

Parametric or Non-Parametric Estimation of Value-At-Risk

Grzegorz Mentel¹

¹ Department of Quantitative Methods, Rzeszow University of Technology, Rzeszow, Poland

Correspondence: Grzegorz Mentel, Department of Quantitative Methods, Faculty of Management, Rzeszow University of Technology, Al. Powstancow Warszawy 8, 35-959 Rzeszow, Poland. Tel: 48-608-59-13-30. E-mail: gmentel@prz.edu.pl

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Abstract

In the financial analyses the fact of predicting future states of the instruments subjected to investments is extremely important. It allows reducing risk and maximizing potential profits. That is why any ways which enable to predict the further negative results of taking decisions are very important and the knowledge about the measures and their efficiency are an additional advantage. Thus, the paper in which value at risk and an assessment of this measure are discussed seems to be of interest.

Keywords: value-at-risk, modeling, risk

1. Introduction

The financial world has its own rules and all decisions result in financial consequences. Therefore, it is crucial to understand the mechanisms that create behaviors in this area, and most importantly, the methods which allow to some extent on their management and possible reduction of their negative effects. Indeed, it is important to have the knowledge of the instruments in which to invest, but even more it is important to know how to deal with risk reduction which appears always with these type financial investments. Any measure, including the value at risk, may not only allow for a full exploration of knowledge about the mechanisms that create the financial markets but, more importantly can be used as a tool to fight over the negative consequences of our decisions.

Thus, in the paper *Value at Risk (VaR)* is presented as an instrument which reduces the risk and defines its scale. The analysis is made among twenty companies of the Warsaw Stock Exchange currently included in the WIG20. Taking into account the multitude of varieties of *VaR* calculation methods, some of them are proposed to carry out some kind of confrontation. Such an approach was designed to assess the effectiveness of simulation methods for more complex parametric methods. As a part of the resultant, which combines both of these groups, also a method which belongs to semi-parametric EKT (Note 1) group is proposed. At the same time the Monte Carlo and historical simulation as non-parametric methods are taken into consideration. The methods developed by the Risk Metrics with different approaches to the modeling of random noise (normal distribution, the Student's t distribution and GED (Note 2) are representatives of the second group). In order to evaluate the full effectiveness of these methods three levels of significance of 0,01, 0,05 and 0,10 are taken into account. The research horizon covers the period from 01.01.2010 to 31.12.2012. In the analyses 151 historical observations are used as the information necessary to estimate these models. In order to estimate the quantity there was used the dependence developed by *Risk Metrics*TM, namely:

$$n = \frac{\ln(\gamma_{tol})}{\ln(\lambda)} \quad (1)$$

where, as it is easy to see the required amount of historical observations is dependent on the specified level of tolerance and the adopted smoothing constant. It is perfectly illustrated in Figure 1.

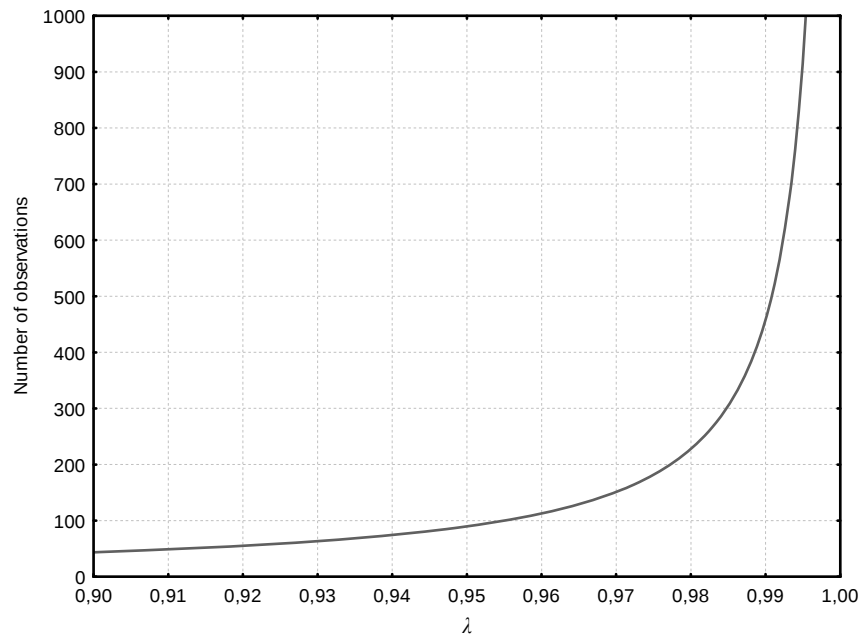


Figure 1. Number of historical observations as the smoothing function λ

Source: own study based upon Risk Grades™ Technical Document, Second edition, February 2001.

2. Parametric Methods

There are many analytical models describing the fluctuation of financial instruments at the time. Most of them are very important because of their practical applications. They are mostly models introduced and successfully used by financial analysts and engineers gathered around a group of *Risk Metrics*™ (Note 3). The core difference between them is mainly due to different approaches to the modeling of random noise (e.g. Normal distribution, t-Student distribution or even GED) and with the same method of calculating VaR. In this case there might be distinguished, inter alia, the models based on the generalized autoregressive processes of conditional variance (such as *GARCH (1,1)*), so-called "reverting to the mean" models (Mean Reversion) and Random Walk, which also are successfully used in many issues of financial engineering (Pisula & Mentel, 2003).

The most common model in this class is *Risk Metrics Normal Drift*, the model with random noise modeled by a normal distribution (Risk Metrics Technical Document, Risk Metrics Monitor, 1996).

In the model it is assumed that the logarithmic returns of stock prices

$$r_t = \ln\left(\frac{P_t}{P_0}\right) \quad (2)$$

where:

P_t - stock price in „t” research period,

P_0 - stock price in the baseline period.

are generated according to the following process:

$$r_t = \mu + \sigma_t \cdot \varepsilon_t, \varepsilon_t : N(0,1), \quad \tilde{r}_t = \frac{r_t - \mu}{\sigma_t} : N(0,1) \quad (3)$$

In this model, the so-called variance conditional of daily returns of stock prices (at the practical assumption that the average value is zero) is calculated as an infinite moving average with exponential

weights: $\sigma_t^2 = \sigma_{t|t-1}^2 = (1-\lambda) \sum_{j=0}^{\infty} \lambda^j \cdot r_{t-1-j}^2$. Approximately for a sufficiently large number of historical

observations ($n \rightarrow \infty$) this dependence can be written as following: $\sigma_t^2 = (1 - \lambda) \sum_{j=0}^{n-1} \lambda^j \cdot r_{t-1-j}^2$, and recursively as:

$$\sigma_t^2 = (1 - \lambda)r_{t-1}^2 + \lambda\sigma_{t-1}^2.$$

For the returns with longer time horizons ($T > 0$) the variance scaling with respect to the horizon length is applied. (Pisula, 2002).

*Risk Metrics*TM applies in the analyses a universal constant smoothing $\lambda = 0.97$ for daily returns.

Estimated on the basis of this model, *VaR* limits (the accepted level of significance α) for the daily time horizon will be the following for the refunds and stock prices:

$$\begin{aligned} \mu + \tau_{N(0,1),\alpha/2} \cdot \sigma_t &\leq r_t \leq \mu + \tau_{N(0,1),1-\alpha/2} \cdot \sigma_t \\ P_{t-1} \cdot \exp(\mu + \sigma_t \cdot \tau_{N(0,1),\alpha/2}) &\leq P_t \leq P_{t-1} \cdot \exp(\mu + \tau_{N(0,1),1-\alpha/2} \cdot \sigma_t) \end{aligned} \quad (4)$$

where:

$\tau_{N(0,1),\alpha/2}$, $\tau_{N(0,1),1-\alpha/2}$ respectively the quantiles of the row in normal distribution

The relevant parameters of the model (λ and μ) are determined by the maximum probability method.

Another model recommended by *Risk Metrics*TM is a *Risk Metrics t-Student* model with random noise modeled by t-Student distribution.

In this model, it is assumed that the returns are generated according to the following process:

$$\begin{aligned} r_t &= \mu + \sigma_t \cdot \varepsilon_t, \varepsilon_t : t\text{-Student}(0,1,\nu) \\ \tilde{r}_t &= \frac{r_t - \mu}{\sigma_t} : t\text{-Student}(0,1,\nu) \end{aligned} \quad (5)$$

VaR limits estimated on the basis of this model (at the accepted level of significance α) for the daily time horizon are appropriate for returns and stock prices as:

$$\begin{aligned} \mu + \tau_{t(0,1,\nu),\alpha/2} \cdot \sigma_t &\leq r_t \leq \mu + \tau_{t(0,1,\nu),1-\alpha/2} \cdot \sigma_t \\ P_{t-1} \cdot \exp(\mu + \sigma_t \cdot \tau_{t(0,1,\nu),\alpha/2}) &\leq P_t \leq P_{t-1} \cdot \exp(\mu + \tau_{t(0,1,\nu),1-\alpha/2} \cdot \sigma_t) \end{aligned} \quad (6)$$

where:

$\tau_{t(0,1,\nu),\alpha/2}$, $\tau_{t(0,1,\nu),1-\alpha/2}$ - respectively the quantiles of the row in t-Student distribution.

The parameter models (λ, μ, ν) are determined as in the previous ones applying the maximum probability method.

Risk Metrics GED model discussed in the article has differently modeled noises. In this case the above noises are modeled by General Error Distribution.

In this model, it is assumed that the returns are generated according to the following process:

$$r_t = \mu + \sigma_t \cdot \varepsilon_t, \varepsilon_t : GED(0,1,\nu), \quad \tilde{r}_t = \frac{r_t - \mu}{\sigma_t} : GED(0,1,\nu) \quad (7)$$

Density function for a generalized error distribution $GED(\mu, \sigma, \nu)$ can be given with the following parameters: location μ , scale σ and shape ν is of the form:

$$f(x, \mu, \sigma, \nu) = \frac{\nu}{\omega 2^{(1+\nu^{-1})} \Gamma(\nu^{-1}) \sigma} \exp\left(-0.5 \left|\frac{x - \mu}{\omega \sigma}\right|^\nu\right) \quad (8)$$

where:

$$\omega = \left(2^{-(2/\nu)} \cdot \frac{\Gamma(1/\nu)}{\Gamma(3/\nu)}\right)^{1/2}, \quad \Gamma - \text{gamma function.}$$

GED distribution is often used in practice, because it has so-called "fat tails". This means that the forecasts constructed on the basis of GED has an easier way to capture the extreme observations (Figure 2). If the shape parameter is $\nu = 2$, then the GED distribution is a normal distribution of $N(\mu, \sigma)$. Estimated on the basis of this model, *VaR* limits of returns and stock prices for daily time horizon can be given as:

$$\mu + \tau_{GED(0,1,\nu),\alpha/2} \cdot \sigma_t \leq r_t \leq \mu + \tau_{GED(0,1,\nu),1-\alpha/2} \cdot \sigma_t \quad (9)$$

$$P_{t-1} \cdot \exp(\mu + \sigma_t \cdot \tau_{GED(0,1,\nu),\alpha/2}) \leq P_t \leq P_{t-1} \cdot \exp(\mu + \sigma_t \cdot \tau_{GED(0,1,\nu),1-\alpha/2})$$

where:

$\tau_{GED(0,1,\nu),\alpha/2}, \tau_{GED(0,1,\nu),1-\alpha/2}$ - respectively the quantiles of the row in GED distribution.

The parameter models (λ, μ, ν) are determined as in the previous ones applying the maximum likelihood method.

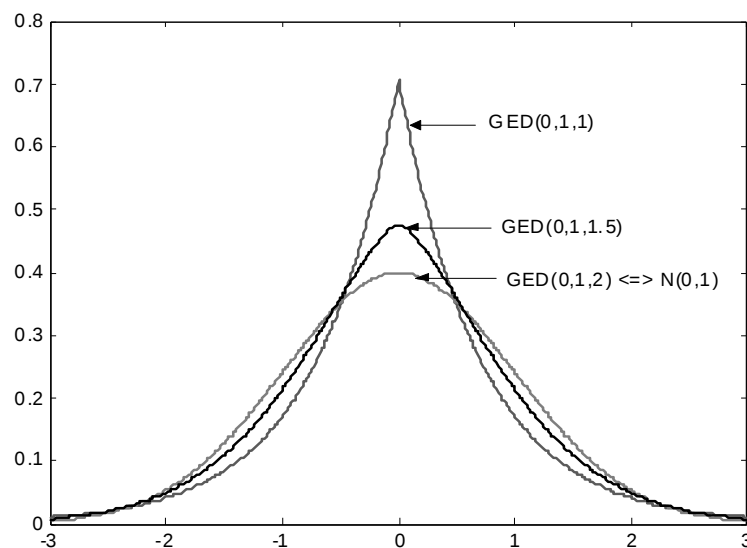


Figure 2. The chart of density function of GED distribution for different parameters

Source: Pisula T., Pisula J., *Możliwość efektywnego przewidywania ryzyka zmian kursów akcji spółek notowanych na GPW, Rynek Terminowy* no. 17/3/2002.

3. Simulation and Nonparametric Models

In the historical simulation method the real data is used to estimate *VaR*, which makes it even better than the covariance method reflects the actual behavior of the market. The main advantage of this method is that it is a non-parametric method. This means that not only there is no restriction resulting from the need for the assumption of normality, but also the estimation of some parameters (such as mean and standard deviation) on the basis of historical data are avoided (Mentel, 2011; Jajuga, 2000).

In the case of "fat tails" in the real distribution of prices, historical simulation method gives a more reliable level of *VaR*. The historical simulation advantage is also the fact that unlike other methods this one is easier to estimate.

Historical approach is a very intuitive method of estimating *VaR*. It is based on historical return rates of the instrument (or portfolio) and their empirical distribution. It is important that the rates of return need to be calculated with the same period of *VaR* (if the investment horizon is one-day, then the rates of return should be determined daily).

While using historical model it is necessary to collect a large series of data. The higher the number, the more accurate, but the data is often very distant which is not important as the less distant data. Sometimes gathering sufficient data is not possible and the use of historical method is then limited.

Historical simulation is also sensitive to extreme return rates included in the distribution. As a result, the size of VaR varies discretely and the size of the risk is often underestimated or overestimated.

Historical model assumes that the development of risk is determined by its historical behavior.

Turning into the second of the considered simulation methods, namely the Monte Carlo method it is worth emphasizing that it is based on a hypothetical stochastic model that describes the evolution of the prices of the financial instrument. The essence of the stochastic processes is that it is not possible to predict the values of the process; one can only determine the probability with which a given value is reached.

In the Monte Carlo method hypothetical model is assumed to describe the mechanism of the formation of prices (or return rates) of the financial instruments. It is often assumed that this process is a geometric Brownian motion. Using as a basis this or other models, a lot of observations of financial instruments prices are generated. This way one gets the distribution of return rates of the financial instrument. Determining of the distribution quantile allows determining VaR in a direct way. The process parameters are usually estimated based on historical data (Jajuga, Kuziak & Papla, 1999).

4. Semi-Parametric Concepts

The resultant of a group of models in estimating value at risk is so-called group of semi-parametric methods. Their common representative is EKT.

This method is a modification of the historical method. It is based on extreme value theory dealing with probability distributions having thick tails (Emmer, Klüppelberg & Trüstedt, 1999). It is quite an important assumption because, in spite of the widely used approach which is the distribution of return rates of financial instruments having the condition of normal distribution, in fact the extreme observations exclude such assumptions. In practice, the greater applications have the distributions catching more detached observations than does the above-mentioned distribution. Much better in that regard is modeling the return rates even by GARCH (1,1) or *t*-Student (Mentel, 2011).

In the discussed method, as defined by the value at risk, it is examined the left tail of the returns distribution from the portfolio, i.e. all the negative return values. Then, the value of these returns is multiplied by -1 receiving at the same time positive values. We sort the values of these phrases: $r_1 \geq r_2 \geq \dots \geq r_M \geq \dots \geq r_L$, where L is the number of observations from the left tail of the returns distribution, and M is a threshold indicator for which all values of r_j , $j = M+1, \dots, L$ are much smaller than the other values r_j , $j=1, \dots, M$ (Talar&Weron, 2001).

The designation of the threshold value r_M is performed with the application of QQ-plot chart which illustrates the observations above and below of the threshold value.

The formula for the estimator of the tail distribution is as follows:

$$1 - p = \frac{M}{L} \left(\frac{r_{M+1}}{x} \right)^{\frac{1}{\alpha}}, x > r_{M+1} \quad (10)$$

where:

p – distribution quantile,

α – the Hill's estimator described by the formula:

$$\alpha = \frac{1}{M} \sum_{i=1}^{M-1} \ln r_i - \ln r_M \quad (11)$$

From the above one gets the value at risk formula:

$$VaR = -r_{M+1} \cdot \left(\frac{M}{L(1-p)} \right)^{\alpha} \cdot \sqrt{T} \quad (12)$$

where: T – time during which one decides to keep their portfolio.

5. Efficiency Assessment

As indicated before VaR is estimated by using three proposed levels of significance. Such an approach is designed to assess more fully the efficiency of the methods in terms of the changing probability. Frequently some methods are "highly effective" in specific circumstances, in this case, under certain established levels of significance.

Along with the change in the value of this index their overall assessment may be subjected either to improvement or deterioration.

Table 1. The percentage of limits exceeded the acceptable values of *VaR* for different methods

	Monte Carlo Simulation			Historical Simulation			EKT Method		
	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$
AssecoPol	2,329	6,489	11,148	1,331	5,824	9,651	0,000	0,998	4,825
Bank Handlowy	1,830	4,493	8,319	0,666	5,990	13,311	0,000	0,998	5,657
Bogdanka	1,498	5,158	7,654	1,830	5,158	11,148	0,000	0,666	5,824
Boryszew	1,498	4,659	6,656	1,165	4,326	8,486	0,000	3,661	18,136
BRE Bank	1,664	4,992	9,318	1,165	4,160	7,654	0,000	0,666	4,493
GTC	0,998	5,158	9,984	2,329	6,988	13,977	0,000	1,165	9,651
Jastrz. Sp. Węg.	1,357	6,335	12,670	0,000	0,905	5,430	0,000	0,000	3,620
Kernel	2,329	5,657	8,985	1,830	6,556	11,314	0,000	1,498	4,659
KGHM	1,830	5,158	8,819	1,664	5,990	10,150	0,000	1,664	7,654
Lotos	2,329	6,156	9,318	1,664	5,657	10,316	0,000	0,832	5,158
PEKAO	1,498	6,323	9,651	1,498	5,491	9,484	0,000	1,165	3,993
PGE	3,161	5,824	9,484	1,664	4,992	9,817	0,000	1,165	6,156
PGNiG	1,498	4,992	8,153	1,331	5,324	10,316	0,000	0,666	4,659
PKN Orlen	1,664	4,825	8,985	1,498	4,659	8,486	0,000	1,664	3,827
PKO BP	1,997	5,657	9,484	0,832	4,326	9,484	0,000	0,832	4,493
PZU	1,953	5,273	8,594	1,563	4,297	10,156	0,000	1,165	5,491
Synthos	2,496	4,160	7,987	1,997	6,988	11,814	0,000	1,331	4,659
Tauron	3,138	6,485	9,414	2,301	7,741	13,180	0,000	1,046	5,230
TP S.A.	2,496	4,992	6,822	1,830	4,825	9,817	0,000	1,331	6,822
TVN	1,498	6,988	11,814	1,664	6,822	13,649	0,000	0,998	5,824
	Risk Metrics Normal Drift Model			Risk Metrics T-student Model			Risk Metrics GED Model		
	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$
AssecoPol	1,498	6,489	10,483	0,666	4,326	8,652	1,165	4,493	10,316
Bank Handlowy	1,830	4,659	8,652	0,998	2,829	5,824	1,498	4,493	11,148
Bogdanka	1,331	5,158	9,151	0,166	1,997	5,324	0,499	4,160	9,318
Boryszew	0,333	0,666	2,329	0,499	3,161	7,488	0,333	3,661	14,309
BRE Bank	2,163	5,824	10,483	0,832	3,494	7,987	1,498	5,657	11,481
GTC	1,165	4,493	8,985	0,666	2,329	7,155	0,666	5,491	11,148
Jastrz. Sp. Węg.	1,810	6,787	12,217	0,000	4,977	10,860	0,905	6,787	13,122
Kernel	1,997	5,491	8,985	0,166	3,494	6,656	0,499	4,160	9,817
KGHM	1,498	3,328	6,323	0,832	3,661	7,987	1,331	5,491	11,814
Lotos	1,997	5,824	10,483	0,832	4,493	8,319	1,165	5,657	11,148
PEKAO	1,498	5,824	9,983	0,998	4,992	8,819	1,165	5,824	10,150

PGE	2,995	6,323	9,484	0,832	4,992	8,153	1,664	6,323	10,815
PGNiG	1,664	5,324	9,151	0,499	3,328	6,156	0,666	4,825	10,649
PKN Orlen	1,664	4,825	9,484	0,998	3,661	7,488	1,165	4,825	10,316
PKO BP	1,997	5,657	10,316	0,499	4,992	8,818	1,498	5,657	11,148
PZU	1,953	5,469	9,180	0,977	3,516	7,031	1,563	5,469	10,156
Synthos	2,829	5,158	9,318	0,666	3,161	6,489	0,832	3,993	9,318
Tauron	2,929	6,904	10,879	0,837	5,439	8,368	1,883	6,485	12,134
TP S.A.	1,165	3,993	5,158	0,499	2,163	4,493	0,832	3,993	8,652
TVN	1,830	6,323	11,814	0,499	3,993	7,987	0,832	5,990	12,978

Source: own study.

Value at Risk imaging is done on the twenty instruments, which in turn eliminates any chance indications that can take place in case of very low study sample. The estimation was done on the basis of the daily return rates of the considered instruments.

The basic summary of the results is illustrated in Table 1, in which there are presented the percentage values of exceeding beyond the *VaR* threshold, resulting in the adopted level of significance. Thus, the levels are 1%, 5% and 10%. So radically different values allow for the good evaluation of the considered models.

While doing a rapid assessment on the basis of that statement, one can have an illusion that it has the EKT method which is highly efficient. This is due to the fact that the interest rates of the acceptable limits are sensational in this case, and it's hard to relate them to other models that are lagging behind. However, as can be seen in Figures 3-5, the method is not very reliable in case of small values of the levels of significance. Then, the differences are significant enough that its use as a measure of limiting the risk is at all unreasonable.

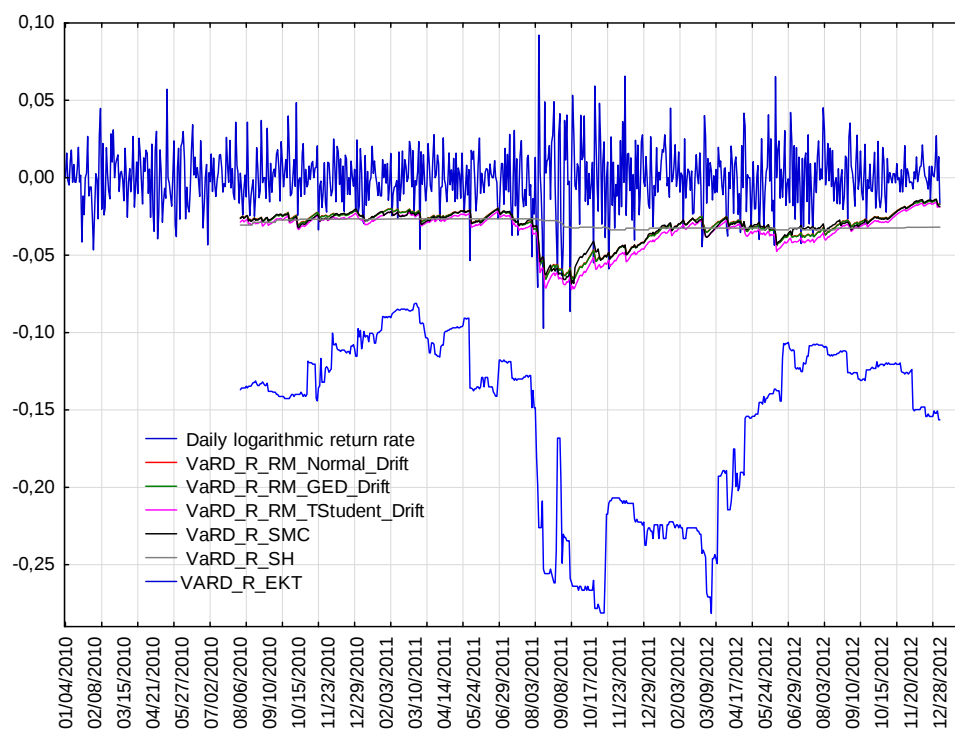


Figure 3. Value at risk for the exemplary daily logarithmic return rates of PEKAO at the significance level $\alpha = 0,01$

Values of that kind are improved with the increase of α , but nevertheless the estimates of value at risk by EKT method overestimate the risk scale compared to other already discussed models.

Quite characteristic method in the context of visual is the historical simulation method whose values are less frequent change. It explains the shape of some kind stepped curve as for the *VaR* values determined by this

method. Changes in the values of the tail occur so little that the estimated value very often maintains a constant level over a longer period of time. Thus, this method is much less flexible in comparison to the others.

All other parametric methods and the Monte Carlo simulation are visually much more positive, mainly due to the fact of the rapid response to market changes, and much weaker aversion to risk.

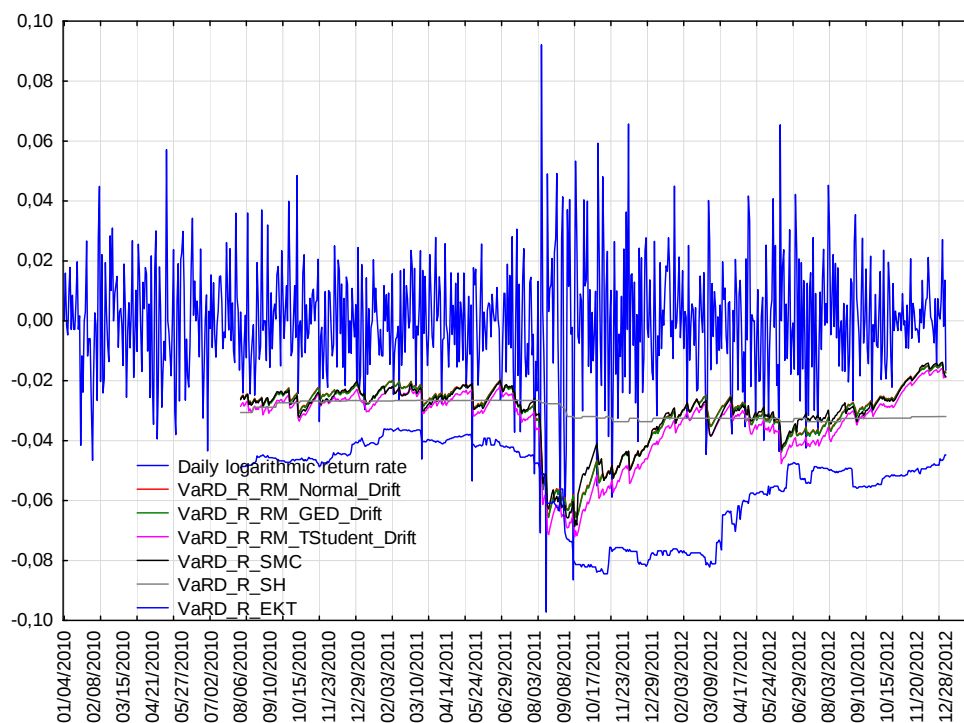


Figure 4. Value at risk for the exemplary daily logarithmic return rates of PEKAO at the significance level $\alpha = 0,05$

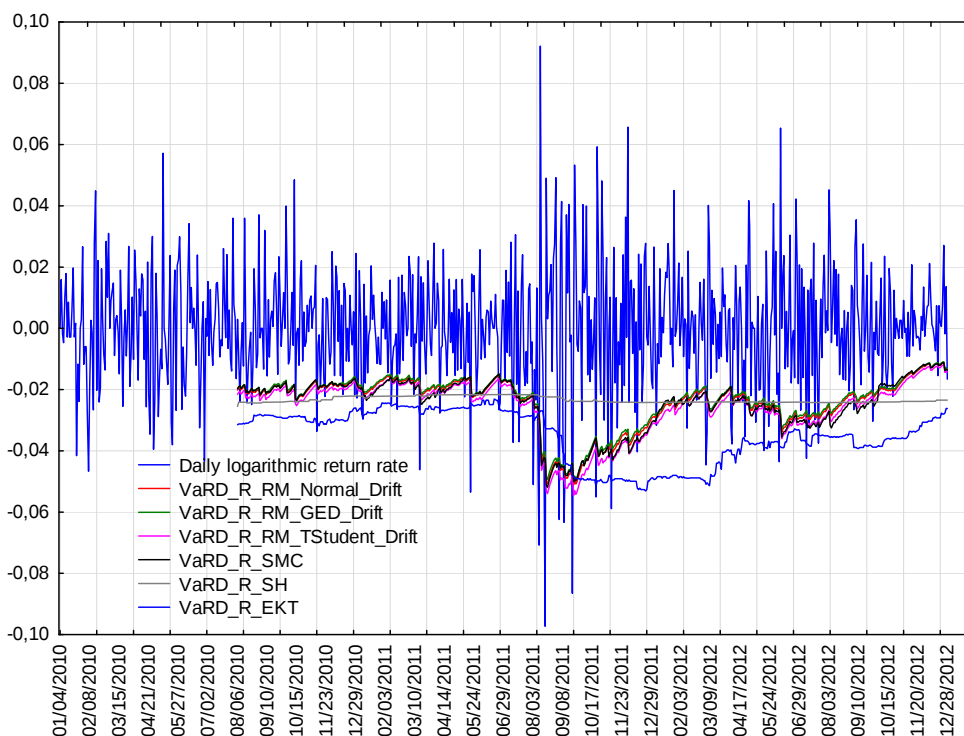


Figure 5. Value at risk for the exemplary daily logarithmic return rates of PEKAO at the significance level $\alpha = 0,10$

It seems that much more meaningful comparison can be made on the basis of average values exceeded the assumed levels of significance and an analysis of their typical ranges of variation (Figure 6).

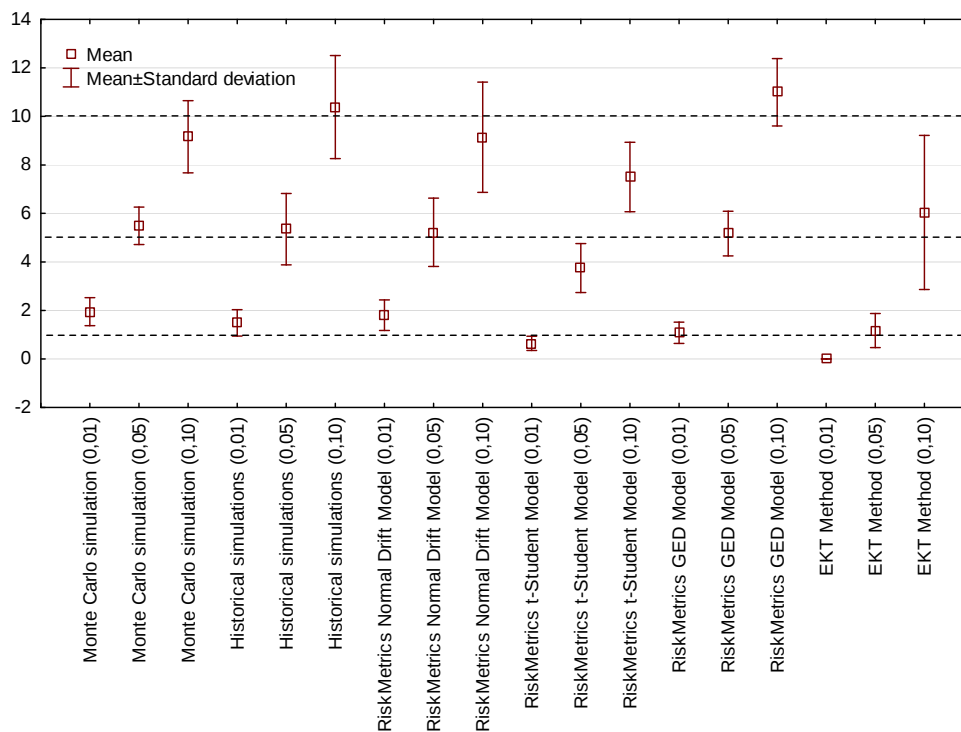


Figure 6. The ranges of the exceeding percentage for the examined methods and levels of significance

Taking into account previous observations that should be emphasized and which are equally important, it should be noted that the highest efficiency of the value-at-risk estimates has the *t-Student Risk Metrics* method. It's fair to say that this method is the most universal one when it comes to the estimated probability values. In each considered option it was much more efficient than it would indicate the threshold values

It seems in this type of analysis the Monte Carlo simulation method performs well. What is important, it is a representative of a group of non-parametric methods.

6. Conclusions

While drawing conclusions it can be argued that the best estimates and the most efficient forecasts, irrespective of the level of significance, provide models of random noise modeled by Student's *t*-distribution. Furthermore, they are characterized by relatively low efficiency variation of the calculated forecasts.

Apart from *EKT* and historical simulation *VaR* forecasts follow market changes. Thanks to this the group of such methods is more flexible and more relevant to the actual market conditions.

Relatively good indications are obtained in the case of simulation methods regardless of the assumed level of significance. Better in this respect the Monte Carlo simulation is characterized by a high stability of indications and thus more resistant to extreme observations. But one cannot say the same about the historical simulation, where the influence of the detached observations is visible.

It may be noted that for small levels the models with random noises modeled by *GED* distribution also give satisfactory forecast, much better than the normal distribution. This is due to the so-called "fat tails", allowing for better catching the extreme observations. However, the situation is becoming worse with the increase of the α level.

Some models, in spite of the significant efficacy, often overestimate the *Value at Risk* and then some indications are much higher than the observed values. An example of such a situation could be here the historical simulation or semi-parametric *EKT* method.

Conclusions and trends observed in the case of *VaR* designated for returns are confirmed by the value at risk

estimated for rates or gains / losses of the analyzed values. Thus, the majority of the dependences are subjected to unification regardless of the calculated *VaR*.

With the increase in the value the assumed for the analysis the level of significance increases the overall efficiency of the analyzed models. An exception may be already mentioned here is *Risk Metrics GED*.

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Notes

Note 1. The name of the method is derived from the first letters of its creators: Emmer, S., Klüppelberg, C., Trüstedt, M.

Note 2. General Error Distribution.

Note 3. *Risk Metrics Group* is an organisation separated from the structure of JP Morgan Bank in 1998 which deals with the risk management processes. It has been listed on the New York Stock Exchange since 2008.