

Application of Monte Carlo Simulation to Find Travel Time of Groundwater in the Iraqi Western Desert

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Abstract

In this study, a computerized mathematical method represented by Monte Carlo simulation was used to predict the travel time of the groundwater flow in the Iraqi western desert. During the run of the simulations, all the hydraulic parameters of Darcy's Law were fixed but the hydraulic conductivity. The input data of the hydraulic conductivity is compared to the triangular distribution function to find the best number of iteration to run the simulations. The results showed that an iteration number of 5000 was enough to achieve best match between the input data of the hydraulic conductivity and the fitted distribution function. In addition, the estimated travel time of the groundwater flow is broadly varied through the entire area and ranges from 1983 years to 113 741 years based on 10 000 m of travel distance. Furthermore, hydraulic conductivity of the aquifer has high impact on the estimated travel time of the groundwater flow. However, head difference of groundwater elevation among the selected wells considerably influences the expected travel time of the groundwater flow.

Keywords: Groundwater flow; Hydraulic conductivity; Monte Carlo simulation; Probability; Travel time.

1. Introduction

Increasingly, reliable predictions of groundwater flow and other hydraulic properties associated with uncertainties have been widely sought. Hence, various stochastic models to evaluate the uncertainty of groundwater problems have been developed. Uncertainty is defined as a situation of having lack of confidence in future outcomes as a result of unknown or inadequate input variables (Singh, Jain, & Tyagi, 2007). In general, there are two types of method to analyze the uncertainty; analytical which uses models to propagate the uncertainty to estimate the probability distributions and empirical which uses computer simulation experiments to generate the distributions of the outputs (Doctor, Jacobson, & Buchanan, 1988). Uncertainty of a system can be caused by natural activities such as those of environmental, hydrological, metrological or other natural processes which cause significant fluctuations in the space and time. In addition, inadequate data for a number of parameters which are required for risk analysis is a major source of the uncertainty in the system (Singh et al., 2007). To find or predict groundwater flow of an aquifer, it is necessary to identify the hydraulic properties of this aquifer. The parameters such as hydraulic conductivity or transmissivity, dispersivity, porosity, and hydraulic gradient, which influence the groundwater flow and transport, are taken into account as the main variables in constructing a stochastic model. However, lack of data of the hydraulic properties as a result of the heterogeneity of the natural porous media leads to the uncertainty of the groundwater flow problems (Neshat, Pradhan, & Javadi, 2015). Additionally, it is well known that such parameters are characterized by spatial distribution throughout the aquifer and their values are randomly varied according to the point measurements (Crestani, Camporese, & Salandin, 2015; Hassan, Bekhit, & Chapman, 2009; Zhang & San, 2000; Hoeksema & Clapp, 1990). To produce a stochastic travel time of groundwater in a natural aquifer, a differential form of Darcy's law which is assumed to be valid for most groundwater flow is used (Zhang, Shi, Chang, & Yang, 2010; Fitts, 2002; Zhang & San, 2000). According to Darcy's law, travel time of groundwater flow is a function of travel distance, porosity, retardation factor, gradient, and hydraulic conductivity. Application of Darcy's law in groundwater flow problems and its advantages and limitations is well elaborated in literature.

Monte Carlo simulation technique is the most general method that deals with uncertainty analyses of complex systems to obtain numerical results (Singh et al., 2007; Li, McLaughlin, & Liao, 2003). The simulation of the

random samples mostly lies within the range of the input distribution. Monte Carlo analysis involves different types of distribution functions to generate a large number of random samples (Doctor et al., 1988). Some of the probability distributions are Triangular, Log Logistic, Beta General, Pearson5, Inve Gauss, and Log normal. In each distribution function, data can be simulated in different iterations up to 10 000 iterations. Several researchers reported different numbers of iterations to represent the sample population. For example, Driels & Shin (2004) indicated that running the simulation for 4684 iterations is accurate to 5% and gives a confidence interval of 95% for weapon effectiveness. Moreover, the number of the simulations also can be changed which results in a higher probability. The simulation is stopped when the desired confidence interval with a specific error has been achieved to produce a clear statistical description of dependent variables. However, using a larger number of simulations to achieve higher accuracy by generating more random numbers and increasing the sample size will generally increase the time consumed by the computer to finish the simulated run, which leads to high costs in complicated transient systems (Pasetto, Guadagnini, & Putti, 2011; Singh et al., 2007). The results of the simulated data can be compared in many ways, one of which is by using fit comparison to the input or output data. A fit comparison graph (density or cumulative curves) combines both the input data and the fitted distribution in one graph in order to determine the location(s) of the best match area of the fitted distribution to the input data. A higher number of iterations gives a closer fit to the data (Palisade Corporation, 2008). The number of simulations can be fixed whenever the difference between the results of two or more simulations becomes insignificant.

Over the years, application of Monte Carlo simulation methods to estimate parameter uncertainty in groundwater hydrology has been widely used. Hoeksema and Clapp (1990), for instance, applied a Monte Carlo simulation to perform model calibration of a groundwater flow. Fu and Gómez-Hernández (2009), in another study, used Monte Carlo Markov chains to assess the uncertainty of groundwater flow and mass transport of a synthetic aquifer with three variables; conductivity, piezometric head, and travel time. They found that all three parameters impacted the uncertainty, and the piezometric head had the highest effect on uncertainty reduction. Similarly, a Monte Carlo Markov Chain method was also used by Hassan et al. (2009) to reduce the range of the input parameter distributions of a two dimensional groundwater flow model. For more studies see Zhang et al. (2010), Hassan et al. (2009), and Pasetto et al. (2011).

In this study, a computerized mathematical method represented by the Monte Carlo technique is used to estimate the likelihood of the travel time of the groundwater flow in several regions of the Iraqi western desert. The simulation included a sampling of hydraulic conductivity of the aquifers in the considered area.

2. Description of Study Area

The boundary of the considered area in this study is the Euphrates River from the east and north east and Iraq's border with Syria, Jordan, Saudi Arabia, and Kuwait from west and south as shown in Figure (1). The Iraqi western desert has several unconfined aquifers which are hydraulically connected to each other as well as some confined aquifers (Al-Fatlawi & Jawad, 2011). The acceptable quality of the groundwater which is easy to access due to its closeness to the surface of the earth makes these aquifers a potential good source of water in this area (Al-Mussawi, 2014). Topographically, the north part of the area which is mostly covered with sandstone, clastic, and marl has a slope of about 0.002 from west to east. However, the southern part of the area which is generally covered with residual soil with calcium carbonate and chert has a lower slope toward the east. Geologically, limestone, dolomitic limestone, dolostone, marly limestone, sandstone, siltstone, claystone, marl, and evaporate are the most common rocks of the aquifers in the region of the Iraqi western desert (Al-Jiburi & Al-Basrawi, 2009; Al-Jiburi & Al-Basrawi, 2007). The rocks' diverse geological formations in the aquifers resulted in a wide range in the hydrogeological properties and hence uncertain hydraulic conductivity throughout this region. Therefore, the hydraulic conductivity of the aquifers in the Iraqi western desert has a broad range which is between 0.03 m/day and 100 m/day. The static groundwater level ranges from several meters to more than 300 m below the ground surface. The main source of the recharge to these aquifers is rainfall. The flow of the groundwater in the designated area is in the direction of the east and northeast (from west and southwest towards the Euphrates River). Additional information on the studied area is described elsewhere (Al-Jiburi & Al-Basrawi, 2009; Al-Jiburi & Al-Basrawi, 2007).

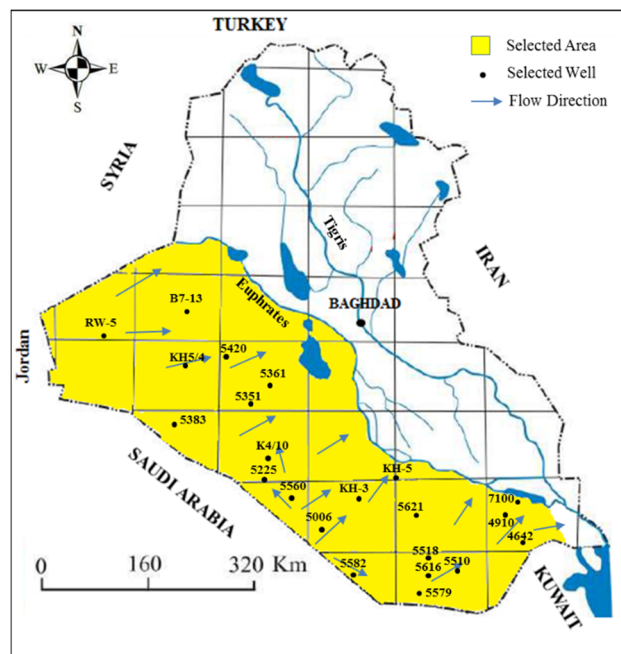


Figure 1. Location of the Considered Area (Adopted from Al-Jiburi & Al-Basrawi (2009) and Al-Jiburi & Al-Basrawi (2007))

3. Methodology

The available data of several wells located in the Iraqi western desert which are designated by the yellow color in Fig. 1 and listed in Table (1) as well as many existing wells in this region, which are described elsewhere (Al-Jiburi & Al-Basrawi, 2009; Al-Jiburi & Al-Basrawi, 2007), were used to run a computerized mathematical method represented by Monte Carlo simulation to estimate the travel time of the groundwater in this area. The locations of the selected wells represent nearly all the area of the Iraqi western desert. As previously mentioned, according to Darcy's law, travel time (T) is the function of the distance (L) for groundwater to travel, hydraulic gradient (dh/dl), porosity, and hydraulic conductivity (K), and is expressed in the following formula (Todd & Mays, 2005):

$$T = -\frac{L}{(K \cdot \frac{dh}{dl})} \quad (1)$$

Table 1. Selected Wells to Estimate Time of Travel for Groundwater in the Iraqi Western Desert

Well No.	Ground surface above sea level (m)	Static water level below ground surfaces (m)	Hydraulic conductivity, K (m/day)
RW-5	622.1	25.7	0.03
B7-13	385.7	205.3	19.4
5383	490.4	296.4	9.2
5351	280.4	90.9	0.7
5383	490.4	296.4	9.2
5361	199.2	103	12
KH5/4	559	314	0.4
5420	430	232	13.1
5225	308	94.0	0.4
K4/10	315	104.3	100
5560	205	7.5	30.9
KH-3	198	6.5	7.3
5006	380	68.1	4.9
5621	170	88	1.6
5579	323	88.2	2.4
5510	200	136	2.0
5616	200	57.0	7.2
4910	10	8.2	19.0

In addition, it is well-known that the most affected parameter by the wide diversity of the natural lithologic material formation is the hydraulic conductivity the values of which cover an extensive range, even with very small hydrologic units. Therefore, in this study, all the parameters of Darcy's Law apart from the hydraulic conductivity were fixed. Again, Monte Carlo simulation can evaluate models with iterations up to 10 000 and it is used to analyze the uncertain propagation of a model. The simulation involved the use of triangular distribution function to form the statistical analysis of the hydraulic conductivity. Triangular distribution is an easy probability function that is used in hydrologic and environmental engineering applications (Singh et al., 2007). To use triangular distribution function, three values, minimum, most likely, and maximum, of the hydraulic conductivity are needed. The hydraulic conductivity of all wells close to the selected wells in Figure (1) was considered in choosing the most likely value of the hydraulic conductivity to run the simulation. For example, wells 5818, 5875, 5987, 5994, 6845, 7055, B4/7, B13/7, BH-5, BH-16, KH5/1, KH5/2, KH5/4, KH5/5, KH5/6, KH5/8, KH7/7, KH9/7, KH12/7, and RW-3 which are reported by Al-Jiburi & Al-Basrawi (2009), and Al-Jiburi & Al-Basrawi (2007) were used to find the minimum, most likely, and maximum of the hydraulic conductivity in the area around and between wells (RW-5) and (B7-13). The other values of the minimum, most likely, and maximum of the hydraulic conductivity as well as the head difference, length (L), and hydraulic gradient (dh/dl) of each selected pair of wells are listed in Table (2) and were used to run the software of the Monte Carlo simulation to predict the travel time of the groundwater. Equation (1) which is used in Monte Carlo simulation to predict travel time of groundwater flow is expressed as follows:

$$Riskoutput("Travel Time, T") + \frac{L}{(Risktriang(minimum, most likely, maximum) * \frac{dh}{dl})} \quad (2)$$

At the beginning, two different wells (RW-5) and (B7-13) were used to run several simulations by using different numbers of iterations to achieve smooth hydraulic conductivity and determine the best number for further simulations. A fit comparison graph for each run was used to determine the best match between the input data and the fitted distribution. If the best match is around the mean or along with tails, good results are most likely achieved (Palisade Corporation, 2008). The best number of iterations was used to assess the travel time of the groundwater for all selected pairs of wells.

Table 2. Hydraulic Parameters of the Selected Pairs of Wells Used in the Monte Carlo Simulation

Well pairs	Hydraulic conductivity, K (m/day)			Head difference (m)	Length, L (m)	Hydraulic gradient, (dh/dl)
	Minimum	Most likely	Maximum			
RW-5	0.03	1	19.4	416	134 000	0.003 10
B7-13						
5383	0.7	6.5	15.3	4.5	109 670	0.000 04
5351						
5383	0.5	5	15.3	97.8	139 000	0.0007
5361						
KH5/4	0.4	5.38	13.1	47	59 355	0.000 79
5420						
5225	0.3	19.5	100	3.3	25 000	0.000 13
K4/10						
5560	0.3	4.5	30.9	6.0	87 500	0.000 07
KH-3						
5006	0.3	1.73	7.3	229.9	116 670	0.00197
5621						
5579	0.1	2.9	17.9	170.8	54 170	0.003 15
5510						
5616	0.1	2.3	22.3	141.2	127 080	0.001 11
4910						

4. Results and Discussion

As previously mentioned, wells (RW-5) and (B7-13) were used to run several simulations with different numbers of iterations to determine the best number that can be used for further simulations. Histograms of

Figures 2, 3, 4, 5, 6, and 7 show the simulations that have been run with iteration number of 100, 500, 1000, 2000, 5000, and 10 000, respectively, to estimate the hydraulic conductivity between well (RW-50) and well (B7-13). The simulations, as previously mentioned, involved the use of the triangular function as a fitted distribution to compare it to the input data of the hydraulic conductivity. However, in Monte Carlo simulation, travel time within given hydrologic parameters most likely follows a lognormal distribution (Doctor et al., 1988). Therefore, lognormal density was used as a fitted distribution to compare it to the input data of the travel time. Histograms of Figures 2 through 6 explained that when the number of iteration was gradually increased, an obvious increase in the match between the input data of the hydraulic conductivity and the triangular distribution function was observed and the best fit was achieved at an iteration number of 5000. However, increasing the number of iteration to 10 000 showed no significant difference to the 5000 iterations as shown in Figure 7 and compared to Figure 6. As a result, a number of 5000 iterations was used to run all the simulations to estimate the likelihood of the travel time of the groundwater flow of each selected pair of wells. Figures A1 through A8 (Appendix A) display fit comparisons between the input data of the hydraulic conductivity and the triangular distribution function for the selected pairs of the wells which are listed in Table (2).

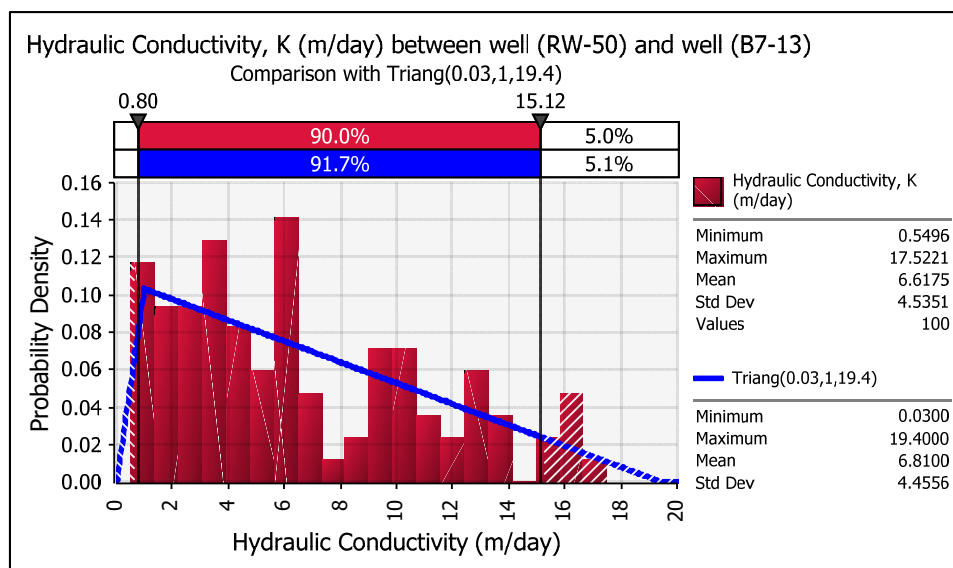


Figure 2. Hydraulic Conductivity between Well (RW-50) and Well (B7-13) with an Iteration Number of 100

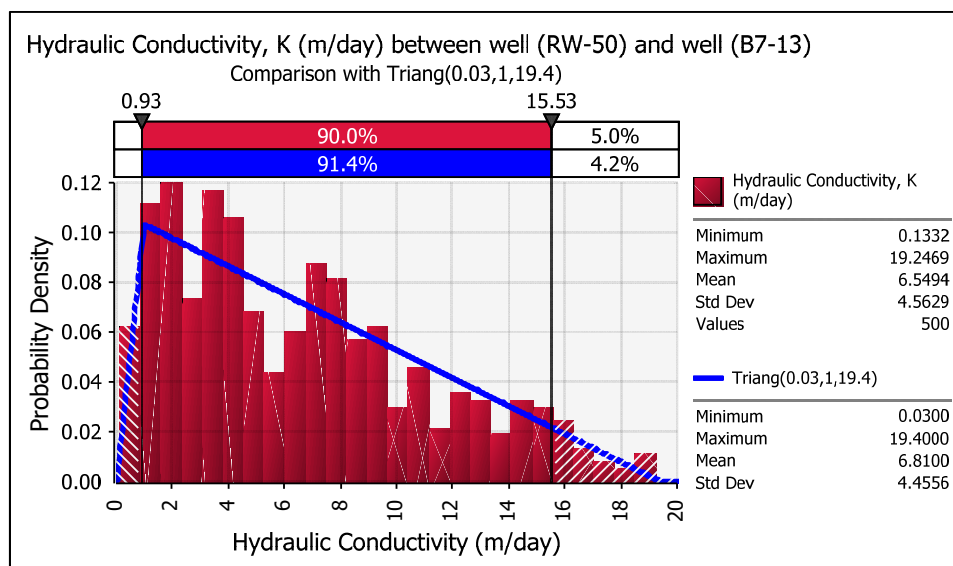


Figure 3. Hydraulic Conductivity between Well (RW-50) and Well (B7-13) with an Iteration Number of 500

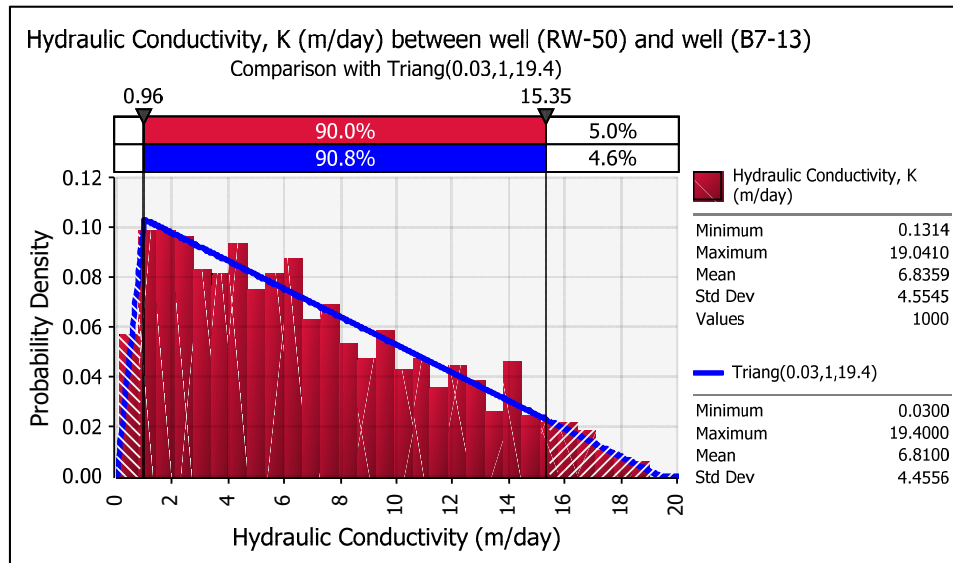


Figure 4. Hydraulic Conductivity between Well (RW-50) and Well (B7-13) with an Iteration Number of 1000

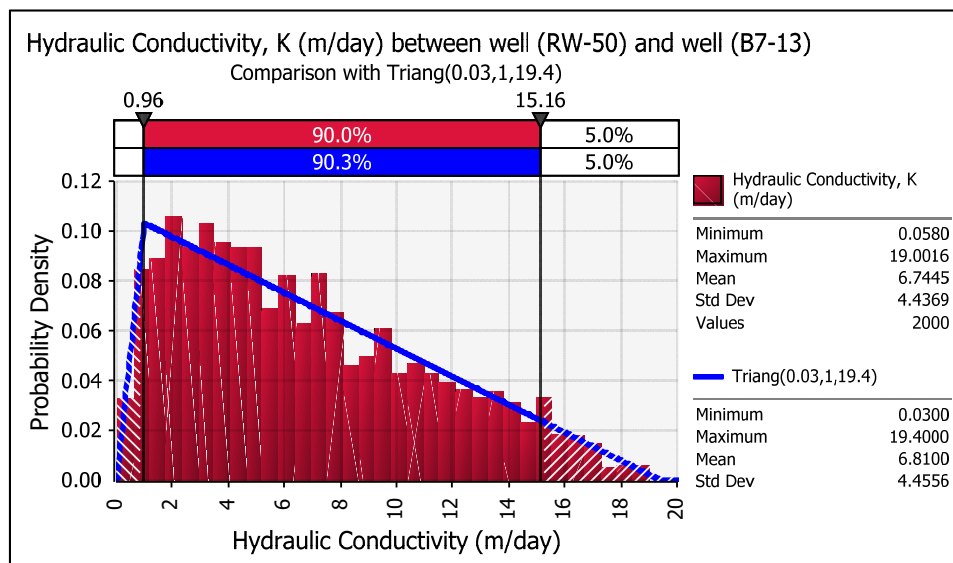


Figure 5. Hydraulic Conductivity between Well (RW-50) and Well (B7-13) with an Iteration Number of 2000

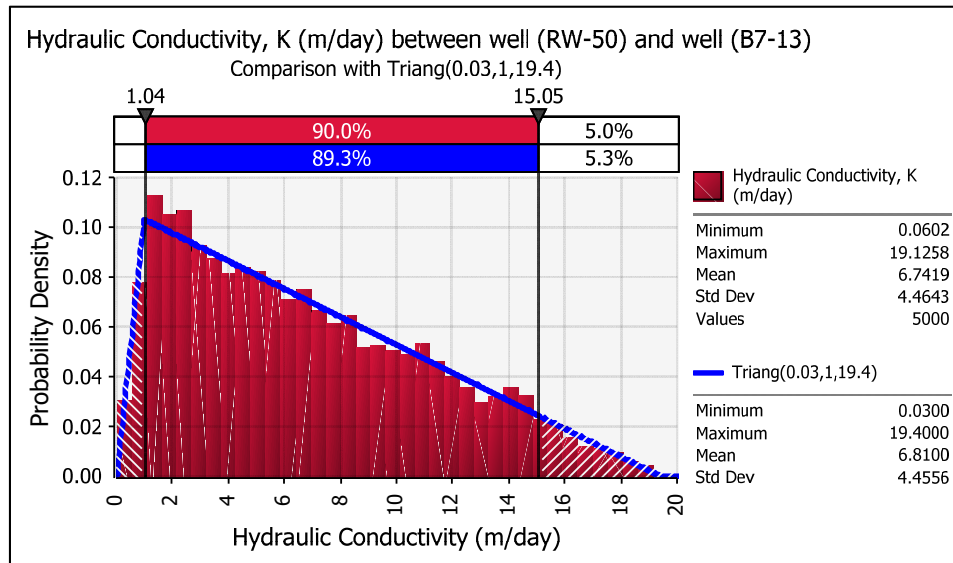


Figure 6. Hydraulic Conductivity between Well (RW-50) and Well (B7-13) with an Iteration Number of 5000

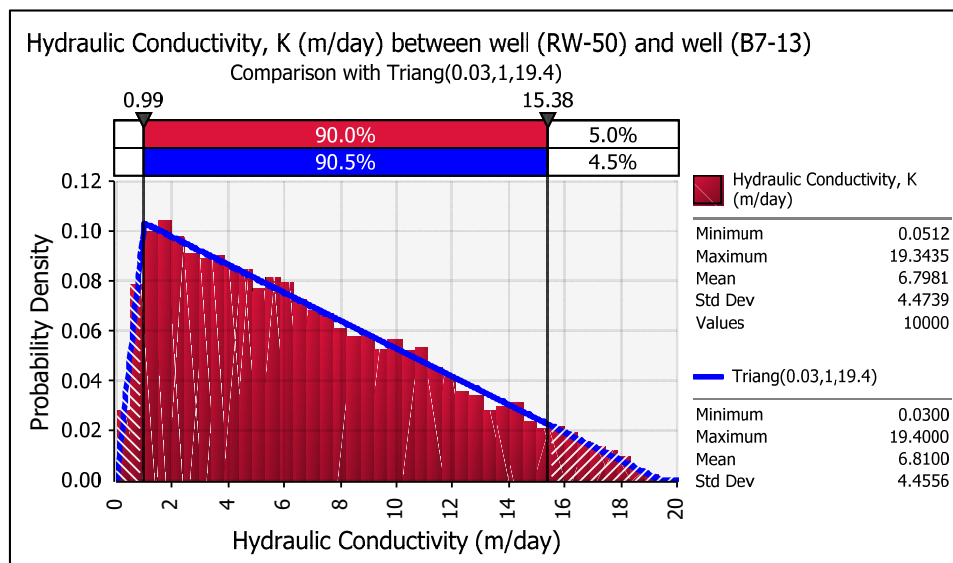


Figure 7. Hydraulic Conductivity between Well (RW-50) and Well (B7-13) with an Iteration Number of 10 000

Figure 8 displays probability distribution of the stochastic travel time of groundwater flow between well (RW-50) and well (B7-13) with an iteration number of 5000. The probability distribution shows that 90% of the relative likelihood of occurrence for estimated travel time of groundwater flow falls within the range of 1560 422 days (4275.129 years) to 410 633 898 days (1125 024 years) with a mean of 12 293 946 days (33 682.04 years) through a travel distance of 134 000 m. For the other selected pairs of wells in the Iraqi western desert, ranges and likelihoods of occurrence for travel time of the groundwater flow are shown in Figures 10 through 17 and the results are summarized in Table (3). The units of the estimated travel time shown in Table (3) which are expressed in days were converted to years to make it easier in the comparison. In addition, a fit comparison graph to determine the best match between the input data of travel time (between well (RW-50) and well (B7-13)) and lognormal density as a fitted distribution is illustrated in Figure 9. Furthermore, Figures B1 through B8 (Appendix B) display fit comparisons for the selected pairs of the wells which are listed in Table (2) and were used to run the software of the Monte Carlo simulation.

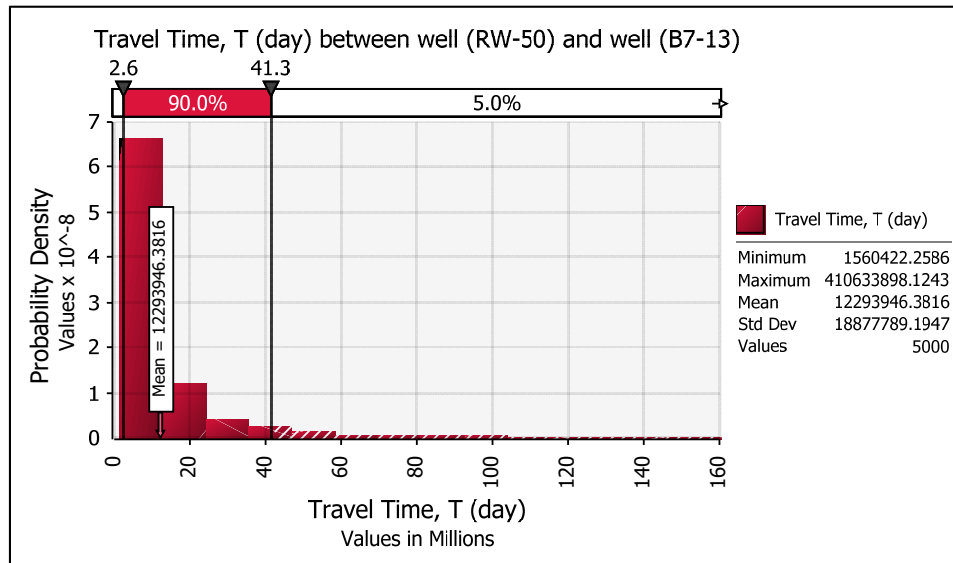


Figure 8. Probability Distribution of Travel Time of Groundwater Flow between Well (RW-50) and Well (B7-13) with an Iteration Number Of 5000

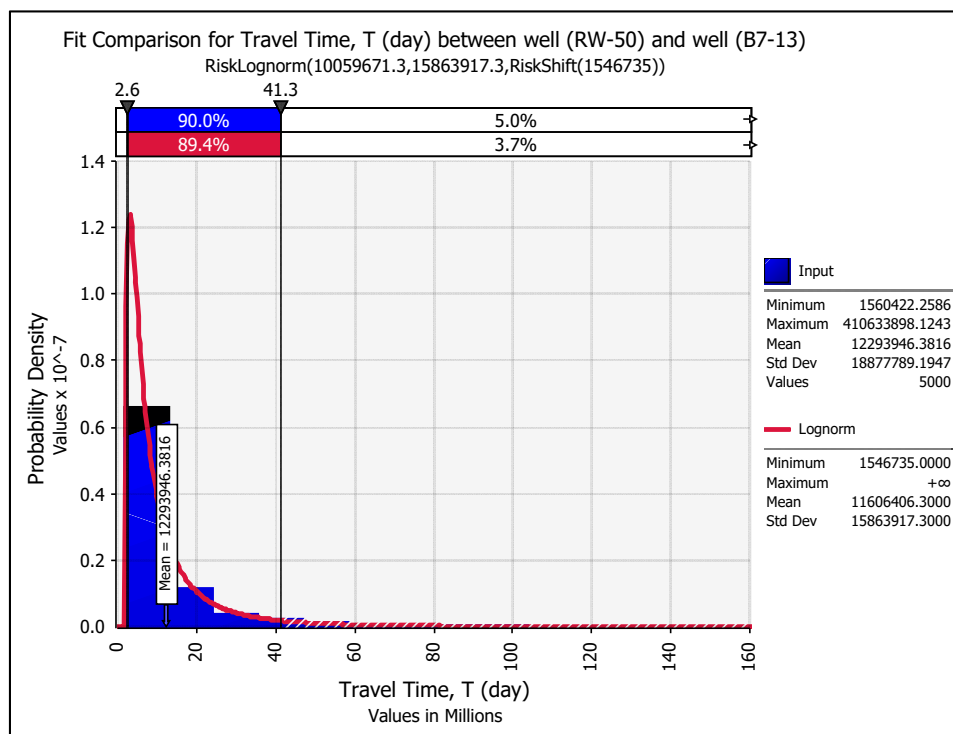


Figure 9. Fit Comparison for Travel Time between Well (RW-50) and Well (B7-13)

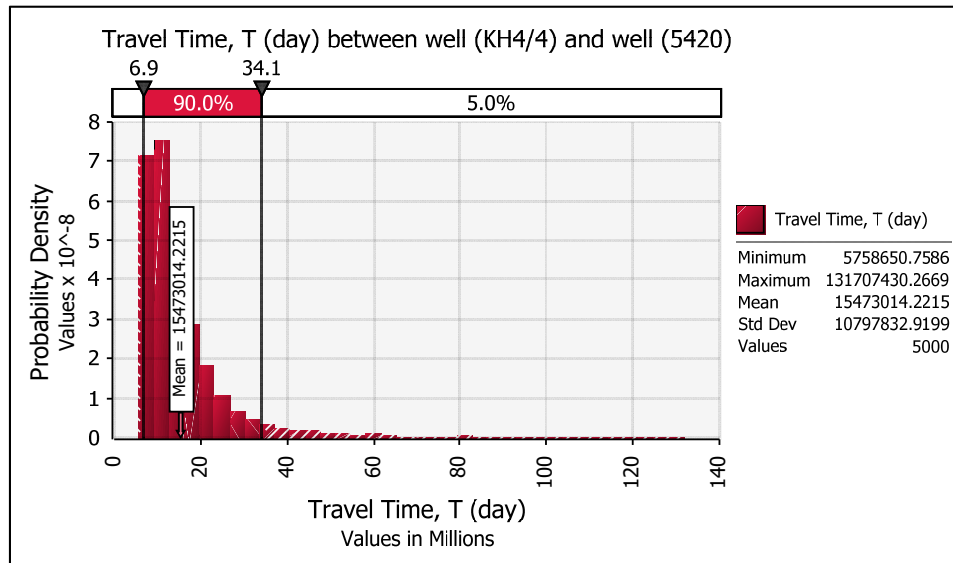


Figure 10. Probability Distribution of Travel Time of Groundwater Flow between Well (KH4/4) and Well (5420) with an Iteration Number of 5000

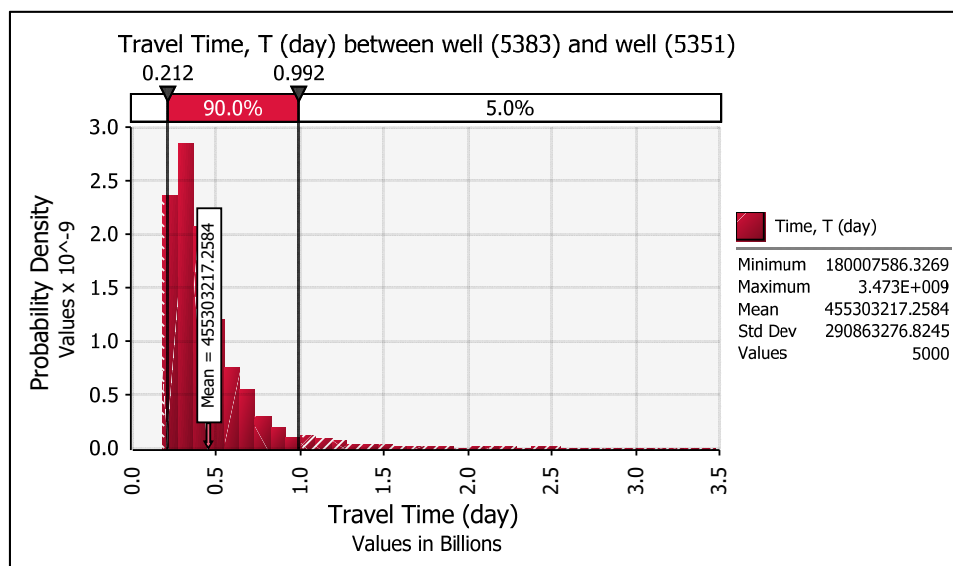


Figure 11. Probability Distribution of Travel Time of Groundwater Flow between Well (5383) and Well (5351) with an Iteration Number of 5000

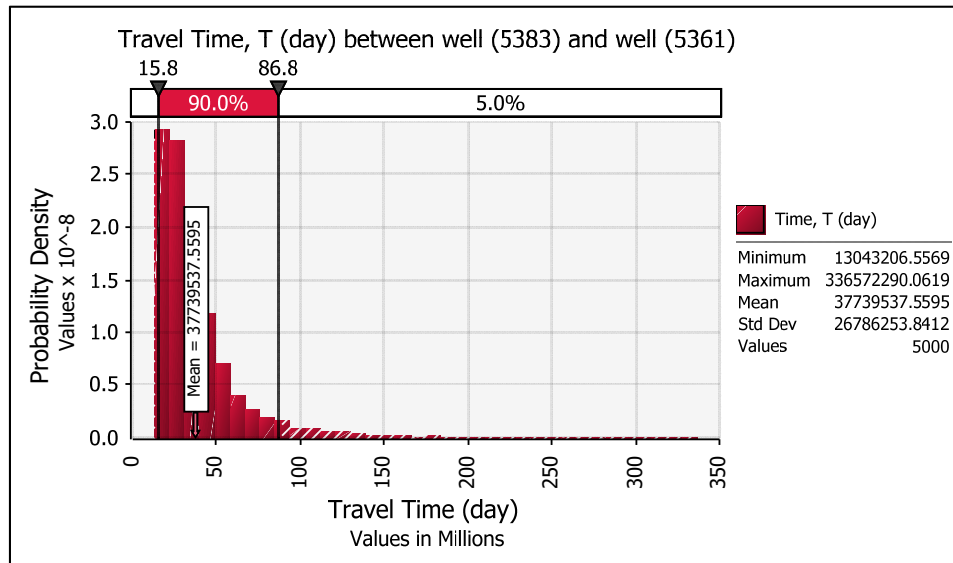


Figure 12. Probability Distribution of Travel Time of Groundwater Flow between Well (5383) and Well (5361) with an Iteration Number of 5000

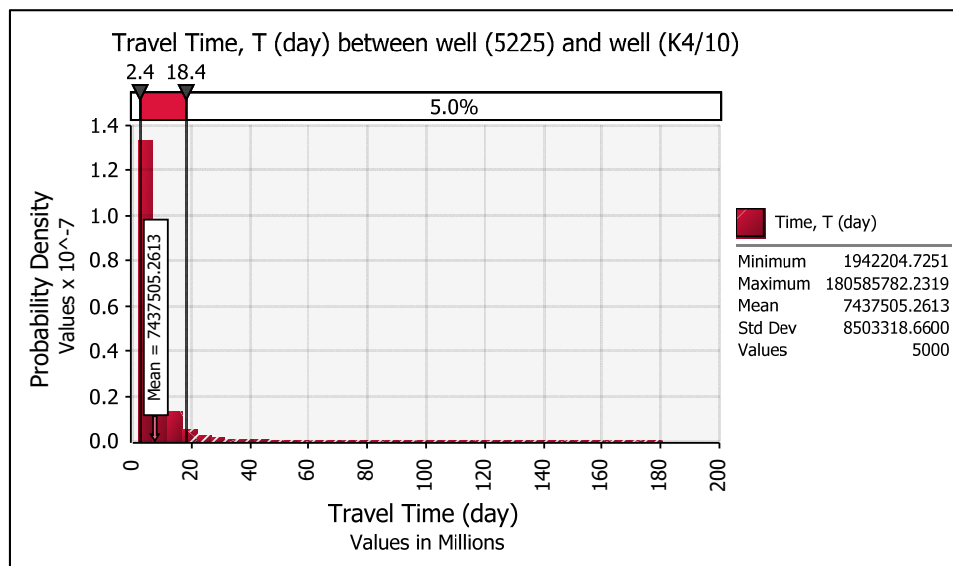


Figure 13. Probability Distribution of Travel Time of Groundwater Flow between Well (5225) and Well (K4/10) with an Iteration Number of 5000

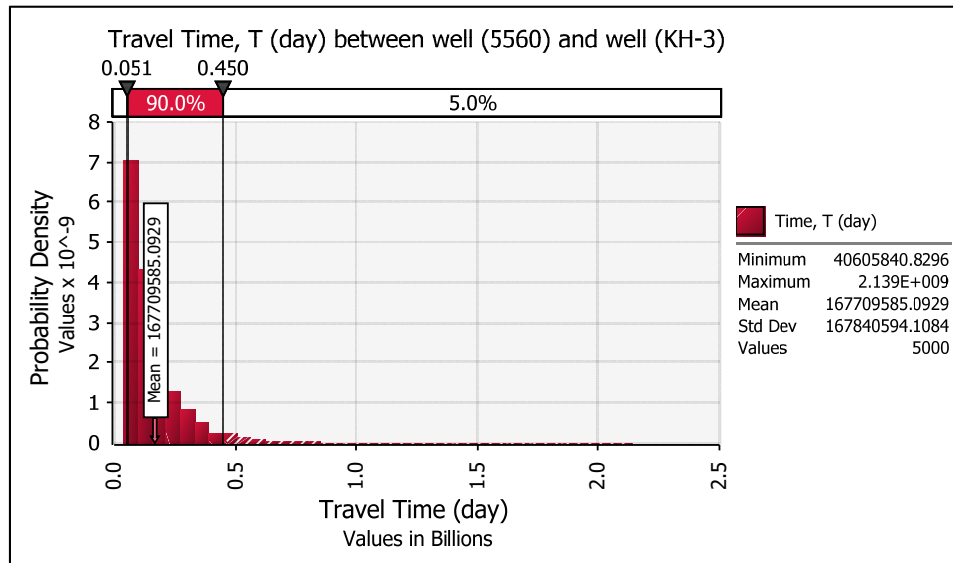


Figure 14. Probability Distribution of Travel Time of Groundwater Flow between Well (5560) and Well (KH-3) with an Iteration Number of 5000

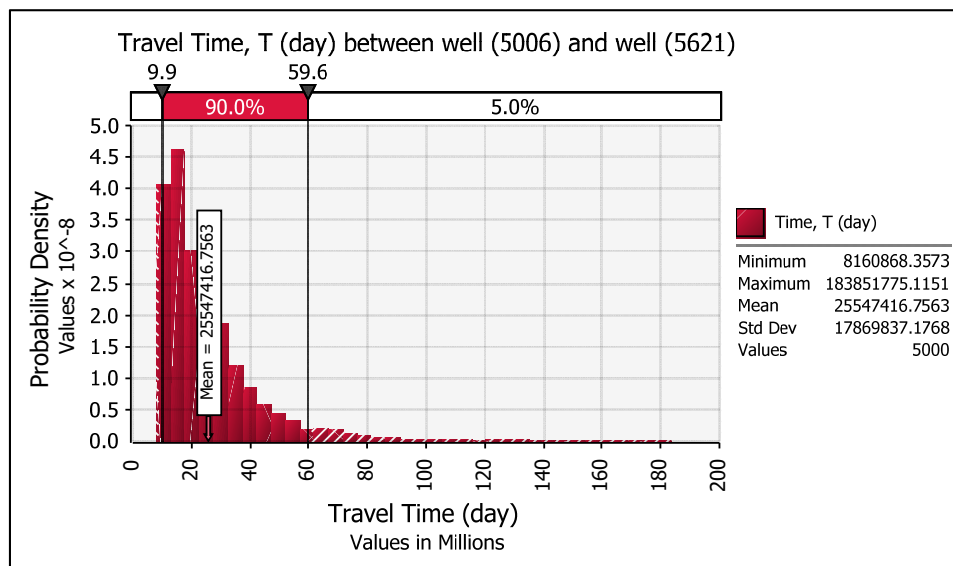


Figure 15. Probability Distribution of Travel Time of Groundwater Flow between Well (5006) and Well (5621) with an Iteration Number of 5000

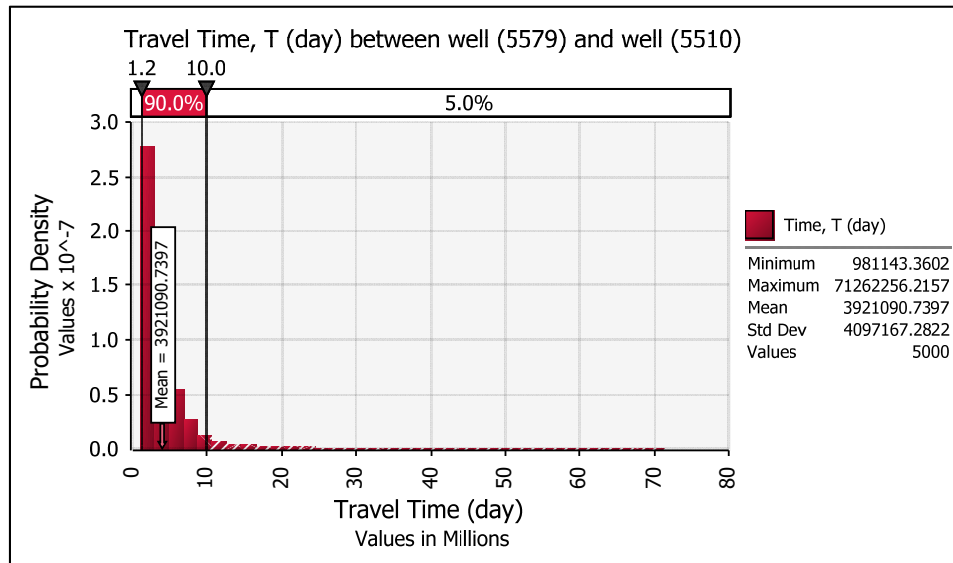


Figure 16. Probability Distribution of Travel Time of Groundwater Flow between Well (5579) and Well (5510) with an Iteration Number of 5000

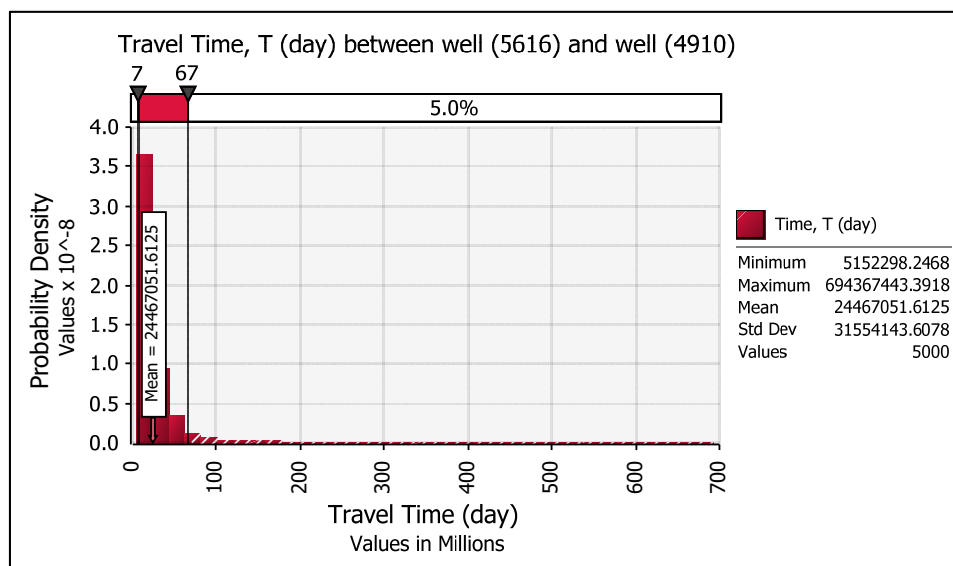


Figure 17. Probability Distribution of Travel Time of Groundwater Flow between Well (5616) and Well (4910) with an Iteration Number of 5000

Table (3) explains that the predicted travel time of the groundwater flow is varied through the entire Iraqi western desert which is the result of the wide range in hydrogeological properties of the geological formation in the aquifers. For example, the estimated mean travel time of the groundwater flow of the north part between well (RW-50) and well (B7-13) (which is 33 682 years with travel distance of about 134 000 m) is about half of the estimated mean travel time of the southern part between well (5616) and well (4910) (which is 67 033 years with travel distance of about 127 080 m). This is attributed to the higher values of the hydraulic conductivity of the geological formation of the southern part of the selected area. Moreover, although the travel distance between well (5383) and well (5361) which is 139 000 m is higher than that between well (5383) and well (5351) which is 109 670 m, the estimated travel time of the groundwater flow between the first two wells (103 396 years) is much lower than that of the second two wells (1247 406 years). This is more likely because of the higher head difference of the groundwater elevation (97.8 m) between well (5383) and well (5361) compared to that of 4.5 m

between well (5383) and well (5351) and as displayed in Table (2). Therefore, in addition to the hydraulic conductivity of the aquifers, head difference of the groundwater elevation highly influences the outputs of the simulations to predict travel time of groundwater flow. Over all, the histograms of the computerized mathematical method which is represented by Monte Carlo simulation revealed that the estimated travel time of the groundwater flow for the selected wells in the Iraqi western desert ranges from 1983 years to 113 741 years based on 10 000 m of travel distance. Moreover, fit comparison graphs to determine the best match between the input data of travel time and fitted distribution function show the same behavior for all selected wells. One of the advantages of assessing groundwater flow time to travel is to estimate travel time for a contaminant to reach discharge boundary. Therefore, it is recommended to apply Monte Carlo simulation on studying contaminant plumes in groundwater transport in this area.

Table 3. Summary of the Predicted Travel Time of Groundwater Flow for the Selected Pairs of Wells Used in the Monte Carlo Simulation

Well pairs	Length, L(m)	Stochastic travel time, (day)			Stochastic travel time, (year)		
		Minimum	Maximum	Mean	Minimum	Maximum	Mean
RW-5 B7-13	134 000	1560 422	410 633 898	12 293 946	4275	1125 024	33 682
5383 5351	109 670	180 007 586	3473 156 375	455 303 217	493 171	9515 497	1247 406
5383 5361	139 000	13 043 206	336 572 290	37 739 537	35 734	922 115	103 396
KH5/4 5420	59 355	5758 650	131 707 430	15 473 014	15 777	360 842	42 391
5225 K4/10	25 000	1942 204	180 585 782	7437 505	5321	494 755	20 376
5560 KH-3	87 500	40 605 840	2139 342 017	167 709 585	111 248	5861 211	459 478
5006 5621	116 670	8160 868	183 851 775	25 547 416	22 358	503 703	69 992
5579 5510	54 170	981 143	71 262 256	3921 090	2688	195 239	10 742
5616 4910	127 080	5152 298	694 367 443	24 467 051	14 115	1902 377	67 033

5. Conclusions

Following conclusions are drawn from this study:

- 1) An iteration number of 5000 was enough to achieve best match between the input data of the hydraulic conductivity and the triangular distribution function.
- 2) As the hydrogeological properties of the Iraqi western desert are significantly varied, the expected travel time of the groundwater flow is widely varied as indicated by the histograms which resulted from Monte Carlo simulations.
- 3) As indicated by several studies, hydraulic conductivity of the aquifer has a high impact on the estimated travel time of the groundwater flow. However, head difference of groundwater elevation among the selected wells considerably influences the expected travel time of the groundwater flow.

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Appendix A

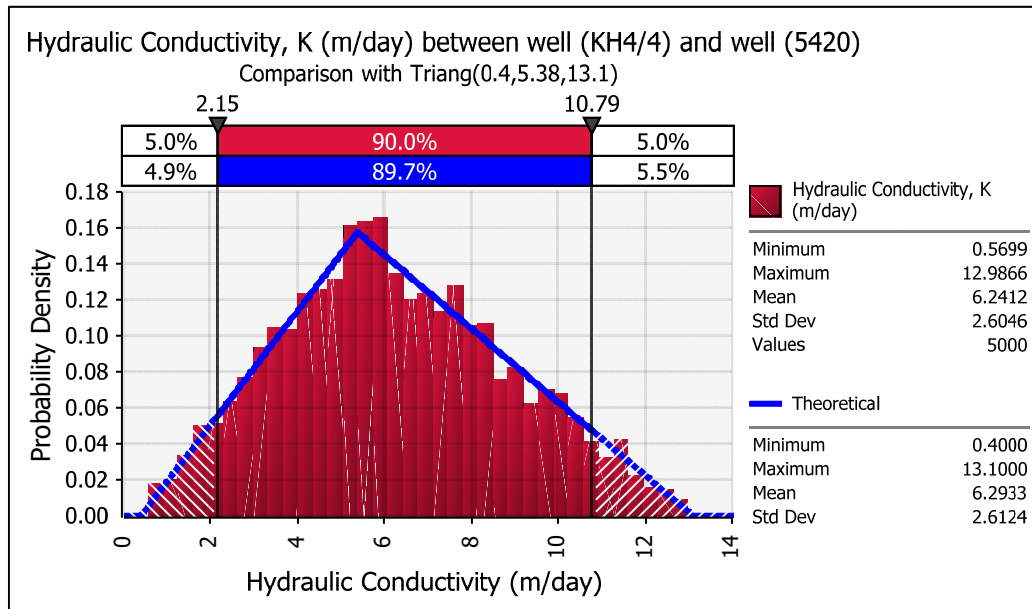


Figure A1. Hydraulic Conductivity between Well (KH4/4) and Well (5420) with an Iteration Number of 5000

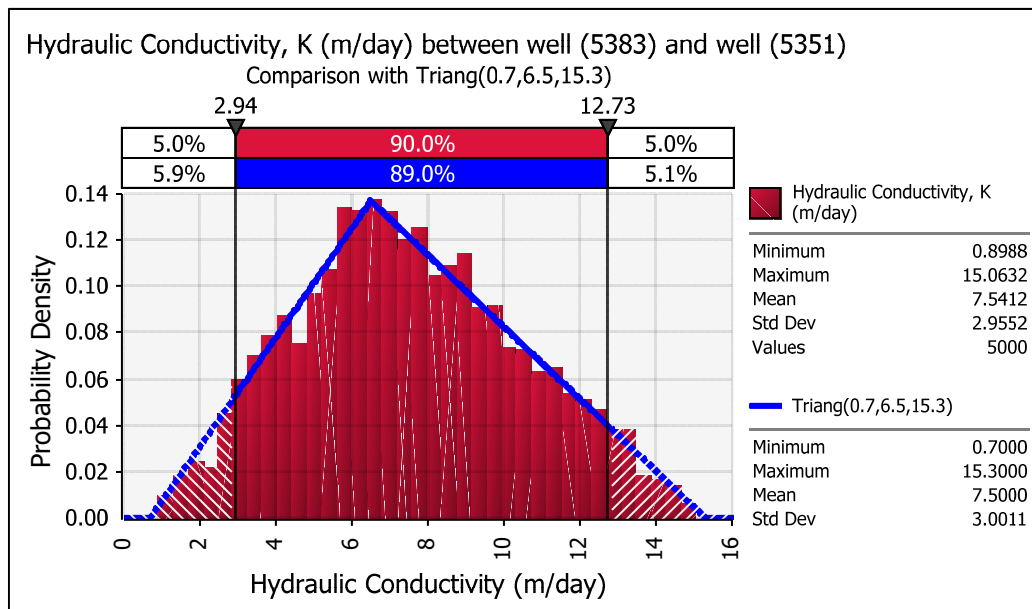


Figure A2. Hydraulic Conductivity between Well (5383) and Well (5351) with an Iteration Number of 5000

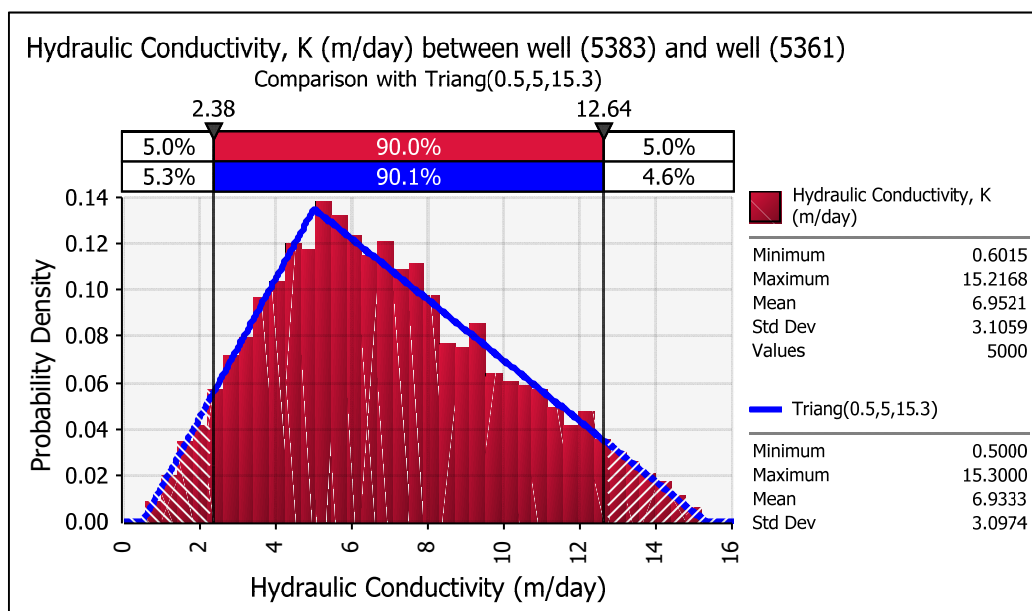


Figure A3. Hydraulic Conductivity between Well (5383) and Well (5361) with an Iteration Number of 5000

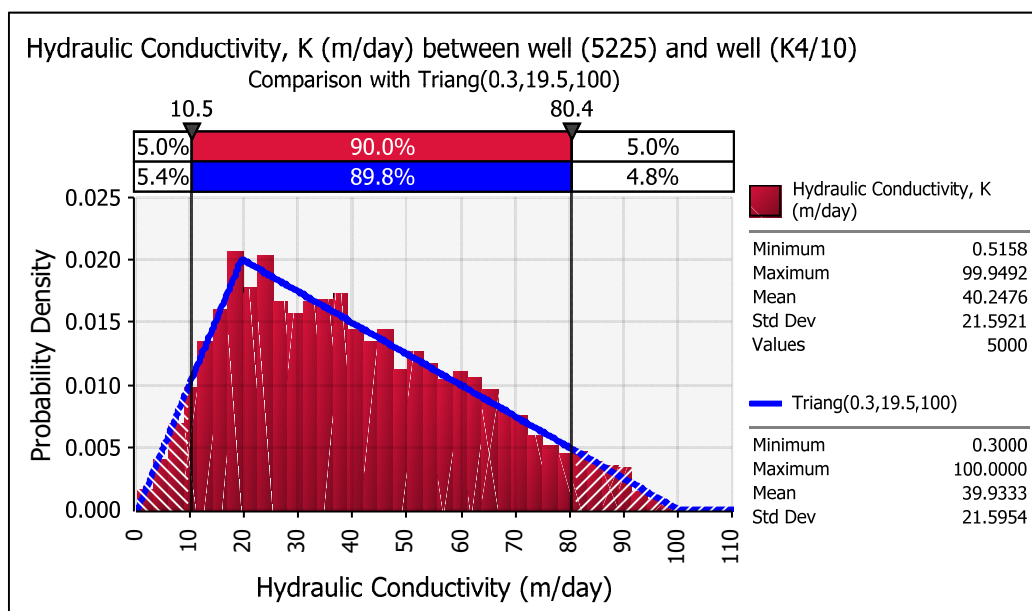


Figure A4. Hydraulic Conductivity between Well (5225) and Well (K4/10) with an Iteration Number of 5000

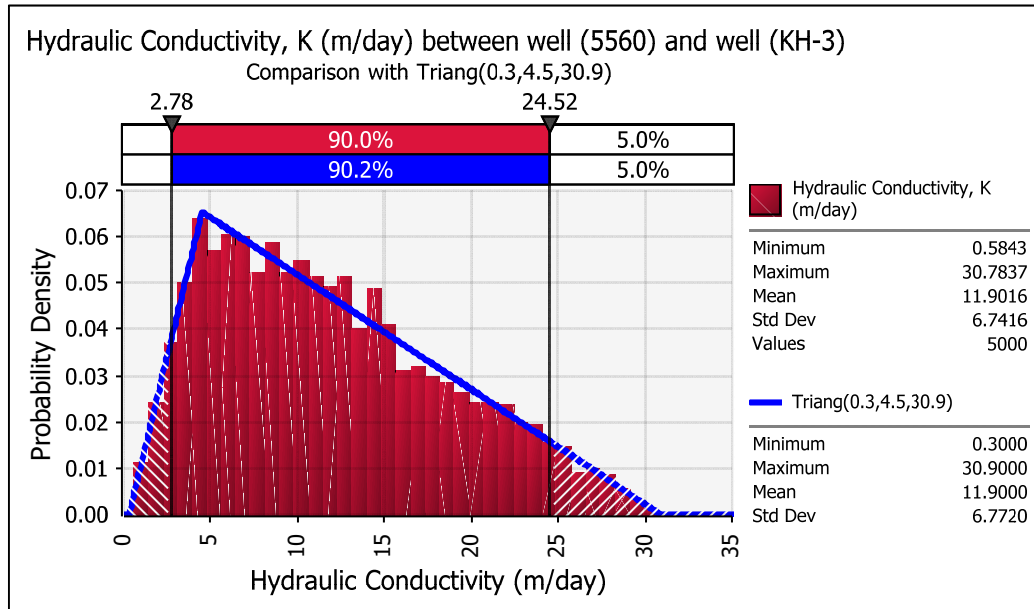


Figure A5. Hydraulic Conductivity between Well (5560) and Well (KH-3) with an Iteration Number of 5000

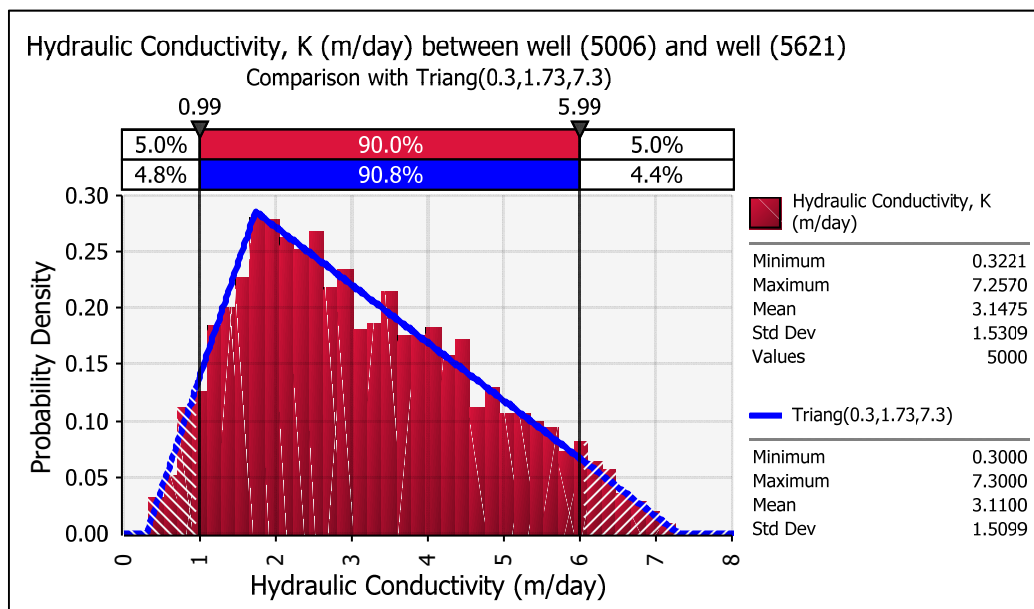


Figure A6. Hydraulic Conductivity between Well (5006) and Well (5621) with an Iteration Number of 5000

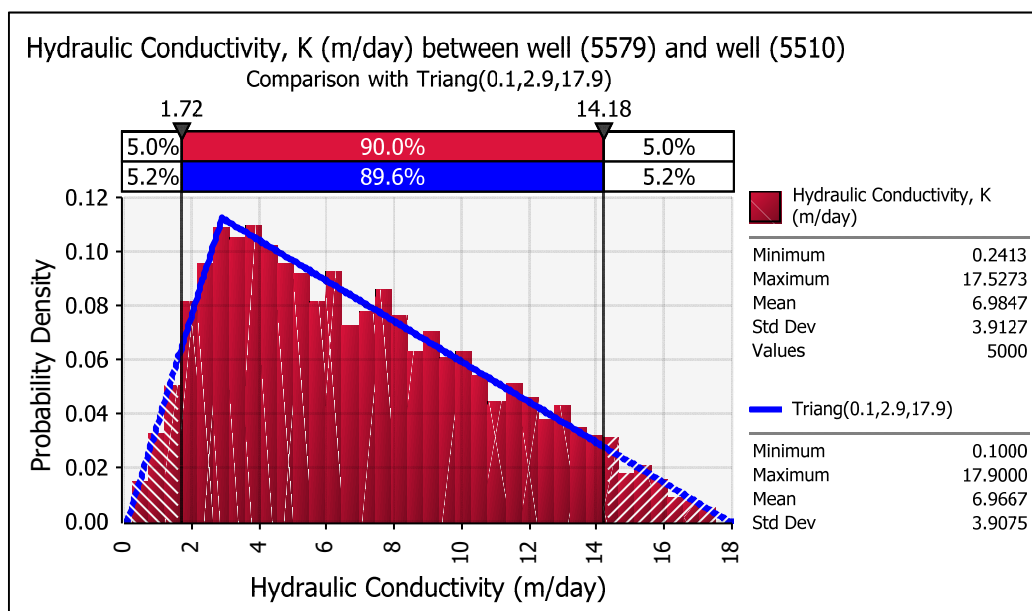


Figure A7. Hydraulic Conductivity between Well (5579) and Well (5510) with an Iteration Number of 5000

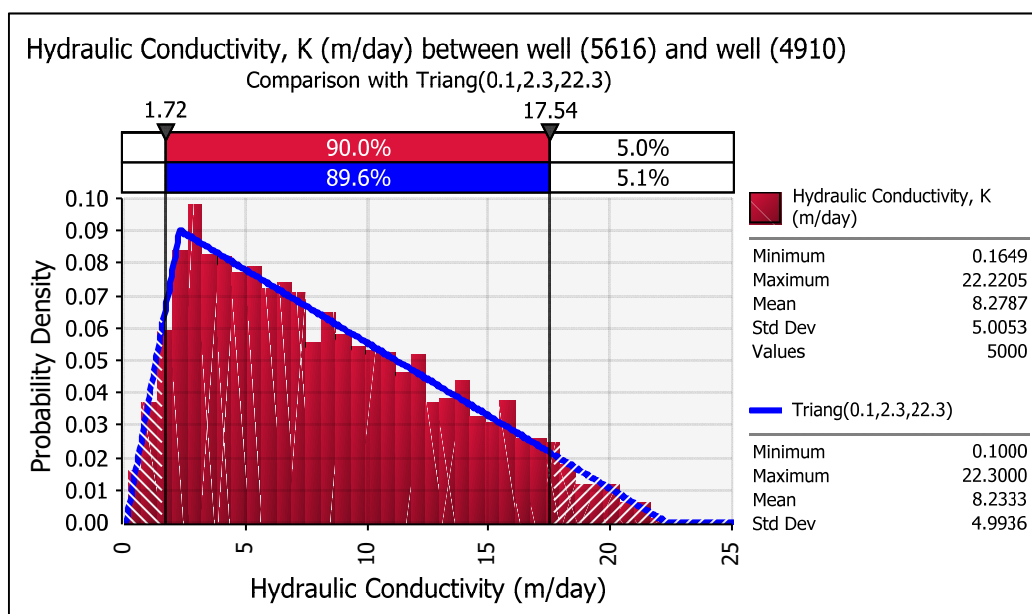


Figure A8. Hydraulic Conductivity between Well (5616) and Well (4910) with an Iteration Number of 5000

Appendix B

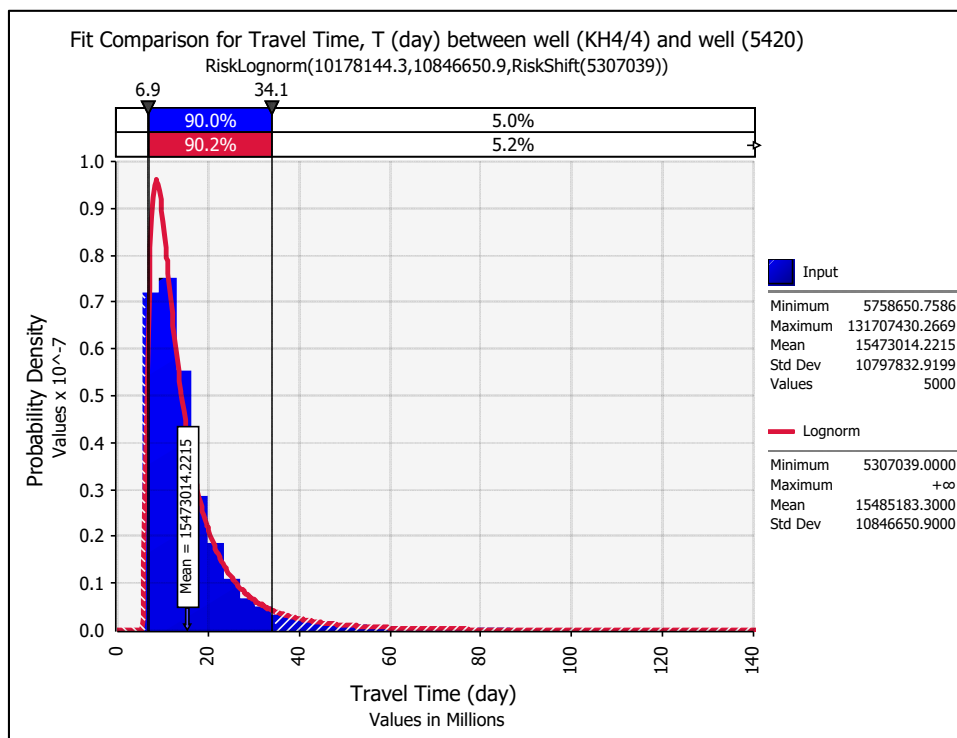


Figure B1. Fit Comparison for Travel Time between Well (KH4/4) and Well (5420)

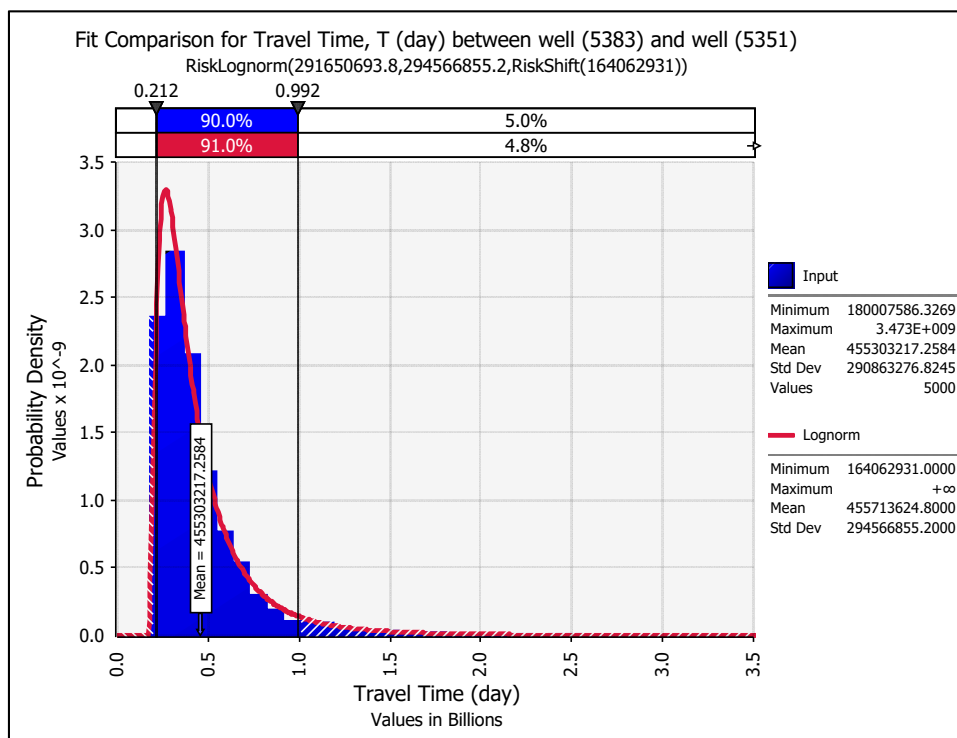


Figure B2. Fit Comparison for Travel Time between Well (5383) and Well (5351)

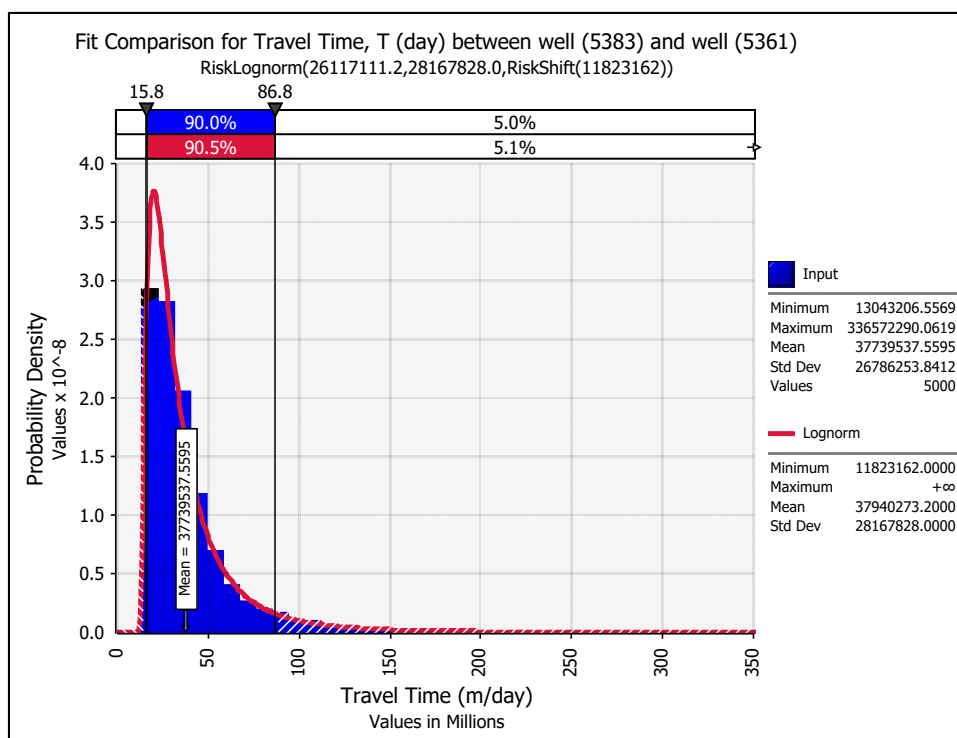


Figure B3. Fit Comparison for Travel Time between Well (5383) and Well (5361)

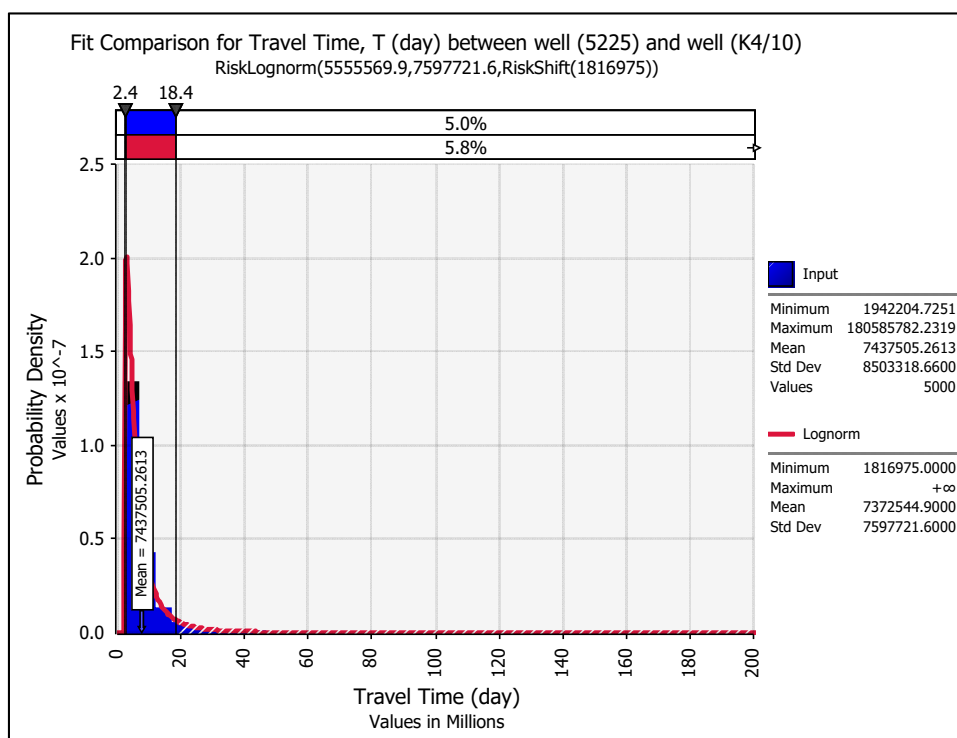


Figure B4. Fit Comparison for Travel Time between Well (5225) and Well (K4/10)

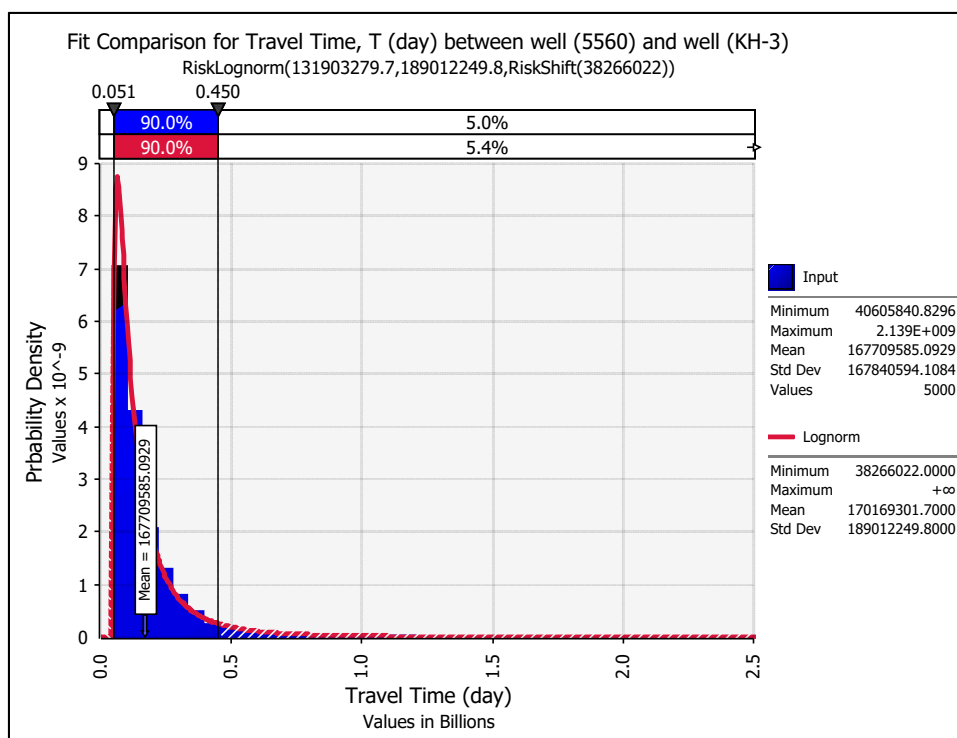


Figure B5. Fit Comparison for Travel Time between Well (5560) and Well (KH-3)

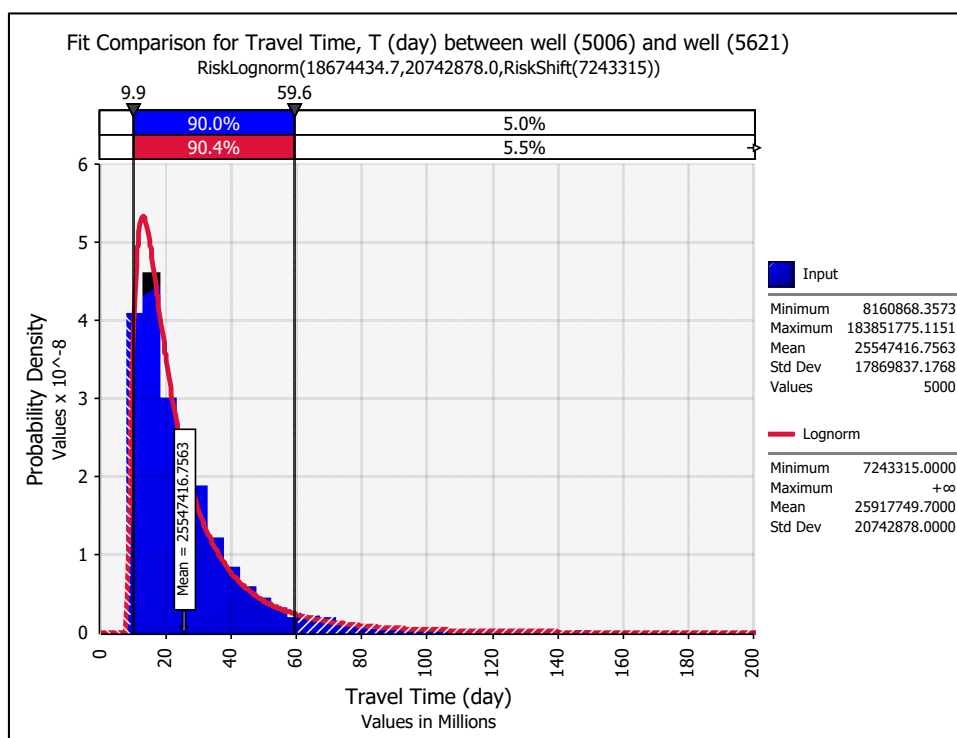


Figure B6. Fit Comparison for Travel Time between Well (5006) and Well (5621)

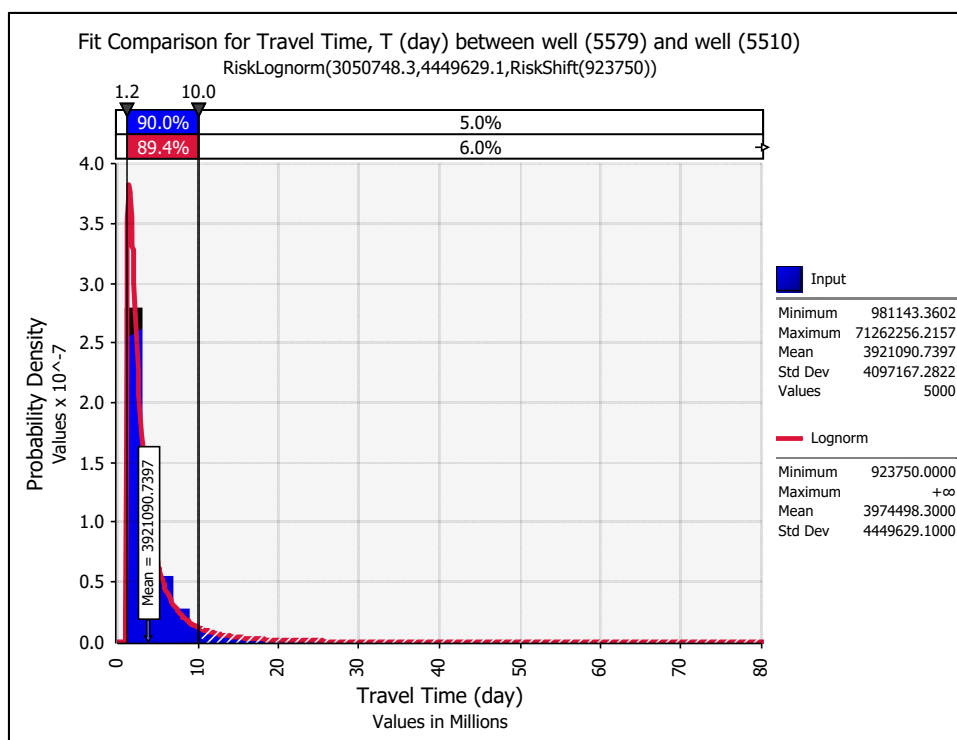


Figure B7. Fit Comparison for Travel Time between Well (5579) and Well (5510)

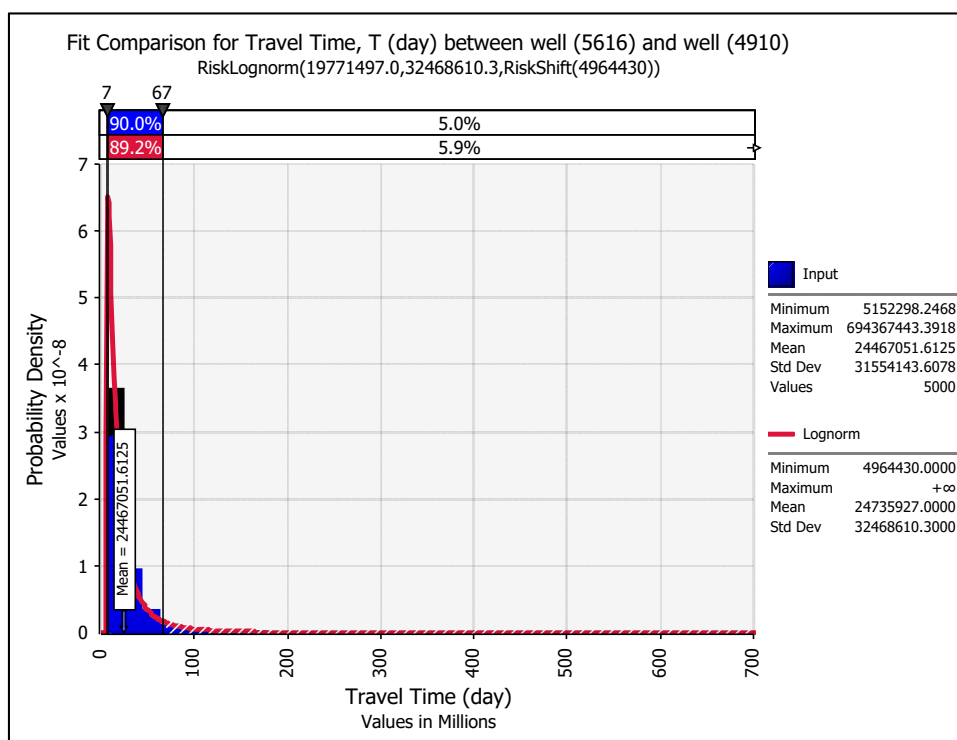


Figure B8. Fit Comparison for Travel Time between Well (5616) and Well (4910)

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