A New Idea about Ricardo’s Comparative Advantage Theory on Condition of Multi-Commodity and Multi-Country

Yiqian Tian
School of Economics, Tianjin University of Commerce
Tianjin 300134, China
E-mail: nkafflatus@yahoo.com.cn

Abstract
Today, many theories prove the imperfection and limits of David Ricardo’s comparative advantage theory. This paper offers necessary explanations and creates a model for the n*n condition (n countries and n commodities) derived from the comparative advantage theory. It is a static equilibrium analysis based on the original model. This paper advances the final distribution methods and ways for the n*n model of multi-commodity trade.

Keywords: Comparative advantage theory, Premise and condition, Multi-commodity trade

1. Introduction
Recent development of the comparative advantage theory is to catch up with the perfection of the mainstream theory, exogenous comparative advantage theory, and its challenges. In recent researches on comparative advantage theory, a prominent phenomenon is: a so-called new mainstream (Helpman & Krugman, 1985; Grossman & Helpman, 1989, 1990) comes into being based on criticizing traditional comparative advantage theory by introducing the economy of scales and product differentiation, represented by Krugman, Helpman, and Grossman. By commenting on this new mainstream, other scholars develop the comparative advantage theory from different angles, including specialization, technology difference, system, and game theory.

These new development is far different from the original comparative advantage theory. This paper advances the viewpoint and model based on Ricardo’s original model. This paper is based on long-term thoughts on the comparative advantage theory, namely the deficiencies of static optimal distribution theoretical model for the multi-country and multi-commodity condition that is seldom studied by others, trying to offer some explanations and creations for the two aspects. In model derivation, three preconditions are necessary: (1) no discussion for intermediate products; (2) no discussion for the probability that small countries could not supply sufficient products for other countries’ demand by specialized production; (3) regardless of any condition that disturbs free trade.

2. Could the comparative advantage theory settle the multi-commodity production and trade?
The following discussions are all about the no ties condition (all commodities have different comparative advantages).

2.1 A brief introduction to relevant explanations in Paul R. Krugman’s International Economics: Theory and Policy, 6e.
In page 26, Krugman analyzes the comparative advantages of many goods under the title “Comparative advantage with many goods”. (The most convincible way is to identify other countries demands for the goods and calculate the average price of the goods in these countries besides the country A. For example, the goods a:

\[ P^a(A, B, C, D,...) = \frac{P^a_B Q^a_B + P^a_C Q^a_C + P^a_D Q^a_D + P^a_E Q^a_E +...}{Q^a_B + Q^a_C + Q^a_D +...} \]

Here for the sake of simple explanation, suppose all countries have same demand for certain goods and calculate the arithmetic average. Ronald W. Jones also employs this assumption in analyzing problems.)

Suppose both country A and B produce n kinds of commodities, namely commodity 1, commodity 2… commodity n. A spends labor aLi (i means any kind of commodity) in producing every kind of commodity. Its wage rate is w. B spends labor a*Li in producing every kind of commodity. Its wage rate is w*.

The direct and final conclusion: as \( w \times a_{Li} < w^* \times a^*_{Li} \), one country focuses on specialized production of certain commodity. As \( w \times a_{Li} > w^* \times a^*_{Li} \), feign countries focus on specialized production of the commodity.

Apparently, \( w \times a_{Li} \) is rightly the cost of producing certain commodity. That is from the definition of Smith’s absolute advantage. In other words, in this model, it is probable to draw a conclusion that contradicts the comparative advantage theory.
2.2 Krugman’s comparative advantage analysis and extension

Suppose \( n=3 \), country A and B produce three commodities, a, b, and c at the same time. Prices (costs) are displayed in table 1 as follow.

Comparing two countries’ costs in producing a, b, and c, then:

\[
\frac{4}{2} < \frac{9}{4} < \frac{7}{3}, \text{ namely } \frac{P_a}{P_a} < \frac{P_c}{P_c} < \frac{P_b}{P_b} \]

"*" stands for the country B.

Obviously, according to the comparative advantage theory, commodity a, and b respectively in two sides of the inequality has comparative advantage and comparative disadvantage. Therefore, even under the condition of \( n>2 \), Krugman’s “comparative advantage” analysis is not convincible.

A has comparative advantage in producing a (choose to producing a by specialization and exporting a) and comparative disadvantage in producing b (choose to import b). But for c, A has comparative disadvantage compared with a, and comparative advantage compared with b. It is hard to determine which country produces c and its trade conditions. Or, according to model deduction, no country chooses to produce c because no matter which country produces c, the country will lose higher returns from producing a, or b (Ricardo’s comparative advantage theory offers detailed reasons. It is unnecessary to explain it again). Therefore, as \( n=3 \), a problem is always there: only two commodities will be produced finally. A just produces a, and B only produces b (that is determined by the fixedness of opportunity cost). For multiple demands, it is impractical. As for this problem, Xiaokai Yang and other scholars do not give better explanations. They only make infra-marginal analysis and even deny the comparative advantage completely. Many experts probe into the multiple demand issue and explain the trade of commodity c. But they fail to analyze this issue from the maximum welfare.

Therefore, traditional comparative advantage theory confronts challenges in the multi-commodity issue. Most theorists, such as Professor Qi Liang, prefer to empirical analysis. They hold a similar opinion: “The vertical analysis is improper for multi-commodity trade. The proper analysis is based on absolute advantage theory and DFS model. (Qi Liang & Erzhen Zhang, 2002)” According to analyses above, with the assumption of comparative advantage, the DFS model is not right.

2.3 An explanation under bilateral trade

The comparative advantage theory cannot explain the multi-commodity trade between two countries, because the direct result is that only two commodities will be produced, what is impractical as a matter of fact. Therefore, for this issue (namely the multi-commodity trade between two countries), the comparative advantage theory is denied.

For this issue, there is an idea. That is, even there are many countries the comparative advantage theory does not allow the existence of more countries than commodities in trade (suppose i refers to the number of countries, j the number of commodities. As \( i<j \), it is inevitable that some commodities could not be produced at last. Finally, \( j \) is equal to \( i \)). In other words, if evaluate Ricardo’s model practically, this model can only explain such a condition that the number of countries is larger than the kinds of commodities in trade. But, whether it is means a denial to Ricardo’s model?

In fact, from daily economic life, we find that comparative advantage is usually an overall index, no matter what it is international trade or inter-provincial trade. In other words, people employ this concept widely. So, the significance of Ricardo’s model in practice is not less than any trade theory. Its essence lies in: for one country or region, how to select production in order to reach optimal welfare. Different editions of Senior International Trade Theory offer excellent analyses on two-commodity trade among \( n \) countries. Get each country’s costs for producing two commodities respectively, calculate the ratio of two commodities’ costs, and get an inequality. Establish the world relative price according to the two countries in two sides of the inequality. Then, countries in the middle of the inequality select their specialized production by comparing their domestic ratios with the world relative price (reference from Wen Hai’s Senior International Trade Theory). Therefore, the internal mechanism of Ricardo’s model is that the number of countries is not less than the kinds of commodities.

Come back to the original issue. Ricardo’s model does not concern the multi-country or multi-commodity condition. Then, how to explain the necessity of people consuming the third commodity? Starting from the internal thoughts of this model, if the consumption of the third commodity is necessary, the third country exists. Therefore, we can explain the commodity c production and trade between two countries. For example, as the country C joins in, a new price table (table 2) is as follow:

Obviously,

\[
\frac{P_c}{P_c} < \frac{P_b}{P_b} < \frac{P_a}{P_a} \]

"*"stands for the country C.

In the bilateral trade between A and C, A will produce and export c. Then, there is the consumption of c among the three countries, what is close to the fact. Therefore, even without the issue of demand diversity, it is possible for the appearance of the third commodity in Ricardo’s comparative advantage theory (of course, if change the numbers or in
the final effective equilibrium the country A may not produce c. But in the optimal equilibrium under different standards, to produce c is inevitable, what is different in theory from an inevitable lack of producing c in two countries). So, with the predetermined assumption, the comparative advantage theory is more reasonable than the comparative advantage identified by people directly.

The final bilateral trade is in table 3 as follow:
From this table, we can notice a clear trade combination. Comparative advantages arrange three countries’ production and trade. However, specific quantitative analysis needs to know the demands of each country for three commodities, and introduce other limits and discussions. For example, in the table 3, B imports a from A and C. Suppose B gives priority to C in imports because the absolute cost of an in C is lower than that in A. As C could not supply sufficient a, B chooses to import a from A. Another example, the assumption of fixed opportunity cost makes us agree that the optimal trade results in each country producing one commodity by specialization but the distribution ways could not be established due to different standards. The bilateral trade that lays particular stress on opportunity cost and the multi-lateral trade that lay particular stress on minimum absolute cost will result in different equilibrium, and even consider the probability of non-specialized production combination (in order to understand the issue of multi-country and multi-commodity trade, we can take reference from Ronald W. Jones’ *Comparative Advantage Theory and Tariffs: A Multi-country, Multi-commodity Model*. The author puts forward some assumptions and mentions that the optimal equilibrium of multi-lateral trade is usually more efficient than that of two kinds of bilateral trade.

That is the condition of bilateral trade, what is not the optimal and efficient production and trade for the 3*3 condition described in Ricardo’s model. Next, we make specific analysis.

3. Model: how to distribute production is more efficient under the n*n condition (n countries and n commodities)?

3.1 The most efficient solution
Firstly understand the difference between optimal efficiency and real equilibrium. In other words, the most efficient distribution is to realize the maximum total welfare of all countries. But the stable equilibrium may result in different distribution. It is only a regular Nash bilateral equilibrium. Here, the standard is: multi-lateral trade is most efficient. Next, we discuss this issue further.

Except the precondition of multi-country and multi-commodity, other assumptions are the same with Ricardo’s comparative advantage model. Suppose j refers to the number of countries, and i the kinds of commodities. Here, we mainly discuss the common condition as i=j (j>=3). In other words, under the condition of n*n, how to distribute is most efficient. For other two conditions, i<j, i>j, the analysis can adopt the method introduced next, because the method does not regulate the relationship of i and j. Here we choose i=j, for the sake of easy explanation.

Here we do not consider the distribution method advanced by Ronald W. Jones, because he introduces two much complex assumptions and he merely makes dynamic analysis on the n*n condition, without a systematic explanation for the form of efficiency. The author of this paper puts forward a definite static analysis method (Ricardo’s model is static) and deduces a universal model according to the most efficient distribution.

Display a conclusion: the best distribution is inevitably to make each country produce one commodity respectively by specialization, which is determined by the fixed opportunity cost.

Next, list the combinations of country and commodity. Countries are A, B, C, ……, and commodities a, b, c, …….

Before the trade, each country produce all commodities and i=j.

The table 4 displays all commodities’ costs (=prices).

According to this table, suppose the method for a country determining its commodity with most comparative advantage. In other words, use all commodities’ prices in one country (such as A) to divide its “average price in other countries” and get n numbers, among which the minimum value is decisive. The process is as follow:

The average price of commodity a: \( P^a = \frac{P_B^a + P_C^a + P_D^a + P_E^a + \ldots}{n-1} \)

The average price of commodity b: \( P^b = \frac{P_B^b + P_C^b + P_D^b + P_E^b + \ldots}{n-1} \)

The average price of commodity c: \( P^c = \frac{P_B^c + P_C^c + P_D^c + P_E^c + \ldots}{n-1} \)

The average price of commodity d: \( P^d = \frac{P_B^d + P_C^d + P_D^d + P_E^d + \ldots}{n-1} \)

……

For the sake of calculation and observation, get the table 5 as follow:

It is insufficient to compare two countries’ commodity with most comparative advantage since there are more than two countries. In table 5, use each commodity’s “price in country A” to divide its “average price in other countries” and get
n groups of rates, what is sufficient multi-lateral comparison. According to the meanings of comparative advantage, the smaller the rate, the more the commodity of the country has comparative advantage. Therefore, by finding out the minimum rate in the group, we can identify the commodity with most comparative advantage in country A (but it does not mean A must produce this commodity by specialization, because A is not necessarily the country that has the greatest advantage in producing this commodity). As another country joins in the comparison mentioned above, this country may possess the greatest comparative advantage in producing the commodity and the rate is maybe smaller. Then, compared with country A, this country is more appropriate for producing the commodity (has greater advantages than A). For example, as:

\[ P_a^b / P^a(b,C,D,\ldots) < P_b^c / P^b(c,B,C,D,\ldots) < P_c^d / P^c(d,B,C,D,\ldots) < \ldots \]

We can conclude: The commodity a in country A possesses the greatest comparative advantage in the world. Then, replace A with B and repeat the process. We can get a group of rates of prices in B to average prices in other countries. Find out the minimum rate. Next, C, D, …

For every country, repeat this process and get n groups of rates finally and get n minimum rates. Then, compare these minimum rates and take the smallest one. For example, if the smallest rate is \( P_b^d / P^d(A,C,D,\ldots) \), B has greatest comparative advantage in producing d (comparing with other commodities). So, B will produce d by specialization. Then, exclude B and d from the comparison (surely, in comparing these rates, besides identifying the minimum rate that determines the specialized production, we may find certain country possesses the greatest comparative advantage in producing other commodity (it happens in the country that possesses greatest comparative advantage in producing different commodities. If two countries possess comparative advantage in producing one commodity, choose the minimum value), which should be excluded. If not, it will not affect the final result, because the fact will not be changed that the country possesses the greatest comparative advantage in producing this commodity) and repeat a series of procedures in other countries and commodities. By this way, determine which country produces which commodity by specialization. By means of repetitive exclusion, get the most efficient distribution.

Then, explain this method further. At the very beginning, we make an assumption: use the prices of commodities in one country to divide the average prices of respective commodities in other countries and get the minimum rate that will determines the commodity with greatest comparative advantage in one country. This assumption is reasonable. Although these rates are meaningless, the \( n^2 \) rates from repeating the process for all countries can precisely reflect the comparative advantages of these countries in producing respective commodities.

Afterwards, generally each country is corresponding to n rates. Then identify the minimum of \( n^2 \) rates, which apparently determines the first country and its commodity with greatest comparative advantage. No matter what it is horizontal or vertical comparison, the country and the commodity represented by this minimum possess the greatest comparative advantage. These comparisons are based on the \( 2^n \) model and the \( n^2 \) model in the comparative advantage theory. Finally, the repetitive exclusions are determined by the fixedness of opportunity cost. And every selection is the combination of one country and one commodity with the greatest comparative advantage among the left. Each selection experiences three procedures: (1) get the rates of prices of commodities in each remaining country to average prices of respective commodities in other remaining countries; (2) compare these rates; (3) select the minimum and determine the country and the commodity with the greatest comparative advantage.

As i is not equal to j, we can adopt this method discussed above to identify each country’s specialized commodity. Then, for remaining countries and commodities, we can adopt the method offered in common trade textbooks to make decision. Here, we mainly develop the essential thoughts of Ricardo model.

That is a way to establish the most efficient distribution for the multi-country and multi-commodity model. By this way, we can deal with the quantitative evaluation and efficiency issue in table 3 (see table 6).

By calculations based on the method discussed above, get the most efficient distribution.

Firstly compare prices in country A with average prices in other countries, so \( 2 \times 4/(2+3), 2 \times 7/(3+7), 2 \times 9/(4+10) \). Then get a group of rates for country A: 8/5, 7/5, 9/7.

Similarly, a group of rates for country B: 4/7, 3/7, 8/19

And a group of rates for country C: 3/3, 7/5, 20/13

Obviously, the smallest rate is 8/19. Therefore, country B possesses the greatest comparative advantage in producing commodity c. So, country B chooses to produce commodity c by specialization. Then, consider the comparative advantages of country A and country C for producing commodity a and commodity b. Apparently, 7/7<4/3. Therefore, country C possesses comparative advantages in producing commodity a. And country A possesses comparative advantage in producing commodity b. Therefore, the most efficient distribution is: country A produces commodity b by specialization, country B commodity c, and country C commodity a. According to analyses above, it is the most
efficient distribution.

3.2 Practical significance

We should know that this distribution is optimal in theory. Practical world is crueler than theory. However, if some countries realize an integration of politics and economy, this theory discussed above will be meaningful since it can help to reach economic efficiency.

In fact, the non-self-governed regions in one country have already realized or have been close to realize the optimal distribution of production. Different endowments in different regions lead to the division and specialization. Some regions (such as some cities in Shanxi province) have rich coal, and some (such as the new seaside zone in Tianjin) have developed manufacturing industry. For example, Shanghai and Shenzhen have advantages in financial services. That is a rudiment of full comparative advantage efficiency.

4. Conclusion

According to the analysis, the comparative advantage theory meets difficulties in analyzing multi-commodity trade. But we should not deny this theory wholly. In analyzing the multi-commodity trade between two countries, by means of the classical model regardless of multiple demands, two countries only produce two commodities at last due to free trade, which can illustrate the importance of demands. But we should not deny Ricardo’s comparative advantage because of something outside assumptions. In analyzing the comparative advantage in multi-country and multi-commodity trade, we can get better results.

The comparative advantage theory relies in comparing opportunity costs. But we should not regard all two-factors as comparative advantages (such as the analysis based on wages and factors demands mentioned in Krugman’s book). Apparently, that is absolute advantage theory in essence.

In general, Ricardo’s comparative advantage theory is still applicable in multi-commodity trade. Its explanations (under the condition of following the assumptions) for commodity production and trade are acceptable. For example, the efficient distribution for the n*n condition deduced by use offers maximum and most efficient welfare by taking full advantage of comparative advantage thought.

Ricardo’s comparative advantage theory is always disputable in academic field. However, it is the disputable theory that makes trade theory more perfect, such as the dynamic comparative advantage theory, and the model combined with factor endowment theory. This paper is merely a tentative exploration, further thoughts and deductions based on Ricardo’s model as the author questions the traditional comparative advantage model. Some questions wait for new solutions.

References


Table 1.

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