_f - \sigma_V^2 / 2)T}{\sigma_V\sqrt{T}} = d_1 - \sigma_V\sqrt{T} \tag{2}$$

The Black-Scholes options model considers $\Phi(d_2)$ as the probability for an option to be exercised, where $\Phi(.)$ is the cumulative distribution function (CDF) of the Normal distribution. In the Merton (1974) model context, $\Phi(d_2)$ is the risk-neutral probability that the firm does not default, so $1 - \Phi(d_2) = \Phi(-d_2)$ is the likelihood that the firm defaults on the debt. An increase in σ_V causes a decrease in d_2. As $(-d_2)$ increases, the risk-neutral probability for the firm to default on the debt $\Phi(-d_2)$ increases as well. We later examine these relations with respect to the volatility of assets' return as derived by different time-series colours of noise.

In the standard Merton model V_t and σ_V are not directly observable but can be postulated. Jones, Mason, and Rosenfeld (1984) find the probability for a public company to default on the debt $\Phi(-d_2)$ by solving two equations with two unknowns. The first equation is the interpretation of the Black-Scholes formula for the value of a firm's equity as in equation (2), and the second necessary equation compares the diffusion coefficients from the equity value dynamic and the one from the Itô's lemma for equity movements:

$$\sigma_E E_t = \frac{\partial E_t}{\partial V_t}\sigma_V V_t = \Phi(d_1)\sigma_V V_t . \tag{3}$$

Hull, Nelken, and White (2004) present a different approach to estimate the volatility of assets' return σ_V by using the implied volatility from existing options on the firm's traded stocks. In practice, however, Moody's avoids solving these simultaneous equations, since the model linking equity and asset volatility holds only instantaneously. Crosbie and Bohn (2003), and Bharath and Shumway (2004) discuss an alternative and preferable method to calculate V_t and σ_V through sequential iterations.

Numerous studies including Ronn and Verma (1986), Kealhofer (2003), and Vassalou and Xing (2004) have adopted the Merton model to assess corporate default risk. We therefore utilize here the Merton model to measure default risk.

3. The Colours of Noise

A stochastic process such as the one describing the market value of corporate assets can be affected by different "colours" of noise, analogous to the colours of a light spectrum. For instance, white noise, like the white light, consists of an even mixture of variation over all frequencies with no memory of past events. This is an entirely stationary dynamic. One can simulate a white noise by adding a normally distributed random variable to the mean value of the parameter in question for each time step, hence to command a zero autocorrelation within the time-series dimension. In the traditional Merton model only changes in market value of assets are subject to stochastic influence; therefore, we examine the behavior of the following AR(1) time-series vibrations:

$$\Delta V_t = \alpha\Delta V_{t-1} + \beta\varepsilon_{t-1} \tag{4}$$

where

$\alpha = 0$ and $\varepsilon_t \sim N(0,1)$

so $\Delta V_t = \beta\varepsilon_{t-1}$

where β denotes a constant parameter scaling the variance of time series, ε is a random effect drawn from the standard Normal distribution, and α is a constant autocorrelation parameter, but in this case it takes no role in the process.

Similar to the red light, a red noise portrays a mixture with a higher proportion of low-frequency thus long-term ingredients. A red noise often describes a dependency on the most recent events. Because of its relations to the Brownian motion, the red noise is also called a brown noise. To simulate a red noise one can compute the parameter in question at every time interval as a combination of the lagged corresponding value and a random variable with zero mean. To assure that the most recent history continues to dominate this time-series of noises, one must select a positive though smaller-than-one constant autocorrelation parameter. In this context, a rising α denotes a stronger dependency on the most recent events, for example right after a clear bubble burst. This setting triggers the following momentum effect on fluctuations in the market value of assets:

$$\Delta V_t = \alpha \Delta V_{t-1} + \beta \varepsilon_{t-1} \tag{5}$$

where

$1 > \alpha > 0$ and $\varepsilon_t \sim N(0,1)$

Between these two extremes, the white and the red noises, lies a whole family of pink noises, which depict more or less reddened frequency spectra. The pink noise generally represents common influences of both historical events and stationary tendencies. It is therefore considered to be a quasi-stationary process, as its variance grows slowly, at a pace proportional to time. One can simulate this class of noises through a wavering autocorrelation parameter within the time-series regression, which can preferably allocate higher weight to the white or the red noises, as follows:

$$\Delta V_t = \alpha_{t-1} \Delta V_{t-1} + \beta \varepsilon_{t-1} \tag{6}$$

where

$1 \ge \alpha_t \ge 0$ and $\varepsilon_t \sim N(0,1)$

and we assign an arbitrary function

$$\alpha_t = \left| \sin\left(\frac{\pi}{1 + \mathrm{mod}(t/36)} \right) \right|$$

The autocorrelation parameter $\alpha_t \in [0,1]$ is no longer fixed, but it varies with time within its quasi-stationary domain. For that purpose we appoint the absolute value of the trigonometric function sine with respect to the ratio between π and one plus the remainder (using the modulo function) of a fraction of the time. Consequently, the autocorrelation cycles repeat themselves once every 36 months. This way, we generate repeated downward sloping convex waves for the autocorrelation function with a greater emphasis on the spectrum of colours from white to light red.

A blue noise often expresses a process with similar properties to that of the red noise, yet with a negative rather than a positive constant autocorrelation parameter. In reality, negatively autocorrelated noise has great importance, when describing for example the mean reversion phenomenon, the investment strategy of contrarians, or common cyclical economic movements. In the Merton model context we conduct the following experiment:

$$\begin{aligned} &\Delta V_t = \alpha \Delta V_{t-1} + \beta \varepsilon_{t-1} \\ &\text{where} \\ &-1 < \alpha < 0 \quad \text{and} \quad \varepsilon_t \sim N(0,1) \end{aligned} \tag{7}$$

A black noise portrays a more persistent dynamic with heavy dependency upon its distant past. Because of that, Bak and Chen (1991) explain that a black noise process could govern various economic catastrophes, and due to its black spectra, if such a major disaster occurs, it could stretch and cluster over a long period of time. We thus utilize the next time-series regression with reliance on three prior changes and thus four preceding assets' values:

$$\begin{aligned} &\Delta V_t = \alpha_1 \Delta V_{t-1} + \alpha_2 \Delta V_{t-2} + \alpha_3 \Delta V_{t-3} + \beta \varepsilon_{t-1} \\ &\text{where} \\ &1 > \alpha_i > 0 \quad \forall i \in \{1,2,3\} \quad \text{and} \quad \varepsilon_t \sim N(0,1) \end{aligned} \tag{8}$$

We would like to further examine how assets' volatility varies across the different noise processes. Chatfield (2004, Chapter 3) shows that the theoretical variance of a stationary time-series of infinite length is:

$$\sigma^2 = \beta^2 / (1 - \alpha^2) \tag{9}$$

In addition, Heino, Ripa, and Kaitala (2000) develop the expected variance of an autocorrelated sample of length T, assuming that T is large enough that initial provisions convey no impact as:

$$E[s^2] = \frac{\beta^2}{(1-\alpha^2)(T-1)} \left[T - \frac{(1+\alpha)^2}{1-\alpha^2} + \frac{2\alpha(1-\alpha^T)}{T(1-\alpha)^2} \right] \tag{10}$$

For a stochastic process with a white noise, the expected sample variance does not depend on the sample length, and in this case $E\left[s^2\right] = \sigma^2 = \beta^2$. If the dynamic exhibits positively (negatively) constant autocorrelated noise, as in the red (blue) spectra analysis, its variance is lower (higher) the shorter the sample. Unfortunately there are no analytical derivations for the variance of the pink or the black noises. In the latter simulations we contrast the theoretical and the expected variances of assets' value during default processes across and within the three relevant noise colours: the white, the red, and the blue.

4. Monte Carlo Simulations

For purpose of robustness and to examine intra-colour and cross-colour behavior, we construct five independent simulations, one for each colour of noise: the white, the red, the pink, the blue, and the black. Every simulator executes 27 different experiments, each one over 60,000 monthly simulated observations. All the simulations derive the risk-neutral default probabilities from the traditional Merton (1974) model, with the following origin values: D = *\$100*, V_0 = *{\$100, \$150,* or *\$200}* to proxy different initial corporate creditworthiness, T = *5* years, r_f = *5%*, and σ_V = *0.1* for the first 12 monthly iterations.[2] We then compute the annual volatility of assets' return directly from changes in V after the first year by multiplying a moving window standard deviation of historical assets' returns $\ln\left(V_t / V_{t-1}\right)$ by the square root of 12.[3] We draw a new random number from the standard Normal distribution in every time interval for ε, and appoint β = *{10%, 20%,* or *30%}* of V_0 in all the simulations as a meaningful variance scaling constant. We further set $\alpha = 0$ in the white noise test, $\alpha = 0.6$ in the red noise test, and $\alpha = -0.6$ in the blue noise test. We also denote $\alpha_1 = 0.3$, $\alpha_2 = 0.2$, and $\alpha_3 = 0.1$ in the black noise analysis. The reader should notice that we select these particular quantities to represent a realistic scenario with gradually decreasing autocorrelation coefficients, and at the same time we preserve comparable tests by inducing $\alpha_1 + \alpha_2 + \alpha_3 = 0.6$ as in the red noise experiment. Within the pink noise simulation we denote a representative function for α as described in equation (6), where we keep tracking the time t.

We let each of the five simulations run over 60,000 monthly iterations or 5,000 collective years of simulated data. In every iteration we compute $V_t = V_{t-1} + \Delta V$, where the changes ΔV can be positive, negative, or zero. We set three default thresholds based on the maximum default probability $Max\{\Phi(-d_2)\} \leq \{0.90, 0.95,$ or $0.99\}$ to represent different levels of corporate risk tolerance.[4] Given the other model parameters, these three default thresholds cover a fairly wide range of ratios between the stochastic market value of assets and the fixed face value of debt.

Furthermore, to prevent any single firm from becoming disproportionally large through a continuous enhancement of assets' value, and by that to consume too many iterations of the simulation, we limit the growth of all firms to three times the initial market value of assets, thus the growth limit = *{\$300, \$450,* or *\$600}*, which varies between three to six times the face value of the debt. These proportions denote very secure firms, with exceptionally low chances to default. To authenticate our findings, in the robustness checks we use other growth limits as well.

Whenever either a default occurs, hence if $\Phi(-d_2)$ exceeds the selected default threshold, or when assets' value touches its upper growth limit, the simulation continues with a new firm having the same initial arbitrary values as with its predecessor. Because of the misbalance between the distances assets' value is allowed to shift, either towards the upper limit (200% up) or to the lower boundary (near 100% down), we expect a relatively high percentage of defaults out of the simulated scenarios. We confirm this presumption when we cap the growth of assets' value at a lower level; thus we place similar constraints on assets' value to rise or decline not more than 100%, and we obtain a more balanced proportion of defaults. Due to the repetitive nature of the robustness tests we do not report them here, but we further utilize different quantities for the remaining time until maturity T, the risk free rate r_f, as well as for the face value of debt D. All of the results remain consistent with the ones drawn from our main settings.

5. Results and Conclusions

In this section we report the key findings of the intra-colour and the cross-colour analyses. We further discuss some important economic conclusions, but we first start with the intra-colour investigations. Within each colour of noise we hold everything else constant, except for the growth limit, which is set to be proportional to V_0, and alternate three variables: the variance scaling parameter β, the initial market value of assets V_0, and the default threshold $Max\{\Phi(-d_2)\}$. Each time, we record how three things vary: the average number of years to default (after mapping monthly observations to annual records), the mean default probability, and the two measures of volatility σ^2 and $E\left[s^2\right]$. We summarize the results of these simulations in **Tables 1 – 5**.

Throughout all of the tests we find consistent relations between the time-series variance scaling variable β and the credit simulated parameters. When β rises the average number of years to default declines, the mean default probability increases, and both volatility measures intensify. These findings suggest that in a more volatile environment firms tend to default more quickly. At the same time, as opposed to a more stable economy, with the presence of strong macroeconomic shocks, which might be either constructive or destructive, firms portray a higher default risk along the way to liquidation. Therefore we can conclude that although default risk rises with the intensity of economic noises, so does the predictive power of traders aiming to forecast defaults.

We also detect coherent results with respect to changes in the original market value of assets. In all of the experiments, when V_0 grows, the average number of years to default increases, and the mean default probability decreases as expected. However, the volatility measures exhibit interesting patterns. While the theoretical time-series variance of infinite length σ^2 remains fixed throughout the white, the red, and the blue noise simulations, the expected variance of length *T* remains constant in the white noise test, rises at the red noise simulation, but declines within the blue noise recreation.

The reason for that lies within the structure of the formula of $E\left[s^2\right]$ given in (10) and in particular the ratio of elements depending on *T*. This ratio is smaller than one at the red noise test but bigger than one in the blue noise simulation. Both ratios converge to one when *T* climbs, and therefore the expected variance of a time-series sample with an average length *T* increases with red and decreases with blue spectra noises. We therefore conclude that when a macroeconomic environment exhibits positive (negative) time-series autocorrelation, the expected variance of assets' value increases (decreases) with the creditworthiness of a firm, which is captured here by the distance between the floating market value of assets and the fixed face value of the debt.

When the default threshold $Max\{\Phi(-d_2)\}$ rises from 90% to 95% and then to 99%, we generally observe a rise in the average number of years to default as expected, yet this effect is less pronounced within the red and the black noise simulations. These two colours of noise tend to intensify already existing trends and therefore force liquidation more quickly even for firms with greater risk tolerance. Furthermore, we detect a tendency of increased mean default probability, but merely minor fluctuations at the blue noise test. Therefore, counter-cyclical economic movements tend to ease default risk for firms with higher sustainability. Although we find no changes in σ^2 when the default threshold varies, we identify a steady $E\left[s^2\right]$ within the white noise test, no clear trend with negligible vibrations at the red noise experiment, but a continuous decline at the blue noise simulation. This decay is associated with the levitation in *T*.

We further conduct cross-colour analyses. We match the variance scaling parameter β, the origin market value of assets V_0, and the default threshold $Max\{\Phi(-d_2)\}$ and compare the average number of years to default, the mean default probability, and the volatility measures across the different simulations. In essence, we examine the impact of changes in the autocorrelation variable α on these credit simulated parameters.

Once we sort the simulations according to their autocorrelation levels we find three consistent relations as follows. When α rises across the noise colours, the average number of years to default as well as the mean default probability simultaneously decrease. This striking evidence suggests that although the chances to default decline, firms tend to default more quickly when a series of economic noises exhibits a higher autocorrelation. This means that when the economy is more reddened, i.e. with a stronger positive autocorrelation within the sequential economic shocks, it would be more difficult for traders to predict corporate failure because of the growing impact of a potential extended catastrophe.

To evaluate whether a single random shock damages a firm's survivability more or less than a series of reduced noises, we compare the red and the black noise simulations. We find that both the average number of years to default and the mean default probability are higher within almost all of the black noise tests. We therefore conclude that a longer memory series with weaker autocorrelation increases default likelihoods but simultaneously allows firms to survive over a longer period of time, as opposed to a shorter memory series having higher autocorrelation.

In addition, when we evaluate the volatility measures at the blue, the white, and the red noise simulations, we observe that when the autocorrelation variable α rises, both the theoretical variance σ^2 and the expected variance $E\left[s^2\right]$ first decrease but then increase. This non-monotonic behavior of assets' volatility suggests a higher likelihood of extreme fluctuations from economic shocks when conditions are either blue or red, compared to markets inherently having random shocks as in the white noise simulation.

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Notes

Note 1. Benjamin Graham (1894 – 1976) is considered one of the first advocates of "value investing" and an early mentor of Warren Buffett.

Note 2. Although in practice the remaining time until maturity of the outstanding debt T is gradually decreasing, we wish to isolate the effects of economic noises on default risk, and therefore we fix this parameter along the simulations, while assuming that firms issue more debt over time and through that maintain a constant weighted average time to maturity on their denominated bond issuances.

Note 3. Alternatively we could have generated daily fluctuations, then scaled it to annual volatility, and further calibrated β to be a smaller variance scaling value. But since it often takes many years for firms to default, this would have obligated us to considerably increase the number of iterations and to face computation difficulties. In addition, we aim the five different colours of noise to represent significant macroeconomic shocks that can alter a firm's market value of assets from various industries and diverse geographic markets, yet these economic shocks rarely happen every day. It is therefore more intuitive and more applicable to consider aggregate monthly noises.

Note 4. Occasionally firms default prior to the moment they are forced to do so by creditors. For example, mall owner General Growth Properties filed for chapter eleven on April 16, 2009, while its assets' value was estimated as $29.5 billion and its outstanding debt was merely $27.3 billion.

Table 1. White Noise Monte Carlo Simulations

α	β	V_0	D	Growth Limit	Default Threshold	Number of Monthly Observations	Total Number of Firms	Total Number of Defaults	Average Number of Years to Default	Average Default Probability	σ^2	$E[s^2]$
0	10	$100	$100	$300	90%	60,000	448	336	14.881	0.269	100.00	100.00
0	15	$100	$100	$300	90%	60,000	1,050	819	6.105	0.345	225.00	225.00
0	20	$100	$100	$300	90%	60,000	1,843	1,444	3.463	0.394	400.00	400.00
0	15	$150	$100	$450	90%	60,000	381	287	17.422	0.206	225.00	225.00
0	22.5	$150	$100	$450	90%	60,000	775	571	8.757	0.268	506.25	506.25
0	30	$150	$100	$450	90%	60,000	1,430	1,040	4.808	0.323	900.00	900.00
0	20	$200	$100	$600	90%	60,000	361	262	19.084	0.164	400.00	400.00
0	30	$200	$100	$600	90%	60,000	733	513	9.747	0.215	900.00	900.00
0	40	$200	$100	$600	90%	60,000	1,366	980	5.102	0.293	1,600	1,600
0	10	$100	$100	$300	95%	60,000	377	284	17.606	0.327	100.00	100.00
0	15	$100	$100	$300	95%	60,000	859	641	7.800	0.386	225.00	225.00
0	20	$100	$100	$300	95%	60,000	1,511	1,140	4.386	0.442	400.00	400.00
0	15	$150	$100	$450	95%	60,000	287	181	27.624	0.209	225.00	225.00
0	22.5	$150	$100	$450	95%	60,000	699	475	10.526	0.296	506.25	506.25
0	30	$150	$100	$450	95%	60,000	1,304	929	5.282	0.356	900.00	900.00
0	20	$200	$100	$600	95%	60,000	306	195	25.641	0.170	400.00	400.00
0	30	$200	$100	$600	95%	60,000	715	487	10.267	0.250	900.00	900.00
0	40	$200	$100	$600	95%	60,000	1,228	869	5.754	0.314	1,600	1,600
0	10	$100	$100	$300	99%	60,000	315	221	22.624	0.342	100.00	100.00
0	15	$100	$100	$300	99%	60,000	664	466	10.730	0.420	225.00	225.00
0	20	$100	$100	$300	99%	60,000	1,266	898	5.568	0.483	400.00	400.00
0	15	$150	$100	$450	99%	60,000	285	194	25.773	0.245	225.00	225.00
0	22.5	$150	$100	$450	99%	60,000	679	489	10.225	0.325	506.25	506.25
0	30	$150	$100	$450	99%	60,000	1,113	767	6.519	0.395	900.00	900.00
0	20	$200	$100	$600	99%	60,000	294	193	25.907	0.192	400.00	400.00
0	30	$200	$100	$600	99%	60,000	618	407	12.285	0.272	900.00	900.00
0	40	$200	$100	$600	99%	60,000	1,136	767	6.519	0.355	1,600	1,600

Table 2. Red Noise Monte Carlo Simulations

α	β	V_0	D	Growth Limit	Default Threshold	Number of Monthly Observations	Total Number of Firms	Total Number of Defaults	Average Number of Years to Default	Average Default Probability	σ^2	$E[s^2]$
0.6	10	$100	$100	$300	90%	60,000	2,680	1,560	3.205	0.167	156.25	144.55
0.6	15	$100	$100	$300	90%	60,000	4,280	2,551	1.960	0.218	351.56	309.71
0.6	20	$100	$100	$300	90%	60,000	5,973	3,658	1.367	0.240	625.00	521.82
0.6	15	$150	$100	$450	90%	60,000	2,601	1,491	3.353	0.115	351.56	326.26
0.6	22.5	$150	$100	$450	90%	60,000	4,377	2,563	1.951	0.164	791.02	696.43
0.6	30	$150	$100	$450	90%	60,000	5,698	3,424	1.460	0.190	1,406	1,187
0.6	20	$200	$100	$600	90%	60,000	2,610	1,499	3.336	0.097	625.00	579.96
0.6	30	$200	$100	$600	90%	60,000	4,249	2,479	2.017	0.145	1,406	1,243
0.6	40	$200	$100	$600	90%	60,000	5,794	3,451	1.449	0.168	2,500	2,108
0.6	10	$100	$100	$300	95%	60,000	2,487	1,460	3.425	0.181	156.25	145.27
0.6	15	$100	$100	$300	95%	60,000	4,216	2,494	2.005	0.241	351.56	310.58
0.6	20	$100	$100	$300	95%	60,000	5,758	3,475	1.439	0.254	625.00	526.42
0.6	15	$150	$100	$450	95%	60,000	2,647	1,508	3.316	0.130	351.56	326.08
0.6	22.5	$150	$100	$450	95%	60,000	4,277	2,498	2.002	0.183	791.02	698.66
0.6	30	$150	$100	$450	95%	60,000	5,834	3,437	1.455	0.205	1,406	1,187
0.6	20	$200	$100	$600	95%	60,000	2,608	1,480	3.378	0.107	625.00	580.51
0.6	30	$200	$100	$600	95%	60,000	4,194	2,426	2.061	0.156	1,406	1,246
0.6	40	$200	$100	$600	95%	60,000	5,778	3,404	1.469	0.182	2,500	2,113
0.6	10	$100	$100	$300	99%	60,000	2,596	1,480	3.378	0.199	156.25	145.13
0.6	15	$100	$100	$300	99%	60,000	4,223	2,434	2.054	0.261	351.56	311.49
0.6	20	$100	$100	$300	99%	60,000	5,776	3,426	1.459	0.279	625.00	527.66
0.6	15	$150	$100	$450	99%	60,000	2,577	1,471	3.399	0.147	351.56	326.68
0.6	22.5	$150	$100	$450	99%	60,000	4,235	2,442	2.048	0.192	791.02	700.58
0.6	30	$150	$100	$450	99%	60,000	5,576	3,298	1.516	0.229	1,406	1,195
0.6	20	$200	$100	$600	99%	60,000	2,560	1,452	3.444	0.120	625.00	581.31
0.6	30	$200	$100	$600	99%	60,000	4,306	2,477	2.019	0.171	1,406	1,243
0.6	40	$200	$100	$600	99%	60,000	5,739	3,355	1.490	0.195	2,500	2,118

Table 3. Pink Noise Monte Carlo Simulations

α_t	β	V_0	D	Growth Limit	Default Threshold	Number of Monthly Observations	Total Number of Firms	Total Number of Defaults	Average Number of Years to Default	Average Default Probability
$\lvert\sin(.)\rvert$	10	$100	$100	$300	90%	60,000	989	688	7.267	0.124
$\lvert\sin(.)\rvert$	15	$100	$100	$300	90%	60,000	1,890	1,360	3.676	0.290
$\lvert\sin(.)\rvert$	20	$100	$100	$300	90%	60,000	3,100	2,235	2.237	0.328
$\lvert\sin(.)\rvert$	15	$150	$100	$450	90%	60,000	816	544	9.191	0.161
$\lvert\sin(.)\rvert$	22.5	$150	$100	$450	90%	60,000	1,704	1,183	4.227	0.234
$\lvert\sin(.)\rvert$	30	$150	$100	$450	90%	60,000	2,620	1,778	2.812	0.266
$\lvert\sin(.)\rvert$	20	$200	$100	$600	90%	60,000	831	542	9.225	0.140
$\lvert\sin(.)\rvert$	30	$200	$100	$600	90%	60,000	1,602	1,041	4.803	0.199
$\lvert\sin(.)\rvert$	40	$200	$100	$600	90%	60,000	2,509	1,679	2.978	0.246
$\lvert\sin(.)\rvert$	10	$100	$100	$300	95%	60,000	833	560	8.929	0.265
$\lvert\sin(.)\rvert$	15	$100	$100	$300	95%	60,000	1,665	1,122	4.456	0.329
$\lvert\sin(.)\rvert$	20	$100	$100	$300	95%	60,000	2,535	1,736	2.880	0.376
$\lvert\sin(.)\rvert$	15	$150	$100	$450	95%	60,000	725	480	10.417	0.199
$\lvert\sin(.)\rvert$	22.5	$150	$100	$450	95%	60,000	1,501	961	5.203	0.250
$\lvert\sin(.)\rvert$	30	$150	$100	$450	95%	60,000	2,451	1,618	3.090	0.307
$\lvert\sin(.)\rvert$	20	$200	$100	$600	95%	60,000	720	482	10.373	0.155
$\lvert\sin(.)\rvert$	30	$200	$100	$600	95%	60,000	1,495	974	5.133	0.227
$\lvert\sin(.)\rvert$	40	$200	$100	$600	95%	60,000	2,326	1,515	3.300	0.271
$\lvert\sin(.)\rvert$	10	$100	$100	$300	99%	60,000	721	458	10.917	0.292
$\lvert\sin(.)\rvert$	15	$100	$100	$300	99%	60,000	1,406	909	5.501	0.382
$\lvert\sin(.)\rvert$	20	$100	$100	$300	99%	60,000	2,368	1,552	3.222	0.426
$\lvert\sin(.)\rvert$	15	$150	$100	$450	99%	60,000	661	414	12.077	0.220
$\lvert\sin(.)\rvert$	22.5	$150	$100	$450	99%	60,000	1,419	913	5.476	0.289
$\lvert\sin(.)\rvert$	30	$150	$100	$450	99%	60,000	2,206	1,394	3.587	0.346
$\lvert\sin(.)\rvert$	20	$200	$100	$600	99%	60,000	710	450	11.111	0.182
$\lvert\sin(.)\rvert$	30	$200	$100	$600	99%	60,000	1,426	915	5.464	0.255
$\lvert\sin(.)\rvert$	40	$200	$100	$600	99%	60,000	2,185	1,397	3.579	0.305

Table 4. Blue Noise Monte Carlo Simulations

α	β	V_0	D	Growth Limit	Default Threshold	Number of Monthly Observations	Total Number of Firms	Total Number of Defaults	Average Number of Years to Default	Average Default Probability	σ^2	$E[s^2]$
-0.6	10	$100	$100	$300	90%	60,000	380	328	15.244	0.325	156.25	156.89
-0.6	15	$100	$100	$300	90%	60,000	997	882	5.669	0.410	351.56	355.46
-0.6	20	$100	$100	$300	90%	60,000	2,004	1,797	2.782	0.456	625.00	639.20
-0.6	15	$150	$100	$450	90%	60,000	225	184	27.174	0.248	351.56	352.37
-0.6	22.5	$150	$100	$450	90%	60,000	632	525	9.524	0.323	791.02	796.22
-0.6	30	$150	$100	$450	90%	60,000	1,364	1,172	4.266	0.371	1,406	1,427
-0.6	20	$200	$100	$600	90%	60,000	206	155	32.258	0.181	625.00	626.21
-0.6	30	$200	$100	$600	90%	60,000	531	420	11.905	0.259	1,406	1,414
-0.6	40	$200	$100	$600	90%	60,000	1,173	978	5.112	0.329	2,500	2,531
-0.6	10	$100	$100	$300	95%	60,000	266	225	22.222	0.360	156.25	156.69
-0.6	15	$100	$100	$300	95%	60,000	745	641	7.800	0.430	351.56	354.39
-0.6	20	$100	$100	$300	95%	60,000	1,421	1,201	4.163	0.505	625.00	634.46
-0.6	15	$150	$100	$450	95%	60,000	169	124	40.323	0.249	351.56	352.11
-0.6	22.5	$150	$100	$450	95%	60,000	550	427	11.710	0.338	791.02	795.25
-0.6	30	$150	$100	$450	95%	60,000	1,058	878	5.695	0.415	1,406	1,422
-0.6	20	$200	$100	$600	95%	60,000	209	153	32.680	0.224	625.00	626.20
-0.6	30	$200	$100	$600	95%	60,000	466	356	14.045	0.280	1,406	1,413
-0.6	40	$200	$100	$600	95%	60,000	1,090	923	5.417	0.378	2,500	2,529
-0.6	10	$100	$100	$300	99%	60,000	175	124	40.323	0.322	156.25	156.49
-0.6	15	$100	$100	$300	99%	60,000	521	387	12.920	0.431	351.56	353.27
-0.6	20	$100	$100	$300	99%	60,000	1,024	829	6.031	0.547	625.00	631.51
-0.6	15	$150	$100	$450	99%	60,000	168	126	39.683	0.254	351.56	352.12
-0.6	22.5	$150	$100	$450	99%	60,000	421	321	15.576	0.388	791.02	794.20
-0.6	30	$150	$100	$450	99%	60,000	873	683	7.321	0.464	1,406	1,418
-0.6	20	$200	$100	$600	99%	60,000	173	133	37.594	0.250	625.00	626.04
-0.6	30	$200	$100	$600	99%	60,000	393	299	16.722	0.322	1,406	1,412
-0.6	40	$200	$100	$600	99%	60,000	799	601	8.319	0.419	2,500	2,519

Table 5. Black Noise Monte Carlo Simulations

$\alpha_{1,2,3}$	β	V_0	D	Growth Limit	Default Threshold	Number of Monthly Observations	Total Number of Firms	Total Number of Defaults	Average Number of Years to Default	Average Default Probability
.3,.2,.1	10	$100	$100	$300	90%	60,000	1,785	1,205	4.149	0.192
.3,.2,.1	15	$100	$100	$300	90%	60,000	3,280	2,182	2.291	0.238
.3,.2,.1	20	$100	$100	$300	90%	60,000	4,648	3,112	1.607	0.269
.3,.2,.1	15	$150	$100	$450	90%	60,000	1,923	1,179	4.241	0.125
.3,.2,.1	22.5	$150	$100	$450	90%	60,000	3,198	2,003	2.496	0.172
.3,.2,.1	30	$150	$100	$450	90%	60,000	4,307	2,735	1.828	0.202
.3,.2,.1	20	$200	$100	$600	90%	60,000	1,974	1,193	4.191	0.109
.3,.2,.1	30	$200	$100	$600	90%	60,000	3,273	1,995	2.506	0.142
.3,.2,.1	40	$200	$100	$600	90%	60,000	4,476	2,759	1.812	0.177
.3,.2,.1	10	$100	$100	$300	95%	60,000	1,966	1,185	4.219	0.192
.3,.2,.1	15	$100	$100	$300	95%	60,000	3,298	2,031	2.462	0.251
.3,.2,.1	20	$100	$100	$300	95%	60,000	4,413	2,831	1.766	0.292
.3,.2,.1	15	$150	$100	$450	95%	60,000	2,098	1,240	4.032	0.128
.3,.2,.1	22.5	$150	$100	$450	95%	60,000	3,248	1,945	2.571	0.182
.3,.2,.1	30	$150	$100	$450	95%	60,000	4,429	2,694	1.856	0.226
.3,.2,.1	20	$200	$100	$600	95%	60,000	1,992	1,177	4.248	0.114
.3,.2,.1	30	$200	$100	$600	95%	60,000	3,285	1,952	2.561	0.157
.3,.2,.1	40	$200	$100	$600	95%	60,000	4,401	2,644	1.891	0.196
.3,.2,.1	10	$100	$100	$300	99%	60,000	2,141	1,254	3.987	0.208
.3,.2,.1	15	$100	$100	$300	99%	60,000	3,182	1,929	2.592	0.278
.3,.2,.1	20	$100	$100	$300	99%	60,000	4,360	2,672	1,871	0.317
.3,.2,.1	15	$150	$100	$450	99%	60,000	2,030	1,171	4.270	0.145
.3,.2,.1	22.5	$150	$100	$450	99%	60,000	3,250	1,928	2.593	0.205
.3,.2,.1	30	$150	$100	$450	99%	60,000	4,262	2,563	1.951	0.250
.3,.2,.1	20	$200	$100	$600	99%	60,000	1,991	1,168	4.281	0.120
.3,.2,.1	30	$200	$100	$600	99%	60,000	3,223	1,883	2.655	0.172
.3,.2,.1	40	$200	$100	$600	99%	60,000	4,143	2,474	2.021	0.216