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An Emerging Credit Risk Framework

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Abstract

The main result of this paper is the establishment of an analytic formula for the estimation capital requirements of an individual loan.

The derived formula, may be considered as a direct analogue to the Basel risk-weight functions for credit risk, first presented in Basel II framework, with the additional advantage of utilizing a lifetime horizon, thus being suitable for IFRS9/GAAP purposes. Essentially, it bridges the gap between Basel and IFRS9 frameworks as:

- The 1-year horizon incorporated in the Basel PD is extended up to maturity, following the IFRS9 rationale;
- The notion of unexpected losses, already supplied by Basel Framework, is added to the IFRS9 logic in an analytic fashion.

Going one step further, the generalization of the risk variables used in the formula, as Kumaraswamy identically distributed variables, allows for the benchmarking of the total loss of a credit portfolio, with the single knowledge of its current non-performing loans percentage.

This conclusion is successfully verified against EBA stress test results for the period 2018-2020.

Keywords: capital requirements, credit risk, credit losses, Kumaraswamy distribution, IFRS9

1. Connection to Current Work

The existing theoretical and practitioner approach to the relatively novel requirement of lifetime expected loss modelling, relies heavily on numerical approximations and computationally intensive procedures. We refer to (Basel Committee on Banking Supervision, 2018; KPMG, 2017; and Kristensen, Kumar, & May, 2019) as are indicative examples of a "population" of numerous studies mainly published by consulting companies, which follow the same computational logic. Essentially, in all known approaches, the discounted cash flow model is applied, with minor assumption differences and variations.

The approach followed in this paper is fundamentally different from current practice and is notionally closer to the Basel IRB methodology for credit risk, where risk parameters must be supplied into a single equation (1). We would like to think of the current effort as an extension of the Basel idea to lifetime horizon.

Additionally, it is vital both for regulators (Task Force of the Market Operations Committee of the European System of Central Banks, 2007) and the financial institutions' management teams, to have quick and accurate estimations of the required capital. Another important concern for regulators, is the harmonization of practices among Banks and the uncertainty regarding model misuse (Wikipedia), and other intentional or unintentional omissions. This harmonization is achieved with general equations (2) and (3).

Concluding, up to the time of writing of the present study, no similar results have come to our attention.

2. Main Results Overview

2.1 Calculation of Single Loan Required Capital

Throughout this paper, it is proven that the required provisions and capital, that is expected and unexpected losses, per 1€ of a single loan obligation, may be determined with the use of the generic formula:

$$Loan\ Credit\ Losses\ \% = LGD \cdot \underbrace{\left[\left(1 - e^{-a \cdot \frac{T}{12}}\right) \cdot (1+\beta) + \frac{a \cdot \beta}{12 \cdot \gamma - a} \cdot \left(1 - e^{\left(\gamma - \frac{a}{12}\right) \cdot T}\right)\right]}_{PD\ Lifetime\ expression} \tag{1}$$

The necessary input is

$LGD =$ Loss given default

$a =$ Obligor risk curve parameter

$T =$ Loan maturity

This result will be gradually developed in section 0

2.2 Benchmarking Bank Stressed Capital Requirements

In Addition, if $npl =$ The percentage of non-performing loans, of a financial institution, then equation

$$Bank\ Credit\ Losses\ up\ limit\ \% = 1 - (1 - npl^{1.35130})^{2.46853} \tag{2}$$

calculated on monthly basis, offers an adequate upper limit benchmark for the stressed losses per 1€ of the Bank's loan portfolio. The accompanying lower limit for the loss estimation is

$$Bank\ Credit\ Losses\ down\ limit\ \% = 1 - (1 - npl^{1.44453})^{1.14213} \tag{3}$$

The portfolio level estimation will be built and tested in section 0.

3. Loss Estimation for a Single Loan

3.1 Definitions & Notation

In the current segment we refer to some basic terminology. What follows is the gradual presentation of the risk parameters that will be included in our credit formula, as presented in sections 0 and 0.

$t_0 =$ Time index indicating current end of month

$t =$ Time distance in months from point t_0.

We assume that an obligor has exposures in two main loan categories (segments), business loans and consumer/mortgage/personal loans, indicated by $s \in \{Business, Retail\}$

$Accruing\ State =$ The situation in which the obligor makes loans payments according to schedule, or not 100% according to schedule, where any minor delays do not indicate that he will not be able to fulfill the terms and conditions of his contract(s).

$Default\ State =$ The situation in which reduced or delayed payments are observed at least in one of the obligor's loans, indicating that he will not be able to fulfill the terms and conditions of his contract(s).

$NonAccruing\ State =$ The situation that usually follows $Default\ State$ in which the obligor does not make any payments, at least in one of his loans, and the financial institution has initiated a recovery process, accompanied by legal procedures.

$ECL_t =$ The expected credit loss (ECL) at time t, the loan amount that under the existing (or expected) economic conditions we expect not to be returned to the financial institution.

$EAD_t =$ Exposure at default, is the on-balance loan amount plus any approved limits that the obligor may use without further notification to the financial institution, if default happens at time t.

3.2 Theoretical Loss Assessment

The total expected loss of a client at current time t_0, is comprised of the loss in each major loan category.

$$ECL_{t_0} = \sum_s ECL_{t_0,s} \quad s \in \{Business, Consumer\} \tag{4}$$

For simplicity reasons during the presentation of the analysis, we will initially focus on the process ignoring separate loan categories (s subscript). So, for a loan in accruing state at present time t_0, the expected losses throughout its remaining life until maturity, calculated at present time $\left(ECL_{t_0}\right)$, should be:

$$ECL_{t_0} = \sum_{t=1}^{N} PD(t, t+\Delta t] \cdot LGD(t, t+\Delta t]|_{\Delta T_{recovery}} \cdot EAD_t \tag{5}$$

$PD(t, t+\Delta t] =$ the probability of the obligor to default during the month interval following month t

$LGD(t, t+\Delta t]|_{\Delta T_{recovery}} =$ The final loss that the Bank will incur, if the obligor defaults during

$(t, t+\Delta t]$, allowing for time $\Delta T_{recovery}$ for legal actions and collateral liquidation.

$EAD_t =$ as defined in previous section 0

3.3 Exposure at Default Modelling (EAD_t)

We start again with the necessary terminology:

$L_{t_0} =$ Loan amount balance at present time t_0;

$r_{loan} =$ The annual loan interest rate;

$T_{mat} =$ The maturity of the loan expressed in months;

$r_{market} =$ The expected annual market rate for equivalent risk loans.

Month adjustments of annual rates:

$$r_l = \frac{r_{loan}}{12} \quad r_m = \frac{r_{market}}{12}$$

$B =$ Equivalent periodic (monthly) installment

Assuming loan initiation at t_0, the monthly installment is calculated as

$$B = L_{t_0} \cdot \frac{r_l}{1 - \frac{1}{(1+r_l)^{T_{mat}}}} \tag{6}$$

The amount at risk, at each point in time t is the remaining number of installments that is approximately $(T_{mat} - t) \cdot B$, which includes both capital and interest.

The present value of the series of the remaining installments at each point in time t $PV(B)_t$, is given by a geometric progression:

$$PV(B)_t = B \cdot \frac{1 - \left(\frac{1}{1+r_m}\right)^{T_{mat}-t}}{r_m} \tag{7}$$

The proper amount at risk A_t is given by the combination of $(6), (7)$

$$A_t = PV(B)_t = L_{t_0} \cdot \underbrace{\left(\frac{(1+r_l)^{T_{mat}} \cdot r_l \cdot [(1+r_m)^{T_{mat}-t} - 1]}{(1+r_m)^{T_{mat}-t} \cdot r_m \cdot [(1+r_l)^{T_{mat}} - 1]}\right)}_{d(t)} \tag{8}$$

Where

$d(t) =$ A decay function which indicates the percentage of the original loan L_{t_0}, at each point in time t, that is at risk, given the agreed interest rate r_l and the prevailing market rate r_m.

The decay function is approached next.

3.3.1 Time Decay Function Approximation $d(t)$

The decay function $d(t)$ could be adequately approximated by a 3-parameter continuous function

$$\Delta(t) \approx \delta - \beta \cdot e^{\gamma \cdot t} \quad \Delta(t) \to d(t) \tag{9}$$

$R =$ The remainder percentage at the last installment which is set at

$$R = 0.001 \tag{10}$$

$p =$ The curvature parameter, which represents in our definition, the percent of initial loan considered as amount at risk at the loan half life

For t_0 current time, a unique solution arises with the parameters specified as

$$\begin{aligned} \gamma &= \frac{2}{(T_{mat} - t_0)} \cdot ln\left(\frac{1-R}{1-p} - 1\right) \\ \beta &= \frac{R-1}{1 - e^{\gamma \cdot (T_{mat} - t_0)}} \\ \delta &= \beta + 1 \end{aligned} \tag{11}$$

Where time differences of $(T_{mat} - t_0)$ and $(t - t_0)$ are expressed in months.

With the valid business assumption that the loan rate will always converge close enough to prevailing market rates, the following detailed approximation may be applied

$$\begin{aligned} r &= \frac{r_l}{12} \approx \frac{r_m}{12} \\ dt &= \frac{T_{mat} - t_0}{12} \ (years) \\ y &= c_0 + c_1 \cdot r + c_2 \cdot t + c_3 \cdot ln(dt) + c_4 \cdot dt^2 \end{aligned} \tag{12}$$

Where

$dt =$ The remaining maturity in years

The coefficients of equation (12) are specified numerically as

$$\begin{aligned} c_0 &= -7.91612 \\ c_1 &= -225.53209 \\ c_2 &= 13.89258 \\ c_3 &= -25.07179 \\ c_4 &= -0.70554 \end{aligned} \tag{13}$$

Finally, the p parameter, appearing in (11) is approximated as

$$p = 0.999 \cdot \left(\frac{1+e^{-y}}{2+e^{-y}}\right) \in [0.4995\ , 0.999] \tag{14}$$

Thus, with the use of relations $(11), (12)$ and (13) we are able to approximate the percentage of a current (t_0) exposure at any point of the remaining exposure life.

3.4 Probability of Default

As we saw earlier, in 0, $PD(t, t+\Delta t]$ represents the probability that the obligor will transfer from Accruing State into a Default State during the semi-open interval $(t, t+\Delta t]$. We may generalize for Δt not to stand for 1 month, but a time fraction expressed in years.

For an obligor with a loan granted at time t_0 the cumulative probability function is considered exponential, given by

$$PD(t_0, t] = \left(1 - e^{-a\cdot(t-t_0)}\right) \quad a > 0 \tag{15}$$

If we consider that $t_0 = 0$ months the expression simplifies to

$$PD(0, t] = PD(t) = (1 - e^{-a\cdot t}) \tag{16}$$

The exponential function described in (eq_10) is suitable for the depiction of cumulative PD as it exhibits the following properties

$$PD(0) = 0 \tag{17}$$

$$\lim_{t\to\infty} PD(t) = 1 \tag{18}$$

$$PD'(t) = \frac{a}{e^{a\cdot t}} > 0 \tag{19}$$

$$PD''(t) = -\frac{a^2}{e^{a\cdot t}} < 0 \tag{20}$$

(17) Expresses the initial condition of non-default.

(18) Is the boundary condition

(19) Shows that the cumulative probability increases with time

(20) Declares that the cumulative probability increases in a decreasing rate, as we approach loan expiration date.

The one period discrete marginal probability applied at specific month points in time are given by

$$PD(t, t+\Delta t]|_{\Delta t=1} = PD(0, t+1] - PD(0, t] = \frac{e^{a}-1}{e^{a\cdot(t+1)}} \tag{21}$$

$a =$ The exponential function risk parameter that defines the obligor PD curve, estimated at each point in time t

The continuous marginal probability is the cumulative function derivative (19)

$$PD(t, t+\Delta t]|_{\Delta t\to 0} = \frac{a}{e^{a\cdot t}} \tag{22}$$

3.4.1 Modelling the Alpha Risk Parameter with the Use of Existing PDs

In this paper we will not argue for the creation of another PD model. We logically adopt the point of view, that there is at least one PD model in a financial institution for every obligor / product type. The existing model scores will be recalibrated as close as possible to the cumulative probability of the form depicted in (16)

Again, providing some necessary definitions:

$df =$ Default frequency

$Model =$ The number of available PD models providing scores

$Score =$ The output of each model

$Segment\ (s) =$ defined in 0

$Type = \{delta, cycle\}$, is the definition of whether all available economic cycle data $(cycle)$ or the most recent annual data $(delta)$ have been used for the calibration of the obligor cumulative PD curve.

$Year =$ The periods (in years) during which default frequencies are observed

$\overline{df}(Segment, Year) =$ Average default frequency of the segment for each Year period, for the non-scored obligors

We calculate the parametric default frequencies

$$df(Segment, Model, Type, Year) = f(Score) \tag{23}$$

Average default frequency is calculated:

- per Segment, Model, Type and Year
- Equal sample fragments are used in each case, using either increasing or decreasing model score in order to have increasing default frequencies df
- A bounded curve in $[0,1]$ is constructed with the use of average df points, which is either monotonous or has one change in monotonicity. In case no more than 1 intervals can be created, all obligors are gathered in one interval
- AR (accuracy ratio) is calculated for each model & year according to the previous interval categorization

The df possible curve general shapes are displayed below:

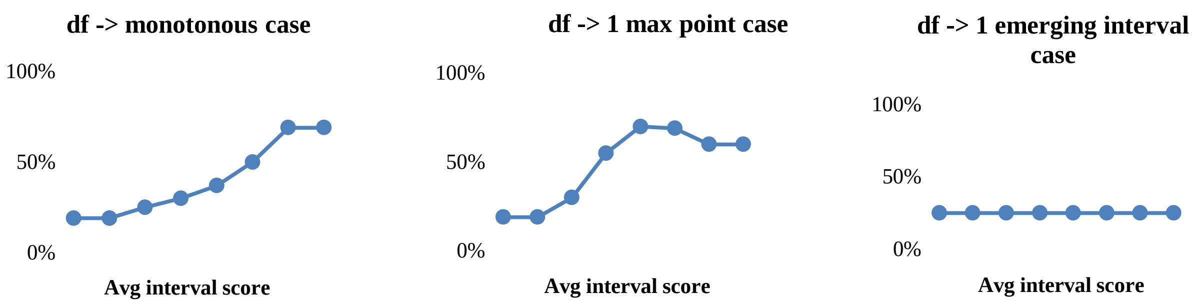


Figure 1. Model default frequency curve forms, as function of model score

Each obligor is evaluated by one or more models in each segment, and receives multiple scores

- In each Segment, every model receives a weight, according to its Accuracy Ratio (AR)
- Each obligor, through his assigned scores, receives the AR-weighted df for every combination of $Segment - Year$
- If the obligor is not a part of the specific segment and has no calculated scores, he receives $\overline{df}(Segment, Year)$

Using least square minimization criterion in segment – obligor level

$$min \sum \left(1 - e^{-a \cdot Year} - df_{[0,Year]}\right)^2 \tag{24}$$

- Two alpha type (ty) parameters are evaluated for each obligor for every segment

So, every obligor receives 4 alpha parameters, 2 in each portfolio

$$a_{s,ty} \quad s \in \{Business, Retail\} \quad ty = \{delta, cycle\} \tag{25}$$

The following conditions are examined:

$a_{s,delta} > a_{s,cycle} \Rightarrow$ The long-term historical default risk is less than the most recent estimated one. Economy is possibly entering a recession phase and so the most conservative estimation is required, $a_{s,delta}$.

$a_{s,delta} < a_{s,cycle} \Rightarrow$ The long-term historical probability of default is enhanced compared to the most recent estimated PD. Economy is possibly exiting a recession phase. The most conservative estimation in this case, $a_{s,cycle}$ will result in reduced and more normalized credit expansion. In that case it would seem more appropriate to use $a_{s,cycle}$ parameter in the process of unexpected credit losses (extra required capital) and not for provision purposes.

Concluding the previous observations, we end up with:

$$a_s = max\{a_{s,delta}, a_{s,cycle}\} \tag{26}$$

3.4.2 Stress the Alpha Risk Parameter with the Macro Environment

The effect of the macroeconomic environment is captured through the historical evolution and the current level of NPL percentage.

$NPL_{s,t} =$ The amount of non-performing loan amounts at point in time (month) t for segment $s \in \{Business, Retail\}$

$L_{s,t} =$ The total amount loans at point in time (month) t for segment s

$npl_{t,s} =$ The percentage of non-performing loan amounts, defined as:

$$npl_{s,t} = \frac{NPL_{s,t}}{L_{s,t}} \tag{27}$$

The time series of $npl_{s,t}$ contains the measurable effect of all macroeconomic factors and their respective lags, on the Bank's segment portfolios. The adversity of macroeconomic conditions' cumulative effect is correctly portrayed through the increase of $npl_{s,t}$.

If:

$E(npl_{t+1,s}) =$ the expected npl value to prevail next month $t+1$

$npl_{t,s} =$ the observed npl value at month t

The maximum expected gross npl change, is:

$\hat{y}_s =$ The maximum expected change of $npl_{t,s}$ through the next month period

$$\hat{y}_s = \frac{E(npl_{t+1,s})}{npl_{t,s}} \tag{28}$$

The maximum change may be analyzed as follows:

$m =$ The set of all macro and market factors, with their respective lags, that have an influence on current NPL percent (npl). We assume $m \in (-\infty, +\infty)$

$npl \in (0,1)$ may be expressed as a sigmoid function of m

$$npl_{s,t} = \frac{1}{1+e^{-(\psi_0+\psi\cdot m)}} \tag{29}$$

The marginal change of $npl_{s,t}$, in real terms the change over a month, is given by

$$\frac{dnpl_{s,t}}{dm} \approx npl_{s,t+1} - npl_{s,t} = \psi \cdot npl_{s,t} \cdot (1 - npl_{s,t}) \tag{30}$$

From (30) it turns out that a time series of

$$\psi_i = \frac{npl_{s,i+1} - npl_{s,i}}{npl_{s,i}\cdot(1-npl_{s,i})} \quad i \in (-\infty, t) \tag{31}$$

may be estimated. We assume that in the long term $E(\psi_i) = 0$ and maintain a conservative value of

$$\psi = 3 \cdot \sigma_{\psi_i} \tag{32}$$

for use in equation (30). The worst expected value for $npl_{t+1,s}$ is given by a rearrangement of (30)

$$E(npl_{s,t+1}) = (1+\psi) \cdot npl_{s,t} - \psi \cdot (npl_{s,t})^2 \tag{33}$$

The worst expected default rate:

$\hat{d}_{s,max} =$ The default rate, including considering the maximum expected deterioration of npl, with the assumption that $L_{s,t}$ remains stable during the period $[t, t+1m]$

$$\hat{d}_{s,max} = \frac{E(NPL_{s,t+1}) - NPL_{s,t}}{L_{s,t} - NPL_{s,t}} = \frac{E(npl_{s,t+1}) - npl_{s,t}}{1 - npl_{s,t}} = \psi_s \cdot npl_{s,t} \tag{34}$$

With the help of $\hat{d}_{s,max}$ we end with an incremental alpha risk parameter estimation, Δa_s for the specific credit portfolio segment (s)

$$\hat{d}_{s,max} = \left(1 - e^{-\Delta a_s \cdot \frac{1}{12}}\right) \Rightarrow \Delta a_s = -12 \cdot ln(1 - \hat{d}_{s,max}) = -12 \cdot ln(1 - \psi_s \cdot npl_{s,t}) \tag{35}$$

3.5 Loss Given Default

As established in 0

$LGD(t, t+\Delta t]|_{\Delta T_{recovery}} =$

The final percentage loss, measured until recovery time $\Delta T_{recovery}$, that the bank will incur if obligor enters into a $Default\ state$ (assumed to be later followed by the $NonAccruing\ State$) during the semi-open interval $(t, t+\Delta t]$

$\Delta T_{recovery} =$ A Bank policy variable, the maximum allowed time interval, measured in months, during which all recovery is accumulated. An indicative value set is $\Delta T_{recovery} = 5\ years$.

Though we do not propose a new LGD model, we consider it logical to adopt the assumption that after entering into $Default\ State$, obligors cannot be efficiently categorized with the use of a classical rating system, since the explanatory power of all atomic "pre-default" characteristics has collapsed.

The only thing that guarantees defaulted loan partial repayments, and perhaps acts as an additional motivation for the obligor to repay, is the level and quality of collateral.

For the downturn LGD value, we will assume that impaired collateral values are applied, in any LGD model calculations, representing the recession (downturn) period of the economic circle.

$LGD_{w,t_0} =$ worst period LGD as defined at time snapshot t_0

This essentially is the same definition implied in the Basel formulas.

3.6 Credit Losses

With the combination of the results provided in 0, 0, 0, 0 we will end up in the specification of equation (1), for expected and unexpected losses, into the final equations(38), (39) and (40).

3.6.1 Expected Credit Losses

The theoretical measurement of expected loss for a single exposure is provided by (5)

Replacing into (5) the relations (9), (22), we construct the continuous time analogue of equation (5), in its general form

$$EL_{t_0} = \int_0^{\Delta T = T_{maturity} - t_0} PD'(t) \cdot LGD_{w,t_0} \cdot \left(EAD_{t_0} \cdot \Delta(t, r.\Delta T)\right) \cdot dt \tag{36}$$

$\Delta T =$ The remaining time in months until maturity

Replace the actual functions into (36)

$$EL_{t_0} = EAD_{t_0} \cdot LGD_{w,t_0} \cdot \int_0^{\Delta T} \frac{a_{delta}}{12 \cdot e^{\left(a_{delta} \cdot \frac{t}{12}\right)}} \cdot \left[\delta(r, \Delta T) - \beta(r, \Delta T) \cdot e^{\gamma(r,\Delta T) \cdot t}\right] \cdot dt \tag{37}$$

The closed formula provided for the calculation of expected credit loss of a single exposure, of a specific segment, is provided by the solution of (37), along with the specification of the functions $\delta(r, \Delta T), \beta(r, \Delta T), \gamma(r, \Delta T)$ in 0 and an LGD approximation as exposed in 0

$$\begin{gathered} EL_{t_0} = EAD_{t_0} \cdot LGD_{w,t_0} \cdot \left[\left(1 - e^{-a \cdot \frac{T}{12}}\right) \cdot (1+\beta) + \frac{a \cdot \beta}{12 \cdot \gamma - a} \cdot \left(1 - e^{\left(\gamma - \frac{a}{12}\right) \cdot T}\right)\right] \\ a = a_{delta} \end{gathered} \tag{38}$$

The amount of EL_{t_0} is the amount to be held in provisions, for one exposure of level EAD_{t_0} at observation time t_0.

3.6.2 Unexpected Credit Losses

For the calculation of losses under stressed conditions, we will apply the result of (35) into (38), and simplify the notation of functions $\delta(r.\Delta T), \beta(r.\Delta T), \gamma(r.\Delta T)$ and the alpha parameter of (26)

$$\begin{gathered} a_w = a\ (26) + \Delta\alpha\ (35) \\ TL_{w,t_0} = EAD_{t_0} \cdot LGD_{w,t_0} \cdot \left[\left(1 - e^{-a_w \cdot \frac{T}{12}}\right) \cdot (1+\beta) + \frac{a_w \cdot \beta}{12 \cdot \gamma - a_w} \cdot \left(1 - e^{\left(\gamma - \frac{a_w}{12}\right) \cdot T}\right)\right] \end{gathered} \tag{39}$$

Unexpected loss, that is excess capital reserve over provisions, for the one exposure, is estimated as the difference:

$$UL_{t_0} = TL_{w,t_0} - EL_{t_0} \tag{40}$$

The amount of UL_{t_0} is the amount to be held in capital for the absorbance of extra-ordinary losses.

4. Portfolio Level Losses with the Use of Aggregate Data

If loan and obligor data are available, the application of (38), (39), (40) will provide an accurate estimation of lifetime losses for 1 exposure which is afterwards summed for the total portfolio.

However, when only aggregate figures are published for a credit portfolio, we will use a generalized version of equation (38) in order to verify and benchmark the loss estimation. For the $j-th$ exposure of a credit portfolio at time snapshot t_0

$$EL_j = E_j \cdot LGD_j \cdot f(PD, EAD)_{life,j} \tag{41}$$

(41) is a generalized version of (38). We identify:

$E_j =$ loan exposure

$LGD_j =$ worst period LGD

$f(PD, EAD)_{life,j} =$ a function of PD and EAD during the remaining life of the exposure

If we further define the portfolio as a single large exposure, with

$E_{p/f} =$ the total loan exposure of the portfolio

(41) may be reformed as

$$ECL_{p/f} = E_{p/f} \cdot LGD_{p/f} \cdot f(PD, EAD)_{life,p/f} \tag{42}$$

In the previous equation, the variables involved represent a different type of risk

$LGD_j \longrightarrow$ Recovery risk

$f(PD, EAD)_{life,j} \longrightarrow$ default risk

Assuming perfectly correlated and identically distributed risk factors, the two risk variables above, may be represented by $x \in [0,1]$, leading to the next equation

$$EL_j = E_{p/f} \cdot x^2 \tag{43}$$

4.1 The Kumaraswamy Distribution Assumption

For a variable $x \in [0,1]$ $A, B > 0$ the probability density and probability cumulative functions are

$$\begin{aligned} pdf&: \quad f(x, A, B) = A \cdot B \cdot x^{A-1} \cdot (1 - x^A)^{B-1} \\ cdf&: \quad F = (\xi, A, B) = \int_0^x f(\xi, A, B) \cdot d\xi = 1 - (1 - x^A)^B \end{aligned} \tag{44}$$

Imposing on (43) $x \longrightarrow Kum(A, B)$

We arrive at the loss of the total portfolio as

$$ECL_{p/f} = E_{p/f} \cdot \int_0^1 x^2 \cdot (A \cdot B \cdot x^{A-1} \cdot (1 - x^A)^{B-1}) \cdot dx \tag{45}$$

Where the expected loss for every 1€ of the performing part of the portfolio is:

$$el_{performing} = \int_0^1 x^2 \cdot (A \cdot B \cdot x^{A-1} \cdot (1 - x^A)^{B-1}) \cdot dx = E(x^2) \tag{46}$$

Is the average loss of the performing, $1 - npl$ portfolio segment. If we consider that $x = 1$ for obligors that belong in the npl segment of the portfolio, npl being the non-performing loan percentage, then the expected loss for every 1€ of the non-performing part of the portfolio is:

$$el_{npl} = \int_0^1 x^2 \cdot (A \cdot B \cdot x^{A-1} \cdot (1 - x^A)^{B-1}) \cdot dx = E(x) \tag{47}$$

The total percentage loss (per 1€ of portfolio value) is

$$el = E(x^2) \cdot (1 - npl) + E(x) \cdot npl \tag{48}$$

The solution of which produces

$$el = B \cdot \Gamma(B) \cdot \left((1 - npl) \cdot \frac{\Gamma\left(1 + \frac{2}{A}\right)}{\Gamma\left(1 + \frac{2}{A} + B\right)} + npl \cdot \frac{\Gamma\left(1 + \frac{1}{A}\right)}{\Gamma\left(1 + \frac{1}{A} + B\right)} \right) \tag{49}$$

$\Gamma =$ the gamma function

$A, B =$ the Kumaraswamy distribution parameters

Equation (49) expresses expected losses for an observed level of npl and a realization of the Kumaraswamy distribution.

4.2 Numerical Approximation of Loss Curves

For the numerical calibration of $el(A, B, npl)$ loss function, we perform the following minimization procedure

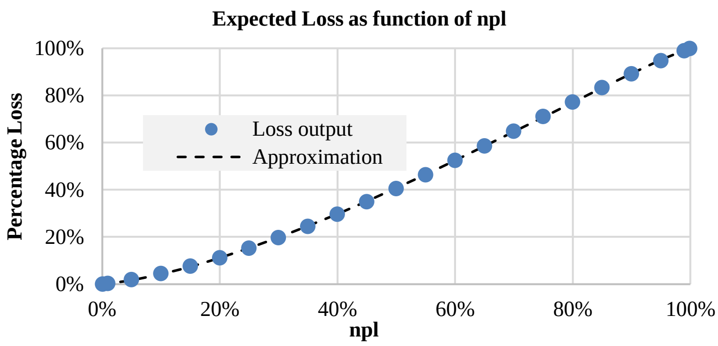
- for j discrete npl values $\{npl_j\}$
- estimate parameters A_j, B_j that maximize expected loss $el_j(A_j, B_j, npl_j)$, as calculated by (49)
- given the constraint $\mathrm{npl_j} = \frac{\mathrm{B_j} \cdot \Gamma(\mathrm{B_j}) \cdot \Gamma\left(1+\frac{1}{\mathrm{A_j}}\right)}{\Gamma\left(1+\frac{1}{\mathrm{A_j}}+\mathrm{B_j}\right)}$, that is the average non-performing percent observed in the portfolio is subject to the same risk distribution imposed on the other correlated risk factors of (43)
- the pairs of $\{npl_j, el_j\}$ are approximated by the cdf function

$$el = 1 - (1 - x^{A_{el}})^{B_{el}} \tag{50}$$

Results are portrayed on the Table 1, along with the relevant graph and the expected loss function (51) inside the table.

For any level of npl of a credit portfolio, (51) yields the percent of the portfolio value that should be kept as provisions.

Table 1. Expected loss based on current npl level



$$el = 1 - (1 - x^{A_{el}})^{B_{el}} \quad \begin{pmatrix} A_{el} = 1.44453 \\ B_{el} = 1.14213 \end{pmatrix} \tag{51}$$

A_j	B_j	npl_j	Loss output el_j	Approximation (el)
0.013	1.687	0.10%	0.03%	0.01%
0.054	1.688	1.00%	0.34%	0.15%
0.161	1.694	5.00%	1.97%	1.51%
0.271	1.692	10.00%	4.54%	4.09%
0.386	1.712	15.00%	7.59%	7.34%
0.497	1.693	20.00%	11.18%	11.09%
0.622	1.688	25.00%	15.17%	15.26%
0.725	1.599	30.00%	19.72%	19.80%
0.861	1.561	35.00%	24.54%	24.65%
1.036	1.562	40.00%	29.60%	29.78%
1.239	1.560	45.00%	34.96%	35.14%
1.480	1.556	50.00%	40.58%	40.73%
1.772	1.553	55.00%	46.41%	46.50%
2.136	1.549	60.00%	52.41%	52.42%
2.583	1.530	65.00%	58.56%	58.47%
3.070	1.439	70.00%	64.81%	64.62%
3.760	1.360	75.00%	71.06%	70.83%
4.853	1.312	80.00%	77.23%	77.06%
6.553	1.233	85.00%	83.30%	83.25%
9.358	1.059	90.00%	89.18%	89.31%
15.169	0.725	95.00%	94.78%	95.09%
24.499	0.173	99.00%	98.99%	99.21%
46.608	0.029	99.90%	99.90%	99.94%

4.2.1 Unexpected Losses Function

If npl_t is the current level of percentage non-performing loans of the portfolio, then the value of (51) should be kept as provisions. However, a higher amount should be set aside in order to cover for unexpected worsening of economic conditions.

We assume that the potential of npl_t in any time snapshot may be described by a sigmoid function

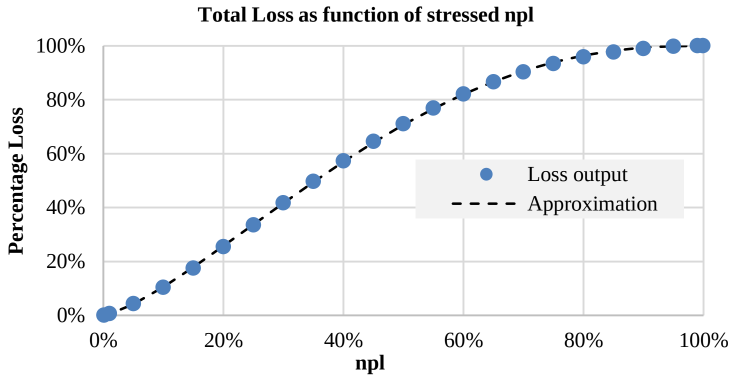
$$npl_t = \frac{1}{1+e^{-t}} \tag{52}$$

The worsening of npl_t over the next monthly snapshot is given by

$$npl_{t+1} = npl_t + \frac{dnpl_t}{dt} = npl_t + npl_t \cdot (1 - npl_t) = npl_t \cdot (2 - npl_t) \tag{53}$$

Repeating the procedure described in 0 but with the use of npl_{t+1} value of (53) we estimate the "total loss curve" tl as specified in Table 2. **Total loss based on worse expected npl level**, where the relevant function (54) and data are included in the table.

Table 2. Total loss based on worse expected npl level



$$tl = 1 - (1 - x^{A_{tl}})^{B_{tl}} \quad \begin{pmatrix} A_{tl} = 1.35130 \\ B_{tl} = 2.46853 \end{pmatrix} \tag{54}$$

A_j	B_j	npl_j	Loss output el_j	Approximation (tl)
0.020	1.688	0.10%	0.06%	0.02%
0.085	1.689	1.00%	0.70%	0.49%
0.265	1.689	5.00%	4.40%	4.25%
0.476	1.702	10.00%	10.41%	10.64%
0.698	1.689	15.00%	17.54%	17.95%
0.894	1.563	20.00%	25.52%	25.75%
1.184	1.558	25.00%	33.60%	33.75%
1.533	1.555	30.00%	41.73%	41.73%
1.957	1.546	35.00%	49.70%	49.55%
2.487	1.537	40.00%	57.32%	57.05%
3.038	1.440	45.00%	64.50%	64.14%
3.760	1.360	50.00%	71.06%	70.71%
4.788	1.315	55.00%	76.93%	76.71%
6.167	1.260	60.00%	82.10%	82.06%
7.903	1.154	65.00%	86.56%	86.73%
10.062	0.993	70.00%	90.32%	90.68%
12.963	0.811	75.00%	93.41%	93.90%
17.132	0.622	80.00%	95.86%	96.39%
19.802	0.351	85.00%	97.70%	98.18%
24.499	0.173	90.00%	98.99%	99.32%
24.405	0.039	95.00%	99.75%	99.87%
180.446	0.011	99.00%	99.99%	100.00%
20003.038	0.012	99.90%	100.00%	100.00%

4.3 Benchmarking with EBA 2018 Stress Test Results

In 2018 EBA performed stress tests on 48 selected European Banks [9]. Based on data of 31/12/2017, 2 scenarios were applied, baseline and adverse, for the following 3-year period up to 2020.

For the purpose of comparison, we will estimate the resulting provisions for each scenario as a function of the restated NPL percent (npl) of 31/12/2017. For each Bank, we calculate the following:

Non-performing loan percentage

$$npl_{scenario} = \frac{NPE^{IRB}_{scenario} + NPE^{STA}_{scenario}}{PE^{IRB}_{scenario} + NPE^{IRB}_{scenario} + PE^{STA}_{scenario} + NPE^{STA}_{scenario}} \tag{55}$$

The percentage loss is

$$Loss_{scenario} = \frac{Provisions^{IRB}_{scenario} + Provisions^{STA}_{scenario}}{PE^{IRB}_{scenario} + NPE^{IRB}_{scenario} + PE^{STA}_{scenario} + NPE^{STA}_{scenario}} \tag{56}$$

Where:

$NPE =$ non-performing exposures

$PE =$ performing exposures

$Provisions =$ Amount necessary to withstand scenario dependent losses

$scenario \in \{2017, 2020\ base, 2020\ adv\}$

$2017 =$ Restated values for original end 2017 figures

$2020\ base =$ End 2020 figures, with baseline scenario assumptions

$2020\ adv =$ End 2020 figures, with adverse scenario assumptions

In the upcoming graphs we will compare the actual stress test results against the results provided by our methodology.

Notation follows:

$Stress\ Test\ 2017 =$ Representation of $Loss_{2017}$ (56) against npl_{2017} (55)

$Stress\ Test\ 2020\ Base =$ Representation of $Loss_{2020\ base}$ (56) against npl_{2017} (55)

$Stress\ Test\ 2020\ Adv =$ Representation of $Loss_{2020\ adv}$ (56) against npl_{2017} (55)

$el =$ Representation of el (51) as function of npl_{2017} (55)

$tl =$ Representation of el (54) as function of npl_{2017} (55)

$EBA\ avg =$ The average Loss against npl_{2017} (55), which reveals on average, EBA policy target.

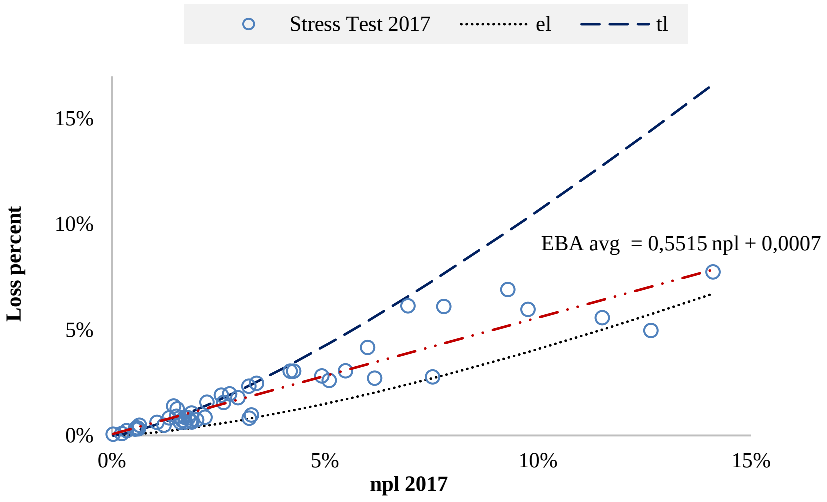


Figure 2. Percentage loss for end 2017 as function of 2017 npl

From Figure 2. Percentage loss for end 2017, it is established that the application of our benchmarking policy equations el, tl provide full coverage for the current year (2017 in our example). The two curves of el, tl may serve as lower and upper limit respectively for the percentage level of losses.

Nevertheless, the value of el, tl goes beyond the current year. The next 2 graphs, Figure 3 and Figure 4, illustrate that for a specific time snapshot (month), if the necessary capital to cover for credit risk losses is calculated according to tl curve, then the portfolio is – on average – adequately covered against extreme macroeconomic fluctuation effects (EBA scenarios) for the next 3-year period.

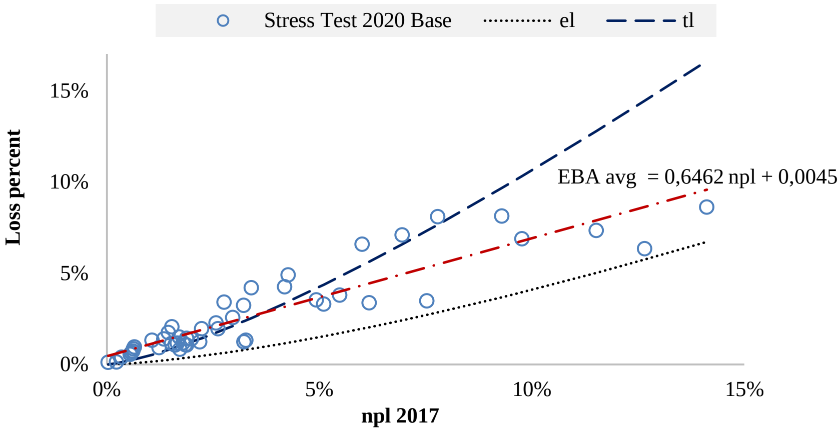


Figure 3. Baseline percentage loss for end 2020 – as function of 2017 npl

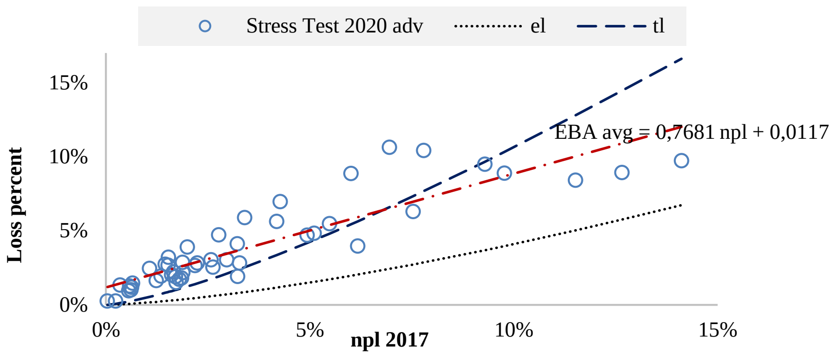


Figure 4. Adverse percentage loss for end 2020 – as function of 2017 npl

However, the continuous (monthly) readjustment of el, tl curve results, based on evolving npl, preserves the characterization of those curves as lower and upper loss boundaries. This is obvious on the next two graphs Figure 5 and Figure 6.

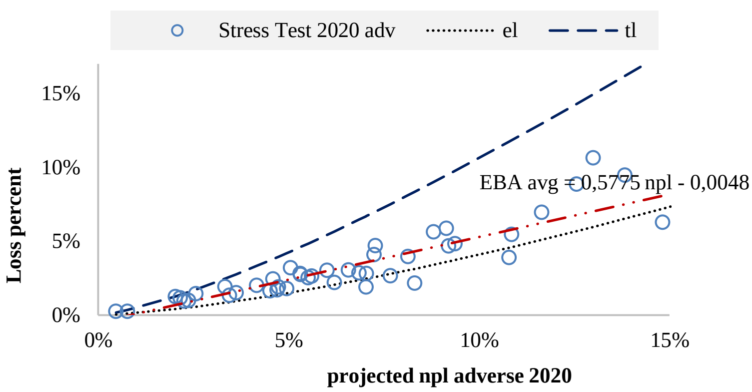


Figure 5. Adverse percentage loss for end 2020 – as function of adverse 2020 npl

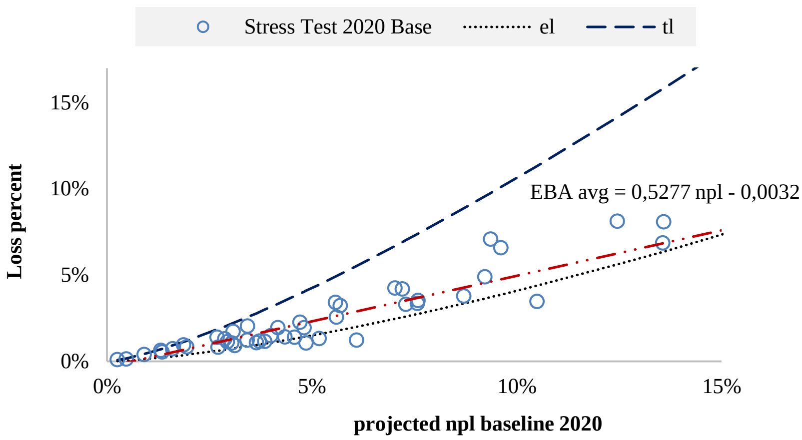


Figure 4. Baseline percentage loss for end 2020 – as function of baseline 2020 npl

5. Conclusion

Throughout this paper, we have demonstrated that analytic solutions can be derived for the estimation of lifetime credit losses, based on a set of business and statistical assumptions. We have approached this issue both form the point of view of the financial institution, developing analytic solution for individual loans, and from the perspective of financial authorities or even investors, providing benchmarks for the level of losses in terms of large credit portfolios.

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