Modification of the Warrants Pricing Model and Validation Analysis

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Abstract

Based on the previous study in this article, modified Black-Sholes model was more suitable for the domestic warrants price, and calculated 6 warrants' price in the listed time, It also confirmed the rationality of the model comparing the pricing theory with the actual market price. The result of the computation of the modified warrant price model shows that though in the warrant market the realistic price deviates from theoretical price, the deviations of realistic and theoretical prices is smaller from May, 2006 to May, 2007.

Keywords: Call warrants, Black-Sholes model, Market present status

1. Introduction

Has been published since the Black-Sholes option valuation model (Black, Scholes, 1973) for its computation convenience and the theoretically consummation, it is widely applied. At the same time the researchers have studied it deeply because of its strict supposition and expanded its assumptions greatly and made it more realistic. Now the warrant pricing theory research is generally based on the B-S option valuation model.

Merton relaxed the assumption on the risk-free interest rate as constant and the pricing model that allows rate is a random variable (Merton., 1973). Under the assumption of the effective capital market, Galai and Schneller (Galai, Schneller, 1987) put forward warrant pricing model considering the flow of the dilution effect through the analysis of single-phase and multi-phase framework and inspected that the warrants issued affected the firm value. Leland (Leland, 1985) developed a hedging strategy which modifies the Black-Scholes hedging strategy with a volatility adjusted by the length of the rebalance interval and the rate of the proportional transaction cost. Boyle and Vorst (Boyle, Vorst, 1992) designed a perfect hedging strategy in the Cox, Ross and Rubinstein binomial model with transaction costs. The perfect hedge is possible due to the assumption of a binomial process for the underlying stock price. Ukhov Andrey (Ukhov, Andrey, 2004) develop an algorithm for pricing warrants using stock prices, an observable variable, and variance of stock returns.

The theoretical study in foreign countries provided a good foundation for the domestic warrants pricing, but in view of the fact that the domestic and foreign transactions environment is different and the products differ, many foreign theories don't apply to the domestic warrants pricing.

Chinese scholars have been studying foreign theories that apply to the domestic market. Tang Bing and Li Hong-rong (Tang Bing, Li Hongrong, 2004) study subscribing for capital stock from the market perspective of warrants pricing. Zhan Shi-guang (Zhan Shiguang, 2005) considered the warrants pricing model with dilution effect, proportional transaction costs, ex rights and ex divided the factor, and did a sensitivity analysis of the model . This article, based on the previous scholars' study, first introduces the model in which stock proportion transaction cost and the division factors are considered, then the warrants pricing is affected by the compound dilution effect and the warrant exercises cost for the warrant modified model. The value of 7 warrants' products which are first introduced in China's warrant market is assessed daily. The two investment portfolios are developed for comparison and validation to verify that the model is rational. At last, the computation is shown that in China's warrant market the actual value's deviation from theoretical value has been well controlled.

Fischer Black and Scholes (1973), in his famous paper "The pricing of options and corporate liabilities", proposed the European call option formula pricing warrant value, a series of assumptions are needed:

(1) Assume that the underlying security price follows the lognormal process-geometric Brownian motion. The parameters, the rate of return μ and the instantaneous variable of the assetσ, are constant.

(2) In a frictionless market there are no taxes and transaction costs. And all assets are entirely unlimited breakdown without restrictions on short selling.

(3) In derivative securities period, from t = 0 to t = T, no cash or no dividend pay for the underlying securities.

(4) In derivative securities period, from t = 0 to t = T, there's the same risk-free rate loans, interest rates on risk-free

compound interest r is calculated as consecutive terms.

The traditional Black-Scholes warrant price formula is:

$$C = SN(d_1) - Xe^{-rT}N(d_2) \quad \text{, Where } d_1 = \frac{\ln(S/X) + (r + \frac{1}{2}\sigma_S^2)T}{\sigma_S\sqrt{T}}, \; d_2 = d_1 - \sigma_S\sqrt{T} \, .$$

The parameters, μ and σ_S, are the instantaneous rate of return and the instantaneous variable of the asset, respectively. The risk free asset earns at the constant rate r. Let S be the price dynamics of the underlying security, C be the price of the warrant , X be the exercise price, and $N(.)$ be cumulative standard normal distribution function.

As calculated on the convenience and strict logic, traditional Black-Sholes model is commonly used to price warrant. But it neglected many factors because of strict assumption, such as various costs and dividends in the listed time. This article will relax these traditional B-S model assumptions restrictions.

2. Modification for Warrant Pricing Model

2.1 Warrants Pricing Model with the Transaction Cost and Dividend

In traditional Black-Scholes model, there are no taxes and transaction costs in a frictionless market. It simplifies the model calculations, but makes certain deviations. Basing on Leland's proportional transaction cost model, we relax the second assumption as follows.

According to China's present relevant provisions, fees are paid as follows in stock transaction: Commission (normally no more than the amount of securities trading 0.3%), Transfer fees (in A-shares, Shanghai Exchange for the deal transaction denomination 0.1%, Shenzhen free transfer fees) and taxes (deal amount 0.1% before May 30, 2007). Costs are paid in warrant transactions: Commission (no more than the amount of securities trading 0.3%, starting at five yuan).

Transaction cost is divided into the stock transaction cost and the warrant transaction cost. The warrant transaction cost has different effects on the theoretical value for buyers and sellers, and reflects indefinitely in market price,so the stock transaction cost is considered here with ignoring the warrant transaction cost. A security traded with a proportional transaction cost rate a is supposed. Based on Leland's the proportional transaction cost warrant valuation model, warrant pricing model and B-S model are similar, but in this paper, the fluctuation rate $\tilde{\sigma}_S$ was defined as

$$\tilde{\sigma}_S = \sigma_S\sqrt{1 + \sqrt{\frac{2}{\pi}} \times \frac{a}{\sigma_S\sqrt{\Delta t}}}$$

Leland expanded the fluctuation rate $\tilde{\sigma}_S$. He considered, when $\Delta t \to 0$, the expanded fluctuation rate offset the transaction cost. Here the ex dividend factor is only considered.

According to discrete dividends B-S model, the new stock price is equivalent to the amount of that the original price of the underlying stocks minus the cash dividend's risk-free rate discount. Then the exercise price's adjustment is a formula according to the stock exchange management:

New exercise price=original exercise price × (shares ex-dividend date reference price/
Ex-dividend one day before the closing price of the underlying stocks)

As the ex-dividend one day before the closing price of the underlying stocks is unknown, shares ex-dividend date reference price could not figure out. So we use the following approximate formula:

New exercise price=original exercise price × (1-Cash bonus discount/
The original price of the underlying stocks)

In order to calculate conveniently, we estimate X_X as $X_X = X \times (1 - \sum_{i, t_i > t} e^{-rt_i} D_i / S_0)$ when $t = 0$.

2.2 Warrant Pricing Model with the Dilution and Exercise

Exercises cost must be paid in warrant exercise. The stock dilution must be considered after exercising in the stock warrant. The dilution and exercise cost interact on each other. These two factors should be considered simultaneously.

Assume that each warrant entitles the owner to receive one shares of stock paying the exercise cost of A. At time T, the agent's profit is: $\max\{k(S_T - X_X - A), 0\}$

Consider valuation of a conventional warrant, issued by a firm for its own stock. Suppose that a company has N shares of common stock and M warrants outstanding. Each warrant entitles the owner to receive k shares of stock upon payment of X dollars. At time T ,the value of the firm is V_T, and when the warrant holders exercise their warrants, the instantaneous value of the firm is: $V_T + MkX_X$, and the instantaneous value of the share is $\frac{V_T + MkX_X}{N + Mk}$.

Therefore,the agent's profit is:

$k(\frac{V_X + MkX_X}{N+Mk} - X_X - A)$,

and the warrant profit is

$\frac{Nk}{N+Mk} Max\{\frac{V_T}{N} - (X_X + \frac{N+M}{N} A), 0\}$

Assume $X' = X_X + \frac{N+M}{N} A$, so the warrant profit is $\frac{Nk}{N+Mk} Max\{\frac{V_T}{N} - X', 0\}$.

We can cognize that the stock warrant is $\frac{Nk}{N+Mk}$ whose profit is $C_T = Max\{\frac{V_T}{N} - X', 0\}$ at time T options portfolio.According to option pricing formula, the warrant values C_t can be obtained arbitrarily.

$$C_t = \frac{V_t}{N} N(d_1) - X' e^{-r(T-t)} N(d_2)$$

Where $d_1 = \frac{\ln\frac{X'}{V_t/N} + (T-t)(r + \frac{1}{2}\sigma^2)}{\sigma\sqrt{T-t}}$, $d_2 = \frac{\ln\frac{X'}{V_t/N} + (T-t)(r - \frac{1}{2}\sigma^2)}{\sigma\sqrt{T-t}}$

So at time t the value of equity warrant calculated as follows:

$$W_t = \frac{N}{\frac{N}{k}+M}\{[S_t - \sum_{i,t_i>0} e^{-r(t_i-t)} D_i + \frac{M}{N} W_t] N(d_1) - X' e^{-r(T-t)} N(d_2)\} \quad (1)$$

$$= \frac{N}{\frac{N}{k}+M}\{\frac{V_t}{N} N(d_1) - X' e^{-r(T-t)} N(d_2)\} \quad (2)$$

Where $d_1 = \frac{\ln[(S_t - \sum_i e^{-r(t_i-t)} D_i + \frac{M}{N} W)/X'] + r(T-t)}{\tilde{\sigma}\sqrt{T-t}} + \frac{\tilde{\sigma}\sqrt{T-t}}{2}$, $d_2 = d_1 - \tilde{\sigma}\sqrt{T-t}$

And W is the price of the equity warrant; M is the number of new warrants; N is the number of stocks; K is the equity warrant rate; S is the stock price; X is the exercise price; r is the risk free rate; T is the expiration date of the warrant; $\tilde{\sigma}$ is the fluctuation rate with transaction cost for the firm value; t_i is time to pay dividends; D_i is the dividends; $N(.)$ is cumulative standard normal distribution function.

An important problem in this model is that $\tilde{\sigma}$ is unknown, we can assess the fluctuation rate using observable variable warrant pricing model by Ukhov, Andrey D. Define $\Omega_S = \frac{\Delta S / S}{\Delta V / V} = \frac{\Delta S}{\Delta V}\frac{V}{S} = \frac{\Delta_S V}{S}$, the elasticity of the stock price with respect to the firm value, where $\Delta_S = \frac{\Delta S}{\Delta V}$ is a hedge ratio for the stock, it measures the price change for the share of stock when the value of the entire firm changes by one. By Ω_S , we get: $\sigma_S = \Omega_S \sigma_V = V \frac{\Delta_S}{S} \sigma_V \Rightarrow \sigma_V = \frac{\sigma_S}{V \frac{\Delta_S}{S}}$. As $V = NS + MW$, so $N\Delta_S + M\Delta_W = \Delta_V = 1$, then $\Delta_S = \frac{1 - M\Delta_W}{N}$. If equation (2) is used, we get $\Delta_W = \frac{\partial W}{\partial V} = \frac{k}{N + kM} N(d_1)$. On the mention above,

$$\sigma_V = \frac{\sigma_S SN}{V\left(1 - \frac{MkN(d_1)}{N + Mk}\right)} .$$

Basing on the proportional transaction cost warrant valuation model by Leland, we may obtain as follows:

$$\tilde{\sigma} = \sigma_V \sqrt{1+\sqrt{\frac{2}{\pi}}\times\frac{a}{\sigma_V\sqrt{\Delta t}}}\ .$$

2.3 Model with Integrating the Above Factors

The above factors integrated, warrant pricing is assessed as follows:

$$W_t = \frac{N}{\frac{N}{k}+M}\{[S_t - \sum_{i,t_i>0} e^{-r(t_i-t)}D_i + \frac{M}{N}W_t]N(d_1) - X'e^{-r(T-t)}N(d_2)\}$$

Where $d_1 = \dfrac{\ln[(S_t-\sum_i e^{-r(t_i-t)}D_i+\frac{M}{N}W)/X']+r(T-t)}{\tilde{\sigma}\sqrt{T-t}} + \dfrac{\tilde{\sigma}\sqrt{T-t}}{2}$, $d_2 = d_1 - \tilde{\sigma}\sqrt{T-t}$,

$$\sigma_V = \frac{\sigma_S SN}{V\left(1-\frac{MkN(d_1)}{N+Mk}\right)}, \tilde{\sigma} = \sigma_V\sqrt{1+\sqrt{\frac{2}{\pi}}\times\frac{a}{\sigma_V\sqrt{\Delta t}}},$$

$$X' = X_X + \frac{N+M}{N}A = X\times(1-\sum_{i,t_i>0}e^{-rt_i}D_i/S_0)+\frac{N+M}{N}A\ .$$

And W_t is the value of the equity warrant at time t; M is the number of warrants; N is the number of stocks; k is proportion each warrant entitles the owner to receive shares of stock; S is the stock price; X is the exercise price; r is the risk free rate; T is the expiration date of the warrant; $\tilde{\sigma}$ is the fluctuation rate with all the transaction costs for the firm value; σ_V is the fluctuation rate on the firm value considering exercise cost; σ_S is the fluctuation rate of the stocks; t_i is time to pay dividends; D_i is the dividends; $N(.)$ is cumulative standard normal distribution function; A is the exercise cost paid by the owner to receive one shares of stock; a is the proportional transaction cost rate.

3. Example for Calculation by Modified Warrant Pricing Model

The warrant mainly has the functions of price discovery and the risk management. It is a tool for effective risk management and the resources allocation. Up to May 30, 2007, the stock markets in China have 33warrants listed. Take Yili CWB1 as example below (Table 1), its value is calculated using the modified warrant price model.

Since May 30, 2007, the securities exchange tax rate from original 0.1% rises to 0.3%, we just get the date by May, 2007 for validation calculation, and transaction stock cost is the amount of transaction cost 0.4%, that's to say, $a = 0.4\%$.

The fluctuation rate is calculated on the historical data. YiLi Ltd 2005 annual dividend distribution: on April 6, 2006, Yili Co.Ltd in "China Securities" and "Shanghai Securities News" published on the distribution of profits notice: A shares of Stockholder's rights registration date is on April 10, 2006, stock dividend day on April 11, 2006. Total cash paid out is 101,728.9 thousand Yuan, occupying the profits which are distributed for the shareholders 23.07%.

The next transaction day selected from April 11, 2006 is from April 12, 2006 to November 14, 2006. During this period, stock rate is calculated in the stock ransaction. And we got 52.13% as the YiLi Ltd's fluctuation rate.

The Yili CWB1 lasts 12 months from November 15, 2006. According to former experience Yili Co.Ltd usually distributed dividends in July. It was estimated that the dividends for 2006 is in July, 2007 for every10 shares as 1 Yuan. In order to consider cash dividend influence on the stock initial price and the exercise price are needed to be adjusted.

After the adjustment the new good power price is $X_X = 7.9556$ yuan, then

$$\sigma_V = \frac{\sigma_S SN}{V\left(1-\frac{MkN(d_1)}{N+Mk}\right)} = 56.6461\%$$

When it exercises warrant only accepts stock transfer fees, for the deal transaction denomination 0.1%. YiLi Ltd stock denomination is 1 yuan, so the exercise cost is

$k\times$stock denomination$\times 0.1\% = 1\times1\times0.1\% = 0.001$yuan,

then $A = 0.001$, and the adjusted exercise price is $X' = X_X + \frac{N+M}{N}A = 7.9569$

The Yili CWB1 lasts one year, a year has 252 transaction days, take $\Delta t = 1/252$, then

$$\sigma\% = \sigma_V \sqrt{1+\sqrt{\frac{2}{\pi}} \times \frac{a}{\sigma_V \sqrt{\Delta t}}} = 59.1251\%$$

From the above calculation method and parameters, we calculated the daily value (Note 2) of Yili CWB1, from April 24 2006 to May 29, 2007. And the traditional Black-Sholes result is compared with the modified model result. The result is shown in Figure 1.

4. Validation on the Modified Warrant Model

In light of the actual situation, the two investment portfolios with the same cost are structured. If the gains and losses for the two investment portfolios differ greatly under the less risk circumstances, the arbitrage opportunities may exist:

Program 1: Stock Investment

Let S_t be the stock price, the ratio of the transaction cost for the stock transaction amount is a, r is the corresponding interest rate. At the time t, the investment amount: $S_t \times (1+a)$; if the dividends is D_i at the time of $t_i (t_i < T)$, then the assets of the investment program at T time is $G_1 = S_T + \sum_i D_i \times [1 + r(T - t_i)]$

Program 2: Warrants and Deposits Investment

Warrant price is W_t', the ratio of the transaction cost for the warrant transaction amount is g. This investment program at the time t, the investment amount is $W_t' \times (1+g)$. For the amount of investment at the time of t as same as the assets of the investment program of Program 1, to save $F_t = S_t \times (1+a) - W_t' \times (1+g)$ in the bank. At the time of T, the bank deposits is $F_T = F_t[1 + r(T-t)]$, so the assets of this program is $G_2 = \begin{cases} S_T + F_T - X - A & S_T > X \\ F_T - S_T a & S_T < X \end{cases}$

The income gap between program 1 and program 2 at the time of T is $G_1 - G_2$, and

$$G_1 - G_2 = \begin{cases} \sum_i D_i \times [1 + r(T - t_i)] - (F_T - X - A) & S_T > X \\ \sum_i D_i \times [1 + r(T - t_i)] - F_T + S_T(1+a) & S_T < X \end{cases}$$

If the Program 1 is better than Program 2, which $G_1 - G_2 > 0$, it was shown that the expected return on the stock market investment is much more than that on the warrant market. The warrant price was overvalued in the actual market, the actual prices of warrant is higher than the theoretical value, so $W_t' - W_t$ and $G_1 - G_2$ should be approximately equivalent.

According to Monte-Carlo simulation, the warrant options expiration at time *T*, random equation of the underlying stock price is: $S_T = S_t \exp\left[(\mu - 0.5\sigma^2)(T-t) + \varepsilon\sigma\sqrt{T-t}\right]$

Where, the variable ε is subject to the standard normal distribution from which the expectation of the S_t is estimated without exercise, and thus $E(G_1 - G_2)$

Taking Yili CWB1 as an example, we constructed investment portfolio. Hereinafter as $W = W_t' - W_t$, $G = G_1 - G_2$, we get a fig of difference value *W* and difference value *G* during existing period and duration:

The difference value *W* basically coincides with the difference value *G* from the above Figure 2. In fact these two variables are in regression analysis, we can get the regression equation as $W = 0.974G + 0.025$. Equation constant and variable G were less than 0.001 significantly.

Referred to the above model and calculation process, the following six products of warrant are analyzed by computing and data analysis, the results are shown as Table 2 and Figure 3.

The estimation for the accuracy of each firm's parameter is different. Each firm has the different effects on the modified B-S model and investment portfolio. According to the above results of the regression, the difference value *W* and the difference value *G* are approximately equivalent. So the above modified B-S model is basically rational.In this paper the price of warrant is mostly calculated from the early 2006 to May 30, 2007. In accordance with the terms of warrant prices map, we can see that there's serious overestimated phenomenon of the price of warrant in the listed time.

A relative error J_t set up to measure level of deviation of the actual prices of warrant from the theoretical price. Actual prices W_t' and theoretical values W_t are known, that $J_t = (W_t' - W_t)/W_t$. According to the data calculated, choose a time of more than four warrants that is from May 24, 2006 to May 10, 2007. The trends figure on the average is shown as follows:

In accordance with option theoretical knowledge, as the exercise date approaches, the degree of actual price deviation from the theoretical value would be smaller. To reduce the impact of this factor, we selected different starting times of warrant products. As shown in Figure 3, the price of warrant was overestimated seriously. But the general trend was that overestimating the price of subscribing for warrant dropped greatly. However, such phenomenon is well controlled in 2007.

5. Conclusion

This paper has presented a warrant pricing with integrating dividends, stock trading costs, warrant exercise cost and dilution of equity warrants. The model is applied to validation analysis. We can see that the market mechanism is not perfect in 2006, investors blindly follow market speculation, and there is an obvious “herding effect”. In 2007, the phenomenon for overestimating price of warrant has been effectively controlled. China stock market matures, legal laws and regulations consummation are much better than before. Warrants’ actual price was close to the theoretical value and it was rational for the investors to understand the warrant.

However, there has certain defects on the modified model developed in this paper: it is not considered that rise stops and fall stops have influence on the warrant pricing; it is not studied in details that if the price dynamics of the underlying security follows the lognormal process, and if the rate of return μ and the instantaneous viable of the asset σ are parameters, and that how the computational method for expected rate of return μ and the instantaneous viable of the asset σ affects the result. Therefore, there will be further improvement on this model, which is more useful for the practice.

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Notes

Note 1. YiLi Ltd(580009)warrant bulletin to market, 2006-11-10

Note 2. Dates from http://cn.finance.yahoo.com/

Table 1. Yili CWB1 Basic Information (Note 1)

Yili CWB1			
Warrant Name	Yili CWB1	Warrant Code.	580009
Underlying Security	YiLi Ltd	Underlying Security Code	600887
Warrant publisher	Yili Co.Ltd	Warrant Type	European stock warrant
Original Stock Price	21.73	Exercise Price (¥)	8
Risk Free Rate	2.52%	k	1:1
Time to Market	2006.11.15	T	12 months
M	154940935	N	516469784

Table 2. Data analysis for Difference value W and difference value G

Warrant Name	Warrant Type	Difference value W and G		Significant Level	
		R2	regression equation	constant term	G
Shouchuang JTB1	Covered Warrants	0.999	W=0.975G+0.032	<0.001	<0.001
Yage QCB1	Covered Warrants	0.978	W=0.879G+0.020	0.018	<0.001
Baogang JTP1	Covered Warrants	0.999	W=0.966G-0.086	<0.001	<0.001
Hangang JTB1	Covered Warrants	0.987	W=0.984G+0.033	<0.001	<0.001
Zhonghua CWB1	European stock warrant	0.990	W=0.961G-0.008	0.68	<0.001
Yili CWB1	European stock warrant	1.000	W=0.974G+0.025	<0.001	<0.001

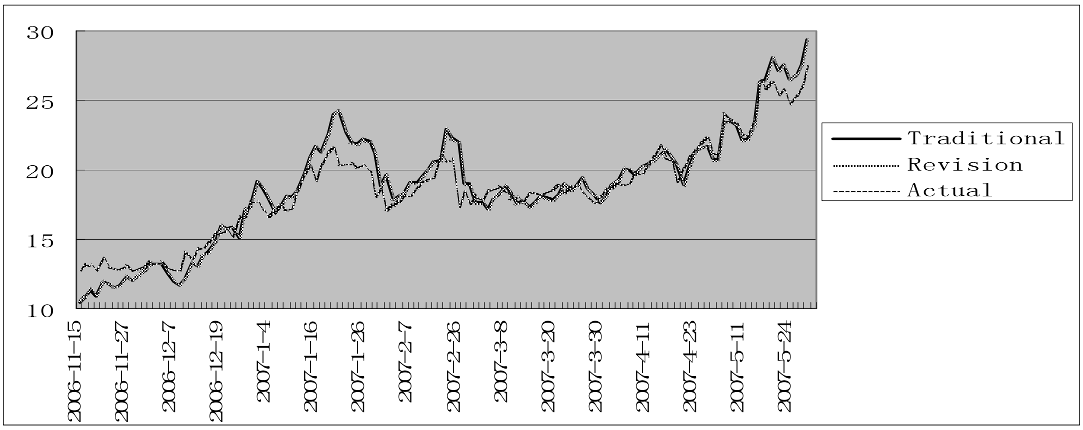

Figure 1. YiLi Ltd actual price and theory price

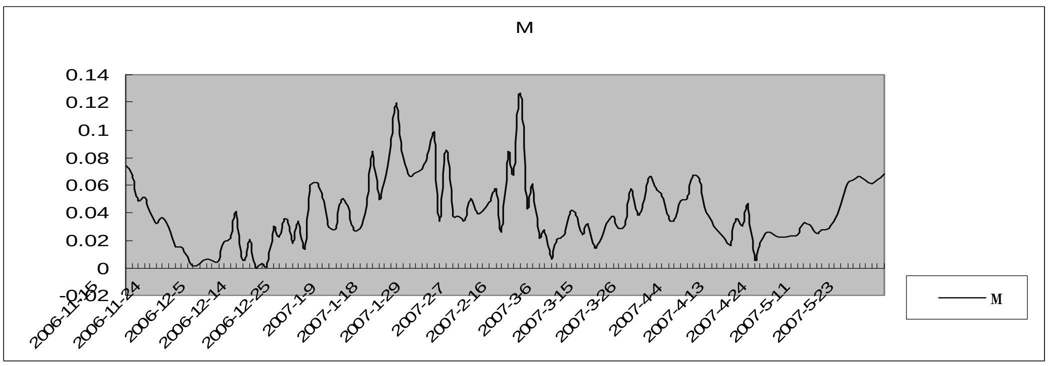

Figure 2. $W_t' - W_t$ and $G_1 - G_2$ differences and similarities

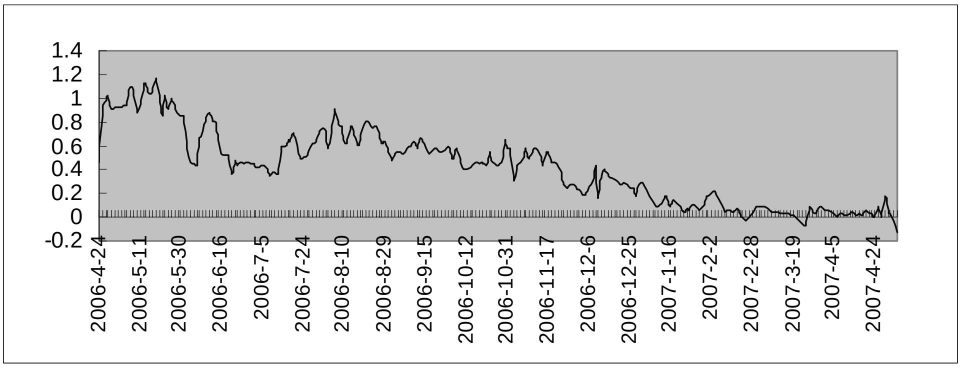

Figure 3. The trends figure of the actual price deviations from the theoretical value