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# Application of Dynamic Programming Model in Stock Portfolio —under the Background of the Subprime Mortgage Crisis

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**Abstract**

Known as "Financial 9.11", the U.S. subprime mortgage crisis causes great shock to the global economy. Meanwhile, global stock markets are in constant turmoil and suffer heavy losses one after another. Stock portfolio can disperse investment risks effectively to maximize investment income. This paper introduces dynamic programming method, establishes dynamic programming model and allocates funds between stocks in stock portfolio reasonably so as to maximize income, thus providing an effective approach to solute similar fund allocation issues.

**Keywords:** Dynamic programming model, Stock portfolio, Subprime mortgage

## 1. Introduction

Over the past year, a financial tsunami caused by structural faults of U.S. subprime debt market sweeps through Wall Street. Series of giant-like financial institutions collapse. The shock waves shake global financial markets, which is named "Financial 9.11". Global stock markets which are also affected by the subprime mortgage crisis shroud in an atmosphere of pessimism faction. If investors can not be informed accurately of investment information, they don't know which stock can bring greater benefits or which one shares smaller investment risk. In such a turbulent stock market, the portfolio can effectively help us circumvent the risk, thus maximize the gross investment return.

Modern portfolio theory originated in Harry Markowitz's paper "portfolio" released in 1952 and its same name monograph published in 1959. In the article and monograph above, Markowitz elaborated on the basic assumptions, theoretical basis and general principles of "portfolio", which laid his historical role—a pioneer of the portfolio theory(Li Guancong, 2006). Stock portfolio refers to the investment project group which is formed when investors consciously decentralized invest funds in a variety of stocks, thus get maximize return on investment. However, how to allocate funds rationally among a variety of stocks so as to make maximum benefit from the portfolio? This is the main issue this paper researches on.

## 2. The basic idea of dynamic programming

The American scholar Berman et al put forward dynamic programming in 1951 which provides an effective approach to such issue as distribution of funds. The unique feature of dynamic programming is that it uses decision-making by stages in the multi-variable complex decision-making issue, and changes it into a decision-making issue of solving several single variables (Liu Song & Wan Junyi, 2005). The basic principle of it is "optimization principle ", namely an optimal program with such a nature—regardless of the initial state and item, relative to the state produced by the initial item, subsequent items certainly constitute the best sub-items. It means any sub-item of an optimal item is always optimal (Liu Tao, 2000).

The key to Dynamic programming method is to write out basic recursive relationship correctly. The first step is to divide the process of the issue into several interrelated stages, select appropriate state variables ,decision-making variables and definite an optimal value function, so that a big problem can be transformed into a hierarchy of congener sub-problems, then solve them one by one. That's to start from boundary conditions and recur the optimal solution stage by stage. Meanwhile, use optimal solution of the anterior sub-problem in each sub-problem solving process in turn, and finally the optimal solution of the last sub-problem is that of the whole issue (Sun Xiaojun, 2002).

## 3. Application of dynamic programming model in stock portfolio

### *3.1 Case Introduction*

Suppose a company decides to invest ¥60,000 to buy 4 stocks. The company hopes to confirm the optimal portfolio through a rational allocation of funds, so as to maximize investment return. After market investigation and experts forecast, the relationship between return (unit: ¥10,000) and investment (unit: ¥10,000) of each stock is as follows.

**Insert Table 1 here**

### *3.2 Establishment of Dynamic Programming Model*

We establish dynamic programming model through dynamic programming method to solve how to allocate funds rationally, so as to maximize return of the portfolio. Due to the special structure of the issue, we will regard it as a multi-stage decision-making issue to solve stage by stage. Therefore, we introduce the following dynamic parameters (Yang Xuezhen, 2000):

(1)$S$—Total investment

(2)$n$—Item number of the portfolio

(3)$u_k$—decision variable, investment assigned to Item k

(4)$g_k(u_k)$—Stage objective function, return of $u_k$

(5)$S_k$—State variables, investment of Item k to Item n

(6)$S_{k+1}= S_k-u_k$ State transition equation

(7)$f_k(S_k)$—maximize return of $S_k$

Therefore, we can get the reverse DP (Dynamic Programming) equation as follows:

$$\begin{cases} f_k(S_k)=\max\left\{g_k(u_k)+f_{k+1}(S_{k+1})\right\} & 0\le u_k\le S_k, k=n,n-1,\cdots,1 \\ f_{n+1}(S_{n+1})=0 \end{cases}$$

Take advantage of the recursive relationship above, we finally solute $f_1(S_1)$ which is the maximum return of the issue, while portfolio allocation scheme is also optimal. This is the "reverse algorithm" of dynamic programming method (Yuan Zining, 2007).

### *3.3 Solving dynamic programming model*

In this case, we regard the process of allocating funds to one or several stocks as a stage. Now we use "reverse algorithm" of dynamic programming method to solve the whole issue stage by stage, given $S=S_1=6$.

3.3.1 The first stage

Given k=4,namely investing $S_4$($S_4$=0,1,2,3,4,5,6)in the fourth stock , in this case,

$f_4(S_4)= \max\{ g_4(u_4)+f_5(S_5)\}$ $0\le u_4\le S_4$

Obviously, if $S_4=0, f_4(0)=0$

if $S_4=1, f_4(1)=60$

if $S_4=2, f_4(2)=80$

if $S_4=3, f_4(3)=100$

if $S_4=4, f_4(4)=120$

if $S_4=5, f_4(5)=130$

if $S_4=6, f_4(6)=140$

Table 2 shows the results above:

**Insert Table 2 here**

3.3.2 The second stage

Given k=4,namely investing $S_3$($S_3$=0,1,2,3,4,5,6) in the third and fourth stocks, which makes maximum return on the investment allocated to the two stocks. In this case,

$f_3(S_3)= \max\{g_3(u_3)+f_4(S_4)\}$ $0\le u_3\le S_3$

(1) If $S_3$=0, $f_3(0)= \max\{ g_3(u_3)+f_4(S_4)\}= \max\{g_3(0)+f_4(0)\}=0$

Optimal item in this case is (0, 0), namely investment allocated to the two stocks is 0, thus optimal return is also 0.

(2) If $S_3$=1, $f_3(1)= \max\{g_3(u_3)+f_4(S_4)\}$, $0\le u_3\le 1$

Namely: $$f_3(1) = \max\begin{Bmatrix} g_3(0)+f_4(1) \\ g_3(1)+f_4(0) \end{Bmatrix} = \max\begin{Bmatrix} 0+60 \\ 50+0 \end{Bmatrix} = 60$$

Optimal item in this case is (0,1), namely investment allocated to the two stocks is 10,000, including investment in the fourth stock is 10,000 while that in the third one is zero. Optimal return at this time is 60,000.

(3) If $S_3$=2, $f_3(2)= \max\{g_3(u_3)+f_4(S_4)\}$, $0\le u_3\le 2$

Namely: $$f_3(2) = \max\begin{Bmatrix} g_3(0)+f_4(2) \\ g_3(1)+f_4(1) \\ g_3(2)+f_4(0) \end{Bmatrix} = \max\begin{Bmatrix} 0+80 \\ 50+60 \\ 120+0 \end{Bmatrix} = 120$$

Optimal item in this case is (2,0), namely investment allocated to the two stocks is 20,000, including investment in the third stock is 20,000 while that in the fourth one is zero. Optimal return at this time is 1200,000.

Empathy, if $S_3$=3,$f_3(3)$=180,Optimal item is$(u_3,u_4)$=(2,1)

if $S_3$=4,$f_3(4)$=230,Optimal item is$(u_3,u_4)$=(3,1)

if $S_3$=5,$f_3(5)$=260,Optimal item is$(u_3,u_4)$=(4,1)

if $S_3$=6,$f_3(6)$=280,Optimal item is$(u_3,u_4)$=(4,2)

The results above are given in Table 3 as follows:

**Insert Table 3 here**

3.3.3 The third stage

Given k=2,namely investing $S_2$($S_2$=0,1,2,3,4,5,6)among the second ,third and fourth stocks, which makes maximum return on the investment allocated to the three stocks. In this case,

$f_2(S_2)= \max\{g_2(u_2)+f_3(S_3)\}$ $\qquad 0\le u_2\le S_2$

Using the same calculation method as the second stage, the final calculation results can be expressed as follows:

**Insert Table 4 here**

3.3.4 The fourth stage

Given k=1, namely investing $S_1$ ($S_1$=$S$=6) among the four stocks, which makes maximum return on the investment allocated to them. In this case,

$f_1(S_1) = \max\{g_1(u_1)+f_2(S_2)\}$ $\qquad 0\le u_1\le 6$

Therefore,

$$f_1(6) = \max\begin{Bmatrix} g_1(0)+f_4(6) \\ g_1(1)+f_4(5) \\ g_1(2)+f_4(4) \\ g_1(3)+f_4(3) \\ g_1(4)+f_4(2) \\ g_1(5)+f_4(1) \\ g_1(6)+f_4(0) \end{Bmatrix} = \max\begin{Bmatrix} 0+310 \\ 40+270 \\ 100+230 \\ 130+180 \\ 160+120 \\ 170+60 \\ 170+0 \\ 130+180 \end{Bmatrix} = 330$$

Optimal item in this case is $(u_1,u_2,u_3,u_4)$=(2,0,3,1),namely investment allocated to the four stocks is 60,000 ,including investment in the first stock is 20,000 , in the second one is 30,000 , in the fourth one is 10,000 , while that in the second one is zero. Optimal return at this time is 3,300,000.

## 4. Ending words

Application of dynamic programming model is designed to help investors select the optimal portfolio among a number of investment items and disperse investment risks effectively in order to get maximize return (Zhang Xiaomin, 2008). In practice, the relationship between return and investment of each stock is mainly judged by information on the stock market grasped by investors and their own experience. Especially under current conditions, the U.S. subprime mortgage crisis brings great uncertainty to the global economy, revenue can hardly reasonable forecast which requires us to conduct an in-depth investigation and accurate predictions. At the same time, there is a positive correlation between risks and returns of a stock to a large extent .The greater the risk, the greater the return, and vice versa. But it does not exclude the possibility that because of some special factors such as force majeure and national macroeconomic policies, risks and returns present a reverse change (Zhou Huaren, 2006). In summary, as for application of dynamic programming model in stock portfolio, there is still much to be improved and supplemented, on which we need further explore and research.

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Table 1. Return and Investment

| Investment \ Return \ Item | $u_1$ | $u_2$ | $u_3$ | $u_4$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 40 | 40 | 50 | 60 |
| 2 | 100 | 80 | 120 | 80 |
| 3 | 130 | 100 | 170 | 100 |
| 4 | 160 | 110 | 200 | 120 |
| 5 | 170 | 120 | 210 | 130 |
| 6 | 170 | 130 | 230 | 140 |

Table 2. Optimal Return and Optimal Item

| $S_4$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f_4(S_4)$ | 0 | 60 | 80 | 100 | 120 | 130 | 140 |
| Optimal item $u_4$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

Table 3. Optimal Return and Optimal Item

| $S_3$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f_3(S_3)$ | 0 | 60 | 120 | 180 | 230 | 260 | 280 |
| Optimal item $(u_3,u_4)$ | (0,0) | (0,1) | (2,0) | (2,1) | (3,1) | (4,1) | (4,2) |

Table 4. Optimal Return and Optimal Item

| $S_2$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f_2(S_2)$ | 0 | 60 | 120 | 180 | 230 | 270 | 310 |
| Optimal item $(u_2,u_3,u_4)$ | (0,0,0) | (0,0,1) | (0,2,0) | (0,2,1) | (0,3,1) | (1,3,1) | (2,3,1) |