International Education Studies; Vol. 10, No. 5; 2017
ISSN 1913-9020 E-ISSN 1913-9039
Published by Canadian Center of Science and Education

How Do Higher-Education Students Use Their Initial Understanding to Deal with Contextual Logic-Based Problems in Discrete Mathematics?

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Received: November 4, 2016 Accepted: December 19, 2016 Online Published: April 29, 2017

doi:10.5539/ies.v10n5p72 URL: https://doi.org/10.5539/ies.v10n5p72

Abstract

Mathematical reasoning in logical context has now received much attention in the mathematics curriculum documents of many countries, including Indonesia. In Indonesia, students start formally learning about logic when they pursue to senior-high school. Before, they previously have many experiences to deal with logic, but the earlier assignments do not label them as logic. Although the students have already experienced much about logic, it does not assure that they have a better understand about it even they purpose to university. Thus, this paper presents several findings of our small-scale study which was conducted to investigate the issues on how higher-education students overcome contextual logic-based problems. Data were collected through pretest, students' written work, video recording and interview. A fifteen-minute test which consisted of four questions was given to 53 student participants in the third semester who proposed mathematics discrete course. The information towards the main issues was required through the analysis of students' written work in the pretest and video recording during the students' interview. The findings indicate that the students' initial understanding, in general, do not help them much to solve logical problems based on context. In our findings, they apply several strategies, such as random proportions, word descriptions, permutation-combination calculations and deriving conclusion through logical premises.

Keywords: logic, mathematical reasoning, context-based problem, initial understanding, discrete mathematics

1. Introduction

At higher mathematics education, logic is a crucial topic that has received many attentions (e.g. Reeves & Clarke, 2003; Flach, 2007; E. Serna & A. Serna, 2015). By the time the students propose to university, they would already have much experience to deal with logic (Chapell & Overton, 2002), even though many earlier assignments at elementary and secondary level do not assign them as logic. In Indonesian educational system, students formally start learning about logics when they are at senior-high school (Depdiknas, 2006). At this level, they learn about various kinds of logics, such as negation, conjunction and disjunction. In addition, the teaching and learning process only focuses on how to draw conclusion based on two or more premises.

Although many assignments have been addressed to make the students used to deal with logical problems; however, there is now a great deal of empirical evidence that children and adults learners have difficulties in reasoning logically (Markovits & Quinns, 2002, p. 696), even in contextual problems. At the same time, Guerrier et al. (2012, p. 369) pointed out that university and college faculty commonly complain that many tertiary students lack the logical competences to learn advanced mathematics, especially proofs and other mathematical activities that require deductive reasoning. This means that although the students have experienced to deal with logical problems; however, it does not guarantee that they understand the essence of logics (Chapell & Overton, 2002; OECD, 2016, p. 5) and logics is still doubtly difficult to study amongst learners (Guha, 2014).

One feasible reason for this situation is that logic is as rigorous as any other analytical study and it is not easy to convince the students that logic is important (Guha, 2014). This might be consciously considered as the way on how logic should be taught in the classroom (Guallart & Nepomuceno-Fernández, 1998). In classroom activity, teachers usually give the students formal notations and draw conclusions deductively (Chapell & Overton, 1998), without providing them its meaning. Thus, the students tend to memorize and apply those notations in order to deal with logical problems formally.

In this case, we do believe that students' formal mathematics understanding about logic can be increased through instructional activities which are based on the students' initial understanding. They need to acquire a deeper insight towards the general logical relationships before they get more formal lessons. Therefore, the main question about "How do higher-education students use their initial understanding to deal with contextual logic-based problems in discrete mathematics?" is explored in this study.

2. Theoretical Framework

2.1 Logic

Many studies have been conducted to explain what logic essentially means and why it is important to study amongst learners. Although it seems difficult to define principally the definition of logic, but one possible definition could be logic as the foundations of mathematics (Pudlák, 2013, p. 66). Thus, there is no doubt that logic is a crucial element in mathematics. The extreme doctrine from group of logicism claims that all mathematics concepts and rules of reasoning can be deductively interpreted to logic (Hintikka, 2012, p. 460).

Logic is a discipline that is studied both in science and mathematics, one possible way to establish the students' mathematical reasoning is by commencing the teaching and learning process with a contextual problem (Nasution & Lukito, 2015, p. 98), including logic. As a consequence, we need to take into account the context in which it is being taught (Guallart & Nepomuceno-Fernández, 1998, p. 45). In classroom practice, logic should be taught from the basic, thus students know how to understand, how to right construct mathematical argumentations and the importance of logic in understanding mathematical reasoning (Rossen, 2012, p. 1). In this part, we will discuss the basic of logic used in this study.

2.1.1 Proposition

The discussion starts with the introduction of the essential element in logic which is called propositions. The proposition can be fundamentally defined as a declarative sentence that is either true or false, but not both (Rossen, 2012, p. 2). For examples:

1) Jakarta is the capital city of Indonesia
2) $2 + 3 = 10$
3) $x + 6 = 7$

The two above sentences are called propositions, where proposition 1 is true and proposition 2 is false. While the last sentence is not a declarative sentence since it is neither true nor false.

Then, suppose that p is a proposition. The preposition states that "p is not true" or "it is not true that p" is called the negation of p which is usually denoted by $\sim p$. For instance, the statement "It is not true that Jakarta is the capital city of Indonesia" is the negation of "Jakarta is the capital city of Indonesia". In this case, we can conclude that if p is a proposition which is true (T), then $\sim p$ must be a false proposition and vice versa.

2.1.2 Conjunction and Disjunction of Two Propositions

If we have two propositions p and q, then the conjunction of p and q is basically denoted by $p \wedge q$. The conjunction of p and q is true when p and q are true and is false otherwise (Rossen, 2012, p. 4). Meanwhile, the disjunction of the two propositions is essentially symbolized by $p \vee q$. The disjunction of p and q is false when p and q are false; otherwise, it is true.

2.1.3 Conditional Statements

In this part, we would like to discuss some other crucial ways where those propositions can be combined. Suppose that there are two propositions p and q. The conditional statements of p and q is denoted by $p \to q$ which is read "*if p, then q*" or "*p implies q*". The conditional statement of p and q is true when p is true and q is false, and true otherwise. In conditional statement $p \to q$, p is called the hypothesis (*antecedent or premise*) and q is called the conclusion (*consequence*) (Rossen, 2012, p. 6).

2.1.4 Converse, Contraposition and Inverse

Converse, contraposition and inverse are also conditional statements formed by its basic conditional sentence $p \to q$. In particular, the proposition $q \to p$ is called the converse of $p \to q$. The inverse of $p \to q$ is denoted by $\sim p \to \sim q$; meanwhile, the contraposition is defined as $\sim q \to \sim p$. In a truth table, the truth value of $p \to q$ is equivalent with the value of its contraposition (Rossen, 2012, p. 8).

2.2 Logical Reasoning

Reasoning has essentially been a crucial topic in Indonesian educational system, including higher education.

According Sumpter (2016), reasoning can be defined as the line of thought that begins with a task (e.g. exercises, tests) and ends with an answer. Thus, reasoning mathematically can be explicitly termed as the line of mathematical thinking which is started with mathematics tests or exercises and ends with a mathematical answer. In this study, we use the term "*Logical Reasoning*" in order to describe reasoning mathematically on how students at higher education in Indonesia solve logical problems based on context.

Many studies have been conducted in order to develop student's logical reasoning at higher education (i.e. Reeves & Clarke, 2003; Flach, 2007; E. Serna & A. Serna, 2015; Furbach et al., 2015; Nkambou et al., 2015). Higher-education level cognition includes the question-answering ability and logical reasoning with common sense (Furbach et al., 2015). Resoning logically with common sense can be commenced by imbedding a context based problem (Nasution & Lukito, 2015). A good context in mathematics is essentially based on logical reasoning (Bakó, 2002; cited in Liu et al., 2015). The students will automatically express their reasoning if only they recognize the situation and there is also a problem in the context.

2.3 Curriculum in Indonesian Higher Education

Based on the Indonesian curriculum, students will learn about logic when they pursue to senior high school (Depdiknas 2006). At this level, they learn about various kinds of logics, such as negation, conjunction and disjunction. In addition, the teaching and learning process only focuses on how to draw conclusion based on two or more premises.

When the students study at university, they will get deeper understanding about logic. At university they will start relearning about logic at the first semester in set and logic course. In this course they will learn more about logic and try to prove some theorems based on logical statement which is essentially called prepositions. At the second and the third semester, the concept of logic is still used in advance mathematics course, such as discrete mathematics. The standard and basic competence that the students learn during studying logic at discrete mathematics can be seen in Table 1 below.

Table 1. Mathematics curriculum for higher education (Lubis et al., 2016, p. 1)

Standard Competence	Basic Competence
	The principles of making conclusions
	Methods to prove theorems
Proof methods in mathematics	Theorem and quantifiers
	Conjectures
	The principles of mathematical induction

Considering the mathematics curriculum at university level, we can explicitly see that the topic logic is in the first chapter in discrete mathematics course. At discrete mathematics course, logic is a prerequisite topic that needs to be taken care of by the students (Lubis et al., 2016). It is a mandatory knowledge for the students that they should have acquired. This is due to many axioms or theorems in discrete mathematics need to be proved based on logical explanations.

2.4 Hypothetical Learning Trajectory

In carrying out this study, we designed several instructional activities aimed to support the students during the teaching and learning process. In designing an instructional activity, we considered the lecturer's actions and also the students' reactions or other conditions that possibly happened during the implementation process in the real classroom. The hypothesis about the teaching and learning process was embedded in each-day teaching experiment which is usually called a Hypothetical Learning Trajectory (HLT).

According to Bakker and Van Eerde (2013), an HLT is a useful instrument in managing an instructional activity and a teaching experiment. Besides, Simon (1995) pointed out that the HLT consists of three main components, such as *the learning goal* that defines the direction, *the learning activities* and *the hypothetical learning process* – a prediction of how students' thinking and understanding are involved in the scope of learning activities. In this study, the HLT was applied as the guideline to implement the teaching experiments.

3. Methodology

3.1 The Participant

This study was conducted in the third semester at State University of Medan, Indonesia. The university was located at suburb area in Medan. During conducting this study, we involved about 53 students who propose discrete mathematics course. Additionally, the lecturer who implemented the teaching and learning process was also involved.

3.2 Data Collection

To retrieve the impressions about how the students deal with contextual logical problems, different types of data collections were essentially used which were also recognized as "data triangulation". Various data sources were included, such as video recordings, students' written works, students' interviews and field notes. In this study, the video recordings were important to record the whole students' activities along the teaching and learning process. During the process of teaching and learning in the classroom, all the students' works and activities were recorded with the purpose of analysis in the retrospective analysis phase.

The students' written works were collected as the data in order to gain the information about the students' strategies to deal with context-based logical problems. For instance, to describe about the starting point towards what the students had already encountered about logics in the first cycle, a four-question pretest was given to the students. The purpose of the pretest was to discover whether the students had already known what we intended them to learn. Besides, interview was conducted to about ten representative students to ask them to explain the strategies that they did to answer the problems in the pretest and communicated their reasons about how they derived the answers.

3.3 Research Design

The aim of this study is to give contributions and to assemble innovations in teaching logic at Indonesian higher education. In order to answer the research question on how university students in Indonesia use their initial understanding to deal with logical problems in discrete mathematics course, we conducted a developmental research which is essentially called design-based research (Gravenmeijer & Cobb, 2006; Bakker & Van Eerde, 2013; Nasution & Lukito, 2015).

According to Bakker and Van Eerde (2013), design-based research is claimed to have a potential to bridge the gap between the educational practice and the theory in which the design of instructional activities, teaching experiments and retrospective analysis are embedded. In this study, we designed the instructional learning materials based on the contextual and authentic problems as a way in order to encourage the students to recognize the problem situations. Consequently, the problems were unquestionable in their mind.

This study was designed within four phases, namely: (1) preliminary design; (2) pilot experiment; (3) teaching experiment and (4) retrospective analysis. In conducting the phase of preliminary design, the students' learning development was conjectured in order to provide an initial HLT. Some learning activities were embedded with a learning line designed to comprehend the students' understanding about logic. Furthermore, the HLT comprised of the students' starting point, students' thinking and lecturer's reactions during the enactment of teaching and learning. The HLT played as the guideline to implement this study and tested it in the preliminary teaching experiment (the first cycle) within 10 students before conducting it in real classroom. During the retrospective analysis phase, the HLT was adapted to the actual students' learning to investigate how it worked in the classroom. The conjectures about students' learning were adjusted in the first teaching experiment. All the findings and the remarks from this cycle were used as the reflections to modify the initial HLT. Then, the revised HLT was subsequently implemented in the real classroom (the second cycle).

3.4 The Problems

In the pretest, four questions were given to the students. In the pretest, the students were asked to solve the following problem:

1) *Three professors are sitting in a restaurant. Then, the waitress comes and asks them: "Does every one want coffee?", the first professor answers: "I do not know". The second professor says: "I do not know". Finally, the third professor says: "No, not everyone wants coffee". Then, the waitress comes back to the professors' table and gives the coffee to the professors who want it.*
   According to you, is the waiter's action right? How did the hostess figure out who wanted coffee?

2) *Badren wants to give two boxes to his friend. Both of the boxes contain present or empty. The first box was written "At least one of these boxes contains present". The second box was written "The first box is empty". Badren tells his friend that both of the writings are true or false.*
   Based on your opinion, which box should his friend choose?

3) *A country has fifty civil representatives. Each representative is either honest or corrupt. Suppose you know*

that at least one of the senators is honest and that, given any two senators, at least one is corrupt.
Based on these facts, can you determine how many representatives are honest and how many representatives are corrupt? If it is possible, what is the answer?

4) *Mr. Andi would like to determine the relative salaries of his three employers (Richi, Alya and Rizky) by using two facts. First, he knows that if Richi is not the highest paid of the three, then Alya is. Second, he knows that if Alya is not the lowest paid, then Rizky is paid the most.*
Based on your opinion, is it possible to determine the relative salaries of Richi, Alya and Rizky? If possible, order the three workers from the highest to the lowest paid.

Problem 1

The first problem aimed to test the students' ability in noticing and explaining about the "universal quantifier", its negation which is also called the "existential quantifier" (Rossen, 2012, p. 42-40) and vise versa.

To deal with this problem, the students can start from the statement of the third professor. From his statement, we can obviously see that this professor do not want the waitress to serve him coffee since he says "No" and the professor adds "not everyone wants coffee". The proposition "not everyone wants coffee" (existential quantifier) is the negation of "everyone wants coffee" (universal quantifier). In logic, the negation of a universal quantifier can be expressed as follows.

If the universal quantifier is *P(x): Everyone wants coffee*, then the negation of *P(x)* should be:

1) *~P(x): Not everyone wants coffee, or*
2) *~P(x): There exists one professor wants coffee, or*
3) *~P(x): Some professors want coffee*

Based on the above explanation, we can obviously see that the waitress' action to serve the professors coffee is right.

Problem 2

This problem aimed to test the students' ability to better understand about the value of two disjunctive prepositions ($p \vee q$).

From problem 2, we can observe that the first box is written "At least one of these boxes contains present" and it is written "The first box is empty" in the second box. The last statement "Badren tells his friend that both of the writings are true or false" can be the beginning way in order to solve this problem. The yielding explanation about this problem as follows.

P(x): At least one of these boxes contains present

Q(x): The first box is empty

In order to check the value of these two statements, see the *truth table* for disjunction of two prepositions in Table 2 below.

Table 2. The truth table for the disjunction of two prepositions

P(x)	*Q(x)*	*P(x)∨Q(x)*
True	True	True
True	False	True
False	True	True
False	False	False

Based Badren's statement "both of the writings are true or false", we can conclude that the value of both preposition (*P(x)* and *Q(x)*) is true. Thus, his friend should choose the second box.

Problem 3

The third problem was aimed to test the student to recognize the context of existential quantifier and disjunction. In this context, they were asked to determine the number of honest representative and the corrupt ones.

In order to solve this problem, the statement "Suppose you know that at least one of the senators is honest and that, given any two senators, at least one is corrupt" need to be considered. The statement means there exists a member of the senators which is honest. The second statement "given any two senators, at least one is corrupt" can be meant that both corrupt. If one is corrupt one the other one is not, then it will yield 24 corrupt representatives and 25 people which are not known whether they are corrupt.

Among those 25 people, if we apply the same algorithm with the previous. Then we will get 12 corrupt representatives and 13 unknown. Then, 13 representatives become 6 corrupt and 7 unknown. From the seven people, three can be corrupt and leave four undefined representatives. Among the four representatives, there will be two more and one of the two will be corrupt. This will result one honest and one unknown. Since there are only two people, one honest and the other one must be corrupt. Therefore, there is one honest and 49 corrupt representatives.

Problem 4

This problem was presented to investigate the students' understanding about implication form and its contraposition.

In order to solve this problem, the students need to transform the statements into implication forms. Based on the first statement "*If Richi is not the highest paid of the three, then Alya is*", we can get two possible conditions, such as:

Condition 1	Condition 2
The first position = Alya	The first position = Alya
The second position = Richi	The second position = Rizky
The third position = Rizky	The third position = Richi

Then, the second statement says that "*If Alya is not the lowest paid, then Rizky is paid the most*". This implies only one condition, which is:

The first position = Rizky

The second position = Alya

The third position = Richi

Therefore, there is no intersecting condition which implied there is no conclusion can be derived. However, if we observe a truth table, then we will find that an implication ($p \rightarrow q$) is equivalent to its contraposition ($\sim q \rightarrow \sim p$). Thus, we could attempt to overcome this problem by using its contraposition.

The contraposition of the first statement is "*If Alya is not the highest paid of the three, then Richi is*" and the contraposition of the second statement is "*If Rizky is not paid the most, then Alya is the lowest paid*". If Alya is not the highest of the three then she could possibly be in the second or the third position and Richi will be the highest. This means that Rizky is not paid the most. Since Rizky is not paid the most, then the possible position of Alya is the lowest one and Rizky is the middle of the two. Therefore, the order of the three employers from the highest to the lowest paid must be Richi > Rizky > Alya.

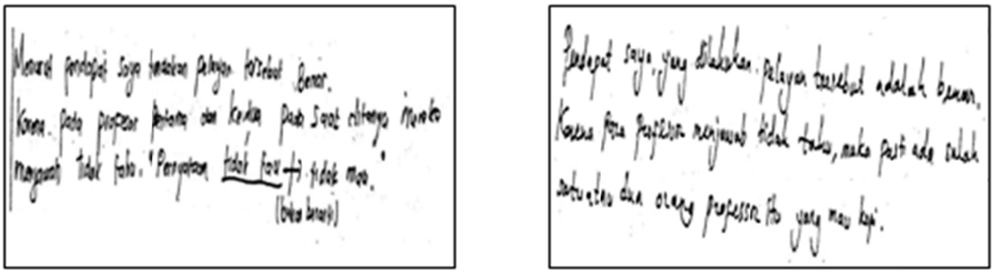
4. Result

Among 53 students involved in the pretest, only a few of them could give correct answers with insufficient explanation. In order to perceive the strategies on how to deal with logical problems based on context, a twenty-minute test at the beginning of this study was conducted. The aim of the pretest was not only to discover their strategies, but also to get the information about their prior knowledge and starting point before conducting classroom implementation. The result of pretest was analyzed in the retrospective analysis phase.

Based on the retrospective analysis of the pretest, the prior knowledge of the students' understanding was discovered. For instance,

Problem 1

In problem 1, some students could determine whether the waitress is right to serve the three professors coffee. However, they did not recognize the element of existential in logic based on the statement of the third professor. They just derive conclusion based on the statement "*I do not know*". The example of this strategy can be seen in Figure 1A and 1B below.



A B

Figure 1. Students' explanation to solve problem 1

Translation of:

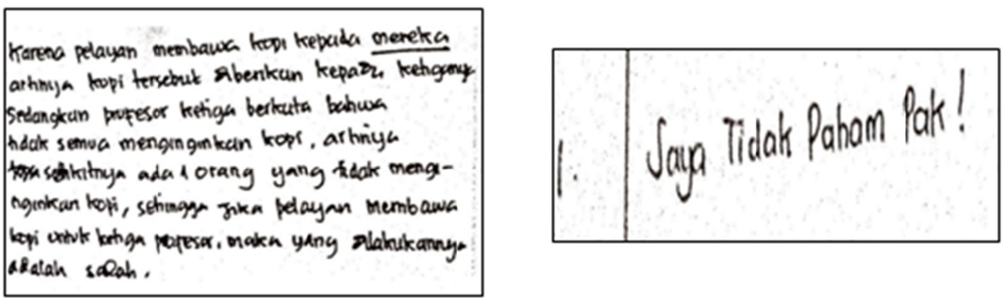
Figure 1A *According to my opinion, the waitress' action is right. Because the first and the second professor answer "I do not know". The statement I do not know does not mean they do not want.*

Figure 1B *According to my opinion, what the waitress has done is right. Because the professors answer "I do not know", then there must be one or two professors want coffee.*

Based on figure 1A, we can explicitly see that both students tried to solve the problem by using what the first two professors say, which is "*I do not know*". Student in figure 1A argued that the statement "*I do not know*" does not mean that the professor does not want to drink coffee. Thus, there must be one or two professors want to drink coffee (Figure 1B). In this case, we did not know why these two students interpret the statement "*I do not know*" to be there exists someone wants coffee since they did not explain them.

In logical description, the preposition "*I do not know*" might have two possible conditions. Firstly, there might be some one or all professors want to drink coffee. Secondly, this statement would possible mean that none of the three professors want to drink coffee. Thus, we cannot derive any conclusions from these two situations.

Furthermore, our pretest-pretest based finding also shows that over 25 students yielded wrong answers because of misconception in logic. In this case, they said the waitress did wrong that she must not serve coffee to the professors. At the same time, the rest of the students leave the question with no answer since they did not understand how to solve it. The examples of such students' written works can be seen in Figure 2A and 2B below.



A B

Figure 2. Students' misconception and misunderstanding

Translation:

Figure 2A *Because the waitress serves coffees to <u>them</u> which mean the coffees were served to the three of them. However, the third professor says that not everyone wants coffee, means at least there is 1 person who does not want coffee, thus if the waitress gives the three professor is wrong.*

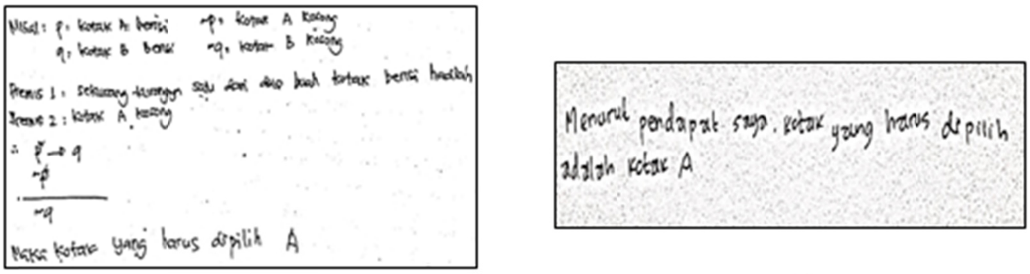
Figure 2B *I do not understand sir!*

In Figure 2A, we can see that the student had already sensed the element of existences in logic since he said that

"… **at least there is 1** person who does not want coffee, …". However, this is still incorrect in order to show the existential quantifier of the universal quantifier. If the universal quantifier "Not everyone wants coffee", then the existential quantifier must be "At least there is 1 person wants coffee" or "Some person want coffee". Furthermore, the student, in figure 2B, did not solve the problem since he/she did not know how to deal with the problem.

Problem 2

According the analysis of the students' written works, almost all students derived incorrect answers and only a few of them could reach the correct answer since there were only two boxes must be chosen. Although a few of them could the correct answer; however, their written works did not show the impression that they really have better understand about this problem. Overall, none of the students could provide right logical explanations which derived a correct answer. In this case, several examples were depicted in Figures 3A and 3B below to show the students' strategies in order to deal with this problem.



A B

Figure 3. Students' strategies to choose box A

Translation of:

Figure 3A *Supposing P: Box A has present* *~P: Box A is empty*

Q: Box B has present *~Q: Box B is empty*

Premise I: At least one of the two boxes has present

Premise II: Box A is empty.

$P \rightarrow Q$

$\underline{\sim P}$

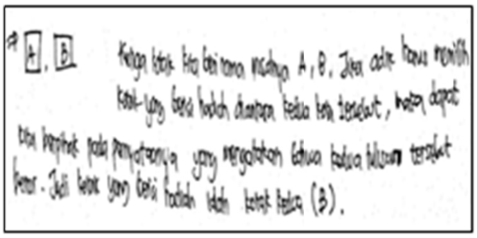
$\therefore\ \sim Q$

Thus, he should choose box A.

Figure 3B *According to me, the chosen box must be box A.*

In Figure 3 A and 3B, it be obviously seen that the answer of both students to choose box A is still incorrect. Student in figure 3A came up with the idea $[(P \rightarrow Q) \wedge \sim P] \rightarrow \sim Q]$ (Rosen, 2012, p. 72). In this case, the student used implication "$P \rightarrow Q$" and latterly concluded "$\sim Q$" which meant that box B is empty, thus box A must be chosen to get the present. Based on that picture, the student did not further explain why he/she used this strategy as this problem essentially did not contain conditional situations. At the same time, student in figure 3B also argued that box A must be chosen. However, the student did not further explain the reason why and how box A should be selected.

Meanwhile, other students came up with different ideas with these students. These students argued that the box B contains present and must be selected. Although they derived correct answer; however, they did not provide sufficient logical explanations. Thus, it does not give us impression that they really have better understand about what we intended them to learn. The examples of these students' works can be seen in Figure 4A and 4B below.



A

B

Figure 4. Students' strategies to choose box B

Translation of:

Figure 4A *Let's say the two boxes with A and B. if his brother must choose a box which contains present, then we can have the statement that says the writing in the two boxes are right. Thus, the box contains present is the second box (B)*

Figure 4B *If the statements are true, then*

	1	*2*
The first box	-	✓
The second box	-	✓

If the statements are false, then

	1	*2*
The first box	✓	✓
The second box	-	-

In conclusion, the most marks are in box 2. Thus, the box must be box 2.

If we reflect on the students' written works in Figure 4A and 4B, then we can see that the student chose box B by assuming the statements "*At least one of these boxes contains present*" and "*The second box was written "The first box is empty*" are true. Since the second box said that box A was empty and his brother should take the second box. These explanations were still insufficient to convince people since there were still other possible conditions, for example both sentences could possibly be wrong, statement one was true and the other one was false and vice versa. Thus, although this student has correct answer; however, the explanations were not sufficient to solve this problem.

In Figure 4B, the student attempted to solve this problem by listing all possible conditions in a table. In the table, the students' interpretation towards the problem situation was still not true. This because the statement in box A means the present could be in box A or in box B. On contrast, the table shows the present must be in box B if the first statement is true. This students' strategy need further explanations about how the student create the table. It was quite difficult for us to understand what the table meant since it lacked of explanations.

Problem 3

Based on the students' written work, most of the students concluded that there were 25 honest representatives and 25 corrupt ones. This case, the students have similar explanations in which interpret the "at least one …" as single. They finally derive 25 corrupt persons since among the given two representatives there is only one corrupt person. The examples of these students' strategies can be explicitly seen in Figures 5A and 5B below.

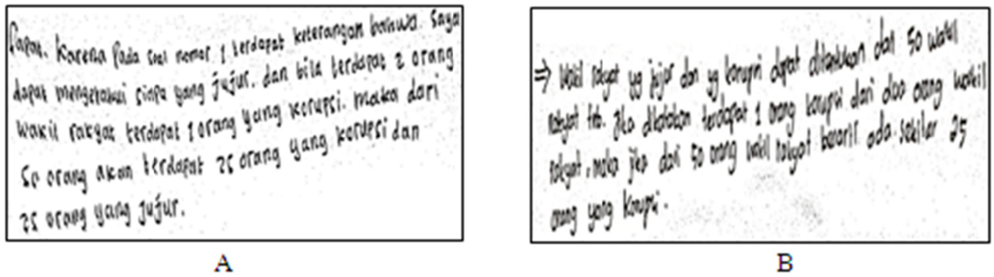


Figure 5. Students' strategies to solve problem 3

Translation of:

Figure 5A We can. Because in problem 1 there is information that I can know who is honest, and if there are 2 civil representatives there will be a corrupt one, then among 50 representatives, there are 25 corrupt and 25 honest people.

Figure 5B *The honest and corrupt representatives can be determined from those 50 people. If there exists one corrupt person among two representatives, then there will be about 25 corrupt within 50 representatives*.

Over half the total students come up with the strategy in Figure 5A and 5B in order to deal with the second problem. Based on Figure 5A and 5B, these students have the similar way of thinking in order to solve this problem. The solutions of these two pupils were still incorrect. They just considered a single corrupt person in any two given person. They did not consider about the other possible conditions if there are one or more corrupt persons within the two people.

Furthermore, some students did random calculations in overcoming this problem. For instance, some students applied the permutation-combination principals and comparisons. Some examples of the students' strategies are provided in Figures 6A, 6B and 6C below to give the impression on how they solve problem 2.

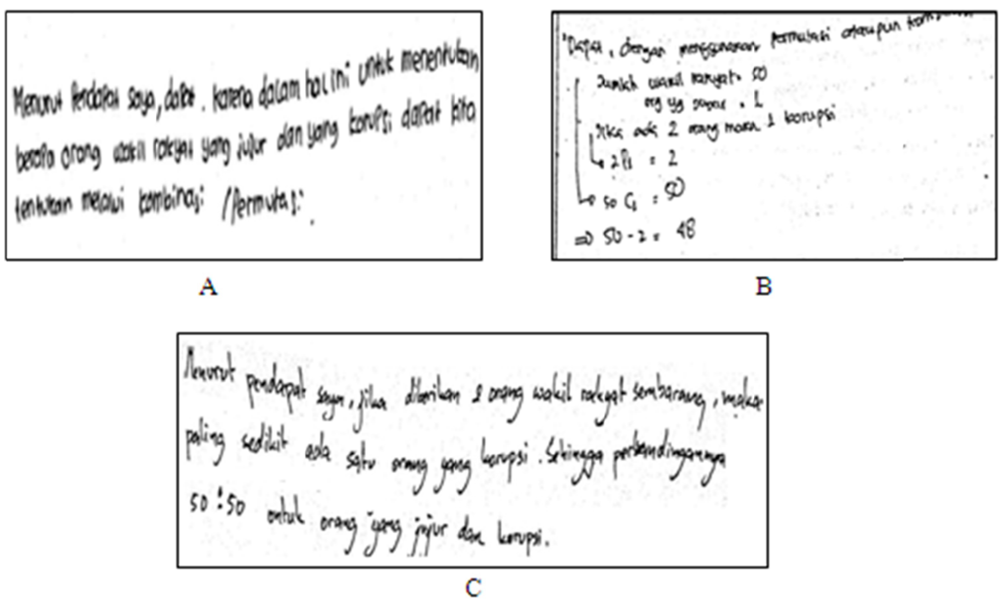


Figure 6. Students' strategies though permutation and combination

Translation of:

Figure 6A *According to my opinion, we can. In this case, we can use determine the number of honest and corrupt civil representatives by using combination / permutation*.

Figure 6B *We can, by applying permutation or combination*.

The number of civil representatives = 50

The number of honest persons = 1

If there are 2 persons and 1 is corrupt, then

$P_1^2 = 2$

$C_1^{50} = 50$

$50 - 2 = 48$

Figure 4C *According to my opinion, if given any two civil representatives, then at least there is one corrupt person. Thus, the comparison is 50 : 50 for the honest and corrupt people.*

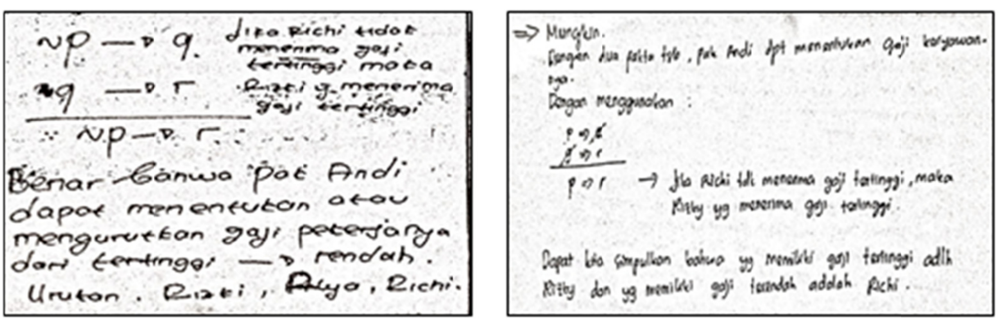
Based on Figure 6A and 6B, we can explicitly see that the two students attempted to solve this problem by using random calculation, such as permutation, combination and even comparison. Student in figure 6A said that the problem can be practically solved by using the principals of permutation or combination. In this case, the student did not give any reason why he/she could overcome the problem with those principals since there were no further explanations about it. Moreover, the strategy reflected by figure 6B shows how the formula of permutation and combination is embedded in this situation. In this case, he/she recognized that the number of civil representatives is 50 and he also knows there is only one honest person. This student straightly calculated $P_1^2 = 2$ since there are 2 persons and 1 is corrupt and also determined $C_1^{50} = 50$. Then, subtracted 2 from 50 and resulted 48. In this case, we did not know what the numbers 2, 50 and 48 mean. This student did not explain why he/she used permutation or combination and the meaning of those numbers.

Based on figure 6C, we can obviously see that the student did comparison strategy. This student considered that there were 50 persons in representatives. Among the 50 persons, there are 25 honest and 25 corrupt individuals. Then, he/she concluded that the comparison of the two sides is 50:50 which showed equality in numbers. However, we did not know why this student did comparison and how it is used in his/her strategy because this student did not explain it.

At the same time, the rest of the students did not answer this problem due to the insufficient information provided in this problem. Thus, they just left this problem with no answers or descriptions.

Problem 4

Aforementioned, this problem was presented to investigate the students understanding about implication form, inverse, converse and contraposition. The retrospective analysis of the fourth problem shows that the students struggled to solve this problem. It can be seen obviously in their written works that they came up with four main different strategies, such as syllogism, listing, drawing picture (diagram) and even no explanation. For further details and explanations, we can see Figures 7A and 7B below.



A B

Figure 7. Students' strategy with syllogism

Figures 7A and 7B show the students' strategy to solve this problem by using the principle of syllogism. These students derived the same conclusion in which Rizky was the one who paid the most and Richi was the least of the three. In this case, this syllogism was still vague where both answers yielded If Richi is not paid the most, then Rizky is. Based on this statement, we cannot straightly put Richi in the lowest position since it is also possible that Richi could be in the second position among the three workers. This this result was still incorrect.

At the same time, there was also student came up with drawing pictures (making diagram) in order to solve this problem. The example of such student's strategy can be seen in the following Figure 8.

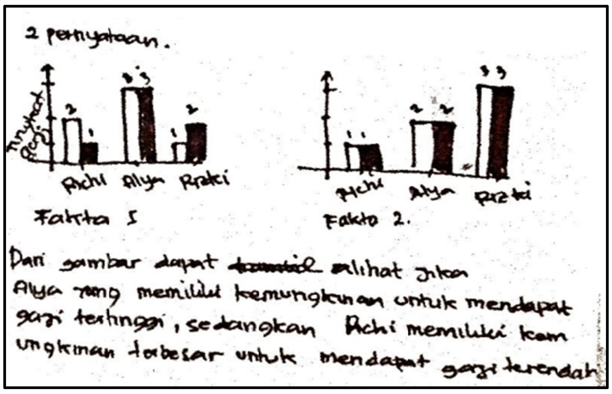


Figure 8. Student's strategy with picture (diagram)

Translation:

Figure 8 *From the figures it can be seen that Alya is probably to get the most paid, while Richi is possible to be the lowest one.*

In Figure 8, we can obviously see that this student drew two pictures (diagrams) in order to rank the three workers (Rizky, Richi, and Alya). We noted that the diagrams consisted of black and white bars. If we interpreted the bars, then the black and white bars would possibly show the possible conditions of the two statements in the problem situation. Based on the first fact in figure 8, we can explicitly see that although Alya is the highest paid of the three however she is not in the second bar (fact II). In this case, we did not know the way how this student concluded that Alya was the highest paid and Richi was the lowest one among the three workers. This is due to the picture lacked of explanations for the student.

While there were some students come up with the ideas of using the principle of syllogism and making diagrams, other students used listing strategy in order to find the answer of the last problem. In this case, the example of such students' strategy can be explicitly seen in Figure 9 below.

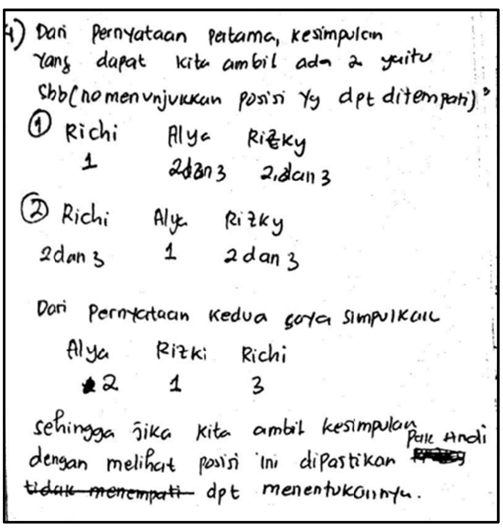


Figure 9. Student's strategy with listing

Translation:

Figure 8	*Based on the first statement, we can conclude 2 possible conditions as follows*		
	1. Richi	*Alya*	*Rizky*
	1	*2 and 3*	*2 and 3*
	2. Richi	*Alya*	*Rizky*
	2 and 3	*1*	*2 and 3*
	Based on the second statement, I can conclude that		
	Alya	*Rizky*	*Richi*
	Thus, if we take a conclusion by using those positions, Mr Andi could determine it.		

Based on figure 8, the student tried to overcome the problem listing the possible conditions for the two statements. Within the first statement, this student concluded that there were two possible situations. Firstly, Richi could be possibly the highest paid of the three workers, meanwhile Alya and Rizky were in the second and the third position. Secondly, Alya was the top person in getting paid. In this case, we did not know how this student concluded that Richi could be the person who earned the highest payment. This student did explicitly explain the reason. In the second statement, this student only resulted that Rizky was the one who got the highest payment and Richi was the lowest.

At the end of the student's worksheet, the student only stated that Mr. Andi could determine the relative salary of the three employers. He/she did not say explicitly which worker was the highest and which one was the lowest for the final answer. This gave us an impression that this students understood the problem well or he/she had no understanding in interpreting logical premises in his/her own words.

5. Conclusion

In order to answer the main research question of this small-scale study, we used the analysis of the data collection, such as students' written work and interview. One possible way to get the students' reasoning on how they solve logical problems is by the commencement of a contextual situation. Starting with contextual situation can make the students recognize the situations where logics are being used. By recognizing the problem, they can possibly reason something by using their own way. Some instances can be derived from the student's case in problem 1 to 4.sccording to

Within the retrospective analysis of students' work, the strategies were discussed. In this case, we can explicitly see that the four questions allow them to reason logically though their own words. In our findings, the students' strategies are mainly words explanation in problem 1, making table for the second problem, using random calculation, such as the principles of combination and permutation strategies and even comparison, to deal with the third problem. The applications of syllogism principle and pictures enactment are also used to overcome the last problem. For example, student in figure 1A tried to solve the problem by using what the first two professors say, which is "*I do not know*". Student in figure 1A argued that the statement "*I do not know*" does not mean that the professor does not want to drink coffee. Thus, there must be one or two professors want to drink coffee (Figure 1B). In this case, we did not know why these two students interpret the statement "*I do not know*" to be there exists someone wants coffee since they did not explain them.

Furthermore, the student in figure 2A had already sensed the element of existences in logic since he said that "… at least there is 1 person who does not want coffee, …". However, this is still incorrect in order to show the existential quantifier of the universal quantifier. If the universal quantifier "Not everyone wants coffee", then the existential quantifier must be "At least there is 1 person wants coffee" or "Some person want coffee". In the third problem, some students did random calculations to determine the solutions of the problem. For example, student in figure 6A said that the problem can be practically solved by using the principals of permutation or combination. In this case, the student did not give any reason why he/she could overcome the problem with those principals since there were no further explanations about it. Moreover, the strategy reflected by figure 6B shows how the formula of permutation and combination is embedded in this situation. In this case, he/she recognized that the number of civil representatives is 50 and he also knows there is only one honest person. This student straightly calculated P_1^2 = 2 since there are 2 persons and 1 is corrupt and also determined C_1^{50}= *50*. Then, subtracted 2 from 50 and resulted 48. In this case, we did not know what the numbers 2, 50 and 48 mean. This student did not explain why he/she used permutation or combination and the meaning of those numbers.

Besides using their own words to discuss, some students, lastly, in figures 7A, 7B and 8 come up with drawing pictures as the model to explain their thinking, listing and drawing conclusion with syllogism. For example, the student, in figure 8, drew two pictures (diagrams) in order to rank the three workers (Rizky, Richi and Alya). We noted that the diagrams consisted of black and white bars. If we interpreted the bars, then the black and white bars would possibly show the possible conditions of the two statements in the problem situation. Based on the first fact in figure 8, we can explicitly see that although Alya is the highest paid of the three however she is not in the second bar (fact II). In this case, we did not know the way how this student concluded that Alya was the highest paid and Richi was the lowest one among the three workers. This is due to the picture lacked of explanations for the student.

6. Recommendation for Future Research

Based on the analysis of the data collection, we may possible conclude that the students have distinctive strategies to solve logic problems with a context. The different way of thinking can be interpreted as they have different levels of initial understanding. The students who could not solve a simple problem, such as conjunction and disjunction, they also will struggle much in determining the solution of more sophisticated problems, such as implication and contraposition. In these findings, we also could see that although some students can determine the correct answer, their reasoning does not reflect that they have better understanding towards the problem situations. Therefore, it is expected that further studies should focus on how to improve the performance and the reasoning of higher-education students in learning contextual logic-based problems.

Moreover, the lecturers' performance in classroom activities plays an important role in order to teach logic at university. The lecturer should be able to handle and to efficiently manage the classroom discussions and to guide the students to the conclusion so that they can derive the right answer. Consequently, future studies should essentially focus on how to support university lecturers in implementing instructional activities in classroom.

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