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A Decentralized Production/Remanufacturing Inventory Model with Multiple Refurbished Components

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Abstract

This study aims to investigate the optimal inventory and pricing policies in a decentralized production/remanufacturing system with multiple refurbished components. A basic inventory problem with deterministic demand rate and return rate is considered. The manufacturer purchases recycled components from the collector, and sells remanufactured products as well as new products to customers. It is assumed that the customers tend to pay different prices for remanufactured products and new products. Both the manufacturer and the collector try to maximize their own profits. The equilibrium of the problem is formulated as a variational inequality. Numerical examples show that the manufacturer and the collector can change the retail prices and return-facilitating costs to influence the product demands and the quantity of returned end-of-use products, such that no surplus or shortage of any products and components will occur.

Keywords: reverse logistics, remanufacturing, inventory management, outsourcing

1. Introduction

In recent years, with the continued trend of environmental protection and sustainable development, companies are devoted to green supply chain management in order to fulfill the international regulation, lower their costs, and improve their corporate images. The environmental protection is often considered as an obligation or voluntary duty. The prevalence of environmental awareness has encouraged companies to collect and recover end-of-life products through the remanufacturing process, which includes disassembly, cleaning, inspection, reconditioning, and reassembly. The studies about remanufacturing have increased rapidly in recent years. Many studies assume that remanufactured products are regarded as good as new; the customers cannot distinguish the remanufactured products from the new ones, or they consider these two products as interchangeable. In addition, most studies only consider single recycled component in a remanufacturing system. Such kinds of assumptions may be too simplified and not practical enough. Therefore it is better to relax these assumptions and to obtain a more realistic solution.

In a production/remanufacturing system, the collecting, refurbishing, and remanufacturing of used products may be performed by the same manufacturer (i.e., a centralized system) or different companies (i.e., a decentralized system). The manufacturer can concentrate on developing core competence by outsourcing the collecting and refurbishing activities. Unfortunately, this topic needs further investigation because the relevant studies are rarely seen.

Therefore, this study aims to investigate the optimal inventory and pricing policies in a decentralized production/remanufacturing system with multiple refurbished components. A basic inventory problem with deterministic demand rate and return rate is considered. The remainder of the paper is organized as follows. Section 2 presents the literature review, and Section 3 describes the model formulation. The solution algorithm is introduced in Section 4. In Section 5, numerical examples are provided with discussions. Conclusions are made in Section 6.

2. Literature Review

The studies about remanufacturing started in 1990s, and have increased rapidly in recent years. In the following, we will review some literature about inventory management in remanufacturing.

Many studies about remanufacturing assume that demand rate and return rate are deterministic and constant. Only a few recent studies consider variation of demand (Kaya, 2010; Hsueh, 2011; Teunter & Flapper, 2011; Wang, Zhao, & Wang, 2011; Shi, Zhang, & Sha, 2011; Chen & Chang, 2012). Kaya (2010) considers a manufacturer producing original products using new materials and remanufactured products using returns from the market where the amount of returns depends on the incentive offered by the manufacturer. The optimal value of this incentive and the optimal production quantities are determined in a stochastic demand setting with partial substitution. Hsueh (2011) investigates inventory control policies in a manufacturing/remanufacturing system during the product life cycle, which consists of four phases: introduction, growth, maturity, and decline. Both demand rate and return rate of products are random variables with normal distribution; the mean of the distribution varies according to the time in the product life cycle. Teunter and Flapper (2011) derive optimal acquisition and remanufacturing policies for both deterministic and uncertain demand. For uncertain demand, they derive optimal newsboy-type solutions for the optimal remanufacture-up-to levels and an approximate expression for the total expected cost given the number of acquired cores. Chen and Chang (2012) deal with a strategic issue of closed-loop supply chains with remanufacturing by developing analytic models under cooperative and competitive settings. The primary goal behind analytic formulation is to investigate under what conditions an original equipment manufacturer (OEM) may take a cooperative approach by participating in remanufacturing. They assume demands are random variables, and the mean is a function of the price. Shi et al. (2011) study the production planning problem for a multi-product closed loop system with uncertain demand and return. The problem is to maximize the manufacturer's expected profit by jointly determining the production quantities of brand-new products, the quantities of remanufactured products and the acquisition prices of the used products. All the above studies, except Hsueh (2011), are single-period stochastic models, which are usually solved by the solution method of newsvendor problem.

In addition to the demand characteristics, it is another important assumption that if remanufactured products are regarded as good as new. Many studies assume that the customers cannot distinguish the remanufactured products from the new ones, or they consider these two products as interchangeable. In many cases this assumption is not so realistic and may cause some problems. Recent studies have taken the difference into account (Jaber & El Saadany, 2009; Ferrer & Swaminathan, 2010; Kaya, 2010; Hasanov et al., 2012). Piñeyro and Viera (2010) investigate a lot-sizing problem with different demand streams for new and remanufactured items, in which the demand for remanufactured items can also be satisfied by new products, but not vice versa. They provide a mathematical model for the problem and demonstrate it is NP-hard, even under particular cost structures. With the aim of finding a near optimal solution of the problem, they develop and evaluate a Tabu-Search-based procedure. Aras et al. (2011) consider a company which leases new products and also sells remanufactured version of the new products that become available at the end of their lease periods. They develop a dynamic programming formulation for determining the optimal price of remanufactured products, and optimal payment structure for the leased products. Other studies also assume that the demand of new products is different from that of the remanufactured products (Chen & Chang, 2012; Robotis et al., 2012; Wu, 2012a, 2012b).

Outsourcing of used-product collection and remanufacturing is an important issue for the manufacturer. However, the relevant studies are rarely seen. Kaya (2010) analyzes three different models in centralized and decentralized settings where the collection process of the returns is managed by a collection agency in the decentralized setting. He also analyzes contracts to coordinate the decentralized systems and determine the optimal contract parameters. Aras et al. (2011) consider a company which leases new products and also sells remanufactured version of the new products that become available at the end of their lease periods. When the amount of end-of-lease items in stock is not sufficient to meet the demand for remanufactured products, the firm may purchase additional cores from a third-party supplier. However, they simply consider it as costs, instead of investigating the relation between two firms. Wu (2012b) considers a supply chain consisting of two manufacturers and a retailer. The first manufacturer is a traditional manufacturer that produces new products, while the second manufacturer operates a reverse channel producing remanufactured products from used cores. Both manufacturers bundle their products with services, including warranty and advertisement, and they sell through the same retailer, which independently determines the sales prices. Chen and Chang (2012) investigate under what conditions an original equipment manufacturer (OEM) may take a cooperative approach by participating in remanufacturing. In contrast, the OEM may take a competitive approach by letting the third-party firm remanufacture the returned cores and remarket in the secondary market that competes with the new product. Although many studies about collaboration and competition in a supply chain have been published, literature about relationship between the actors in the manufacturing/remanufacturing system is not common.

In addition, multiple refurbished components in a remanufacturing system are rarely considered in the literature, except El Saadany and Jaber (2011). They assume that each unit of a used product is collected and disassembled into components, which are fed back into the production-remanufacturing process. Different from our study, they assume that the returned components are remanufactured to represent a second source of as-good-as-new units of the product.

According to the above literature review, we summarize some issues that need further investigation: (1) the customers perceive that the remanufactured products are different from the new ones, and the demands for the two kinds of products affect each other, (2) the collecting and refurbishing of used products is outsourced to another company, and (3) multiple refurbished components are considered. These issues are the basic assumptions of this study.

3. Model Formulation

3.1 Problem Description

A production/remanufacturing system with a manufacturer and a collector is considered. The framework of the system is illustrated in Figure 1. The manufacturer produces new products as well as remanufactured products, and sells to customers at different prices. The manufacturer may use refurbished components, which are provided by collectors, in the remanufactured products. The collector collects end-of-use products, refurbishes them to gain some valuable components, and sells to the manufacturer. There are different stocks in the system, including remanufactured products, new products, and different refurbished components. Both the manufacturer and the collector try to maximize their own profits, and it will eventually result in system equilibrium. The manufacturer determines optimal prices and optimal production lot size of new products and remanufactured products, as well as optimal order quantity of refurbished components. The collector determines the sale quantity of refurbished components and the return-facilitating costs. Assumptions of the model are listed in the following.

1) New products and remanufactured products have different values. Their demands are functions of all prices, i.e., $d_N = f_N(\rho_N, \rho_R)$ and $d_R = f_R(\rho_N, \rho_R)$.

2) Return rates are functions of the demands and return-facilitating costs.

3) Multiple refurbishable components are used, which may have different availabilities.

4) New products and remanufactured products are produced by the manufacturer in the same assembly line.

5) No disposal cost or salvage value are considered.

6) Remanufactured products cannot be refurbished again.

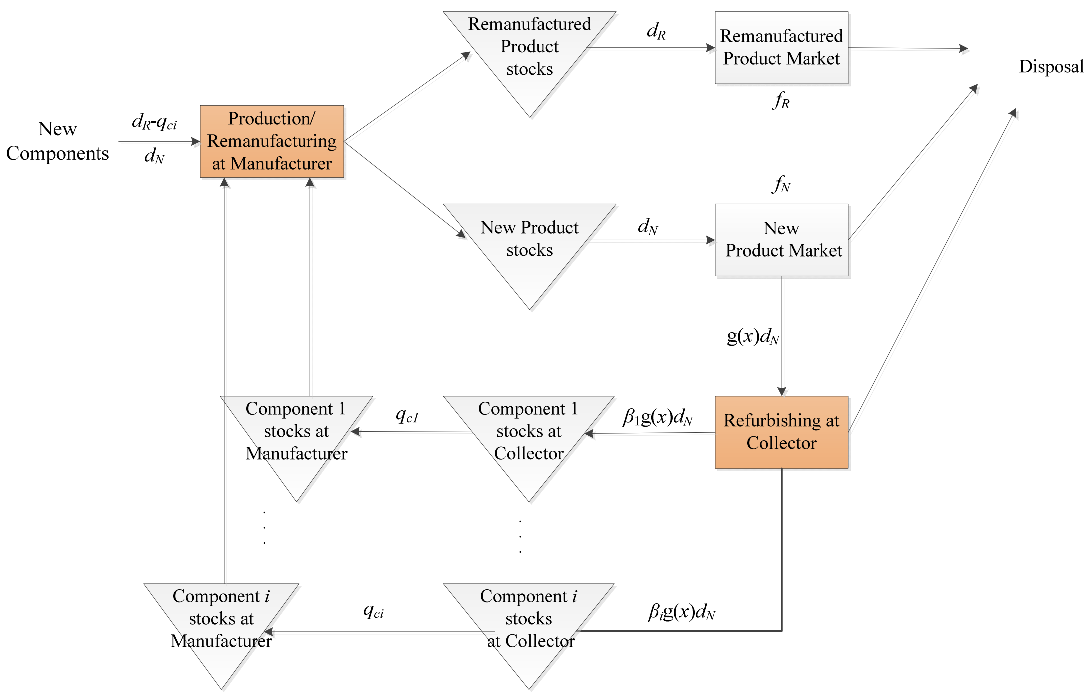


Figure 1. System framework

Notations that will be used in the model are introduced in the following:

Set:

Φ : Set of refurbished components

Decision variables:

ρ_N : Price of new products

ρ_R : Price of remanufactured products

d_R : Supply/demand quantity of remanufactured products per year

Q_N : Production lot size of new products in each production cycle

Q_R : Production lot size of remanufactured products in each production cycle

Q_{Ci} : Order quantity of refurbished component *i*

q_{Ci} : Purchase quantities of refurbished component *i* per year

x : Return-facilitating costs

Dependent variables:

d_N : supply/demand quantity of new products per year

Parameters:

h : holding cost of products

h_{Ci} : holding cost of refurbished component *i*

p : production rate

K : setup cost

c_m : production cost of product

c_o : ordering cost of refurbished component

ρ_{Ci} : price of refurbished component *i*

ρ_{ci}^{new} : price of new component *i*

ρ_c^{new} : sum of prices of all non-refurbishable components

c_{ci}^{m} : collecting and refurbishing cost of component *i*

β_i : availability of returned component *i*

Functions:

$f_N(\rho_N, \rho_R)$: demands of new products

$f_R(\rho_N, \rho_R)$: demands of remanufacturing products

$\mathrm{g}(x)$: return ratio of products

3.2 Model Formulation

In this section we will investigate inventory level of different stocks first in order to derive the inventory holding costs. After that, the profit-maximizing models for the manufacturer and the collector are proposed, respectively. Both the manufacturer and the collector will try to maximize its own profit. However, these two models interact with each other, which are unable to solve directly. The system will eventually arrive at an equilibrium status, which complies with the first order conditions of the previous models. The equilibrium solution can be obtained by solving the variational inequality model that derived from these optimal conditions.

3.2.1 Analysis of Inventory Level

Since the remanufactured products and the new products are produced in the same assembly line, the manufacturer produces these two products sequentially, followed by a period of idle time. Figure 2 illustrates inventory levels of these two products. In order to make sure that this process is repeated smoothly, the cycle times of the two products must be identical. Based on Figure 2, the inventory holding costs can be derived as follows:

Inventory holding costs of the remanufactured products= $\frac{Q_R}{2p}(p - d_R)h$.

Inventory holding costs of the new products= $\frac{Q_N}{2p}(p - d_N)h$.

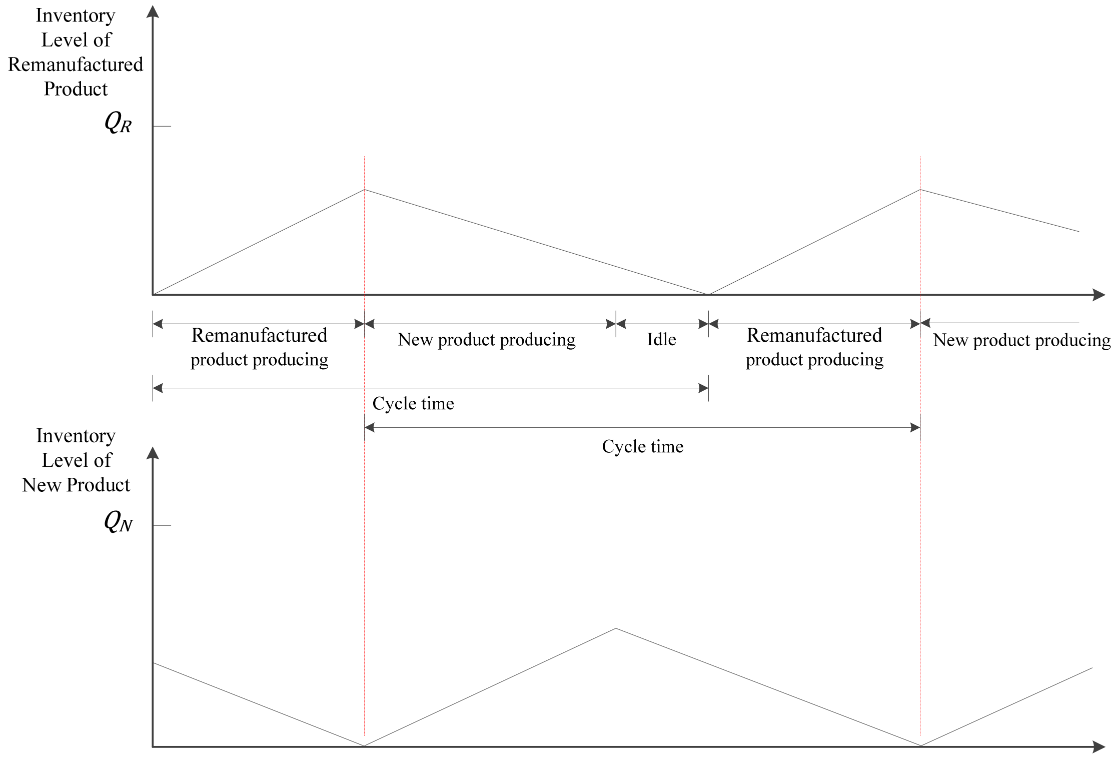


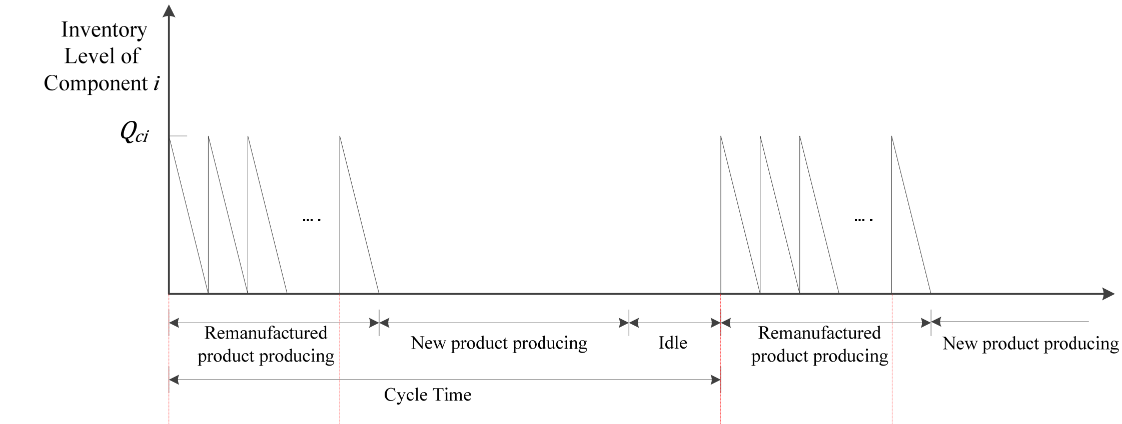
Figure 2. Inventory levels of remanufactured products and new products

During the period of producing remanufactured products, the manufacturer purchases refurbished components from the collector periodically and keeps zero stocks of refurbished components during the periods of producing new products and idle time, in order to reduce the inventory holding cost. The inventory level of refurbished components of the manufacturer is illustrated in Figure 3(a). The inventory holding cost can then be derived as:

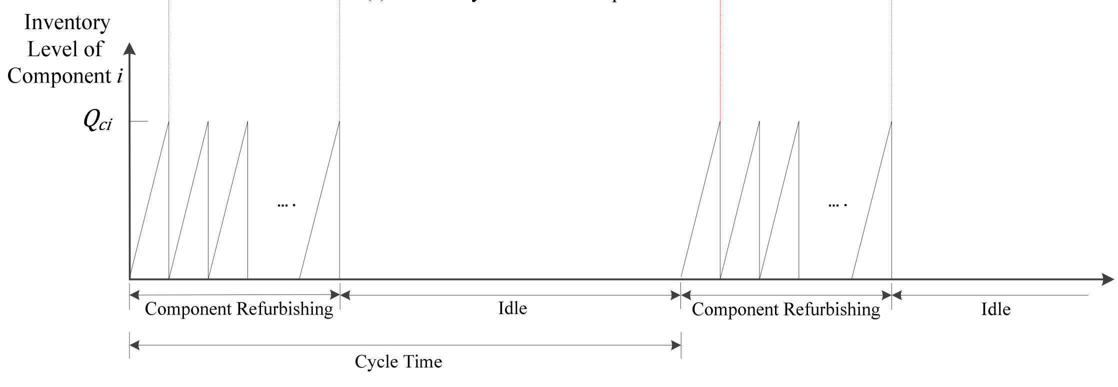
Inventory holding cost of refurbished components i of the manufacturer $= \frac{1}{2} Q_{ci} h_{ci} \left(\frac{Q_R}{p} / \frac{Q_R}{d_R}\right) = \frac{d_R}{2p} Q_{ci} h_{ci}$.

The collector collects and refurbishes the components, corresponding to the demand from the manufacturer. The inventory level of refurbished components of collector is illustrated in Figure 3(b). The inventory holding cost can then be derived as:

Inventory holding cost of refurbished components i of the collector $= \frac{d_R}{2p} Q_{ci} h_{ci}$.



(a) Inventory Level of Component i at the Manufacturer



(b) Inventory Level of Component i at the Collector

Figure 3. Inventory level of refurbished components

3.2.2 Profit-Maximizing Model of the Manufacturer

After deriving the inventory holding cost, the profit-maximizing model of the manufacturer can be formulated as follows:

$$\max\ \pi_M = \rho_R d_R + \rho_N d_N - \frac{Q_R}{2p}(p-d_R)h - \frac{Q_N}{2p}(p-d_N)h - \left(\frac{d_R}{Q_R}+\frac{d_N}{Q_N}\right)K - (d_R+d_N)c_m - \frac{d_R}{2p}\sum_{i\in\Phi} Q_{ci}h_{ci} - c_o\sum_{i\in\Phi}\frac{q_{ci}}{Q_{ci}} - \sum_{i\in\Phi}\rho_{ci}q_{ci} - \sum_{i\in\Phi}\rho_{ci}^{new}(d_R - q_{ci}) - \rho_c^{new}(d_R+d_N) \quad (1)$$

$$\text{s.t.}\quad \frac{Q_N}{d_N} = \frac{Q_R}{d_R} \quad (2)$$

$$d_R \le f_R(\rho_N, \rho_R) \quad (3)$$

$$d_N \le f_N(\rho_N, \rho_R) \quad (4)$$

$$q_{ci} \le d_R \quad (5)$$

$$\prod_{i\in\Phi}(d_R - q_{ci}) = 0 \quad (6)$$

$$d_R + d_N \le p \quad (7)$$

$$\rho_N, \rho_R, d_N, d_R, Q_N, Q_R, Q_{ci}, q_{ci} \ge 0 \quad \forall i \in \Phi \quad (8)$$

Eq. (1) is the objective function, which maximizes the manufacturer's profit, and the costs in the Eq. (1) include holding cost of remanufactured products, holding cost of new products, setup cost, production cost, holding cost of refurbished components, ordering cost, and purchase cost. Constraint (2) requires that the production cycle time of new products must be equal to the cycle time of remanufactured products. Eq. (3) states that the sale quantity of remanufactured products is less than its demand, which is affected by the prices of both products. Eq. (4) states that the sale quantity of new products is less than its demand, which is also affected by the prices of both products. Eqs. (5) to (6) indicate that total quantity of remanufactured products equals the maximum value of purchase quantities of all refurbished components, i.e., $d_R = max_i(q_{ci})$. Eq. (7) is a capacity constraint, and

Eq. (8) is a nonnegative constraint.

For deriving the optimal conditions, a Lagrange function is defined as

$$L_M = \pi_M + v_1\left(\frac{Q_R}{d_R} - \frac{Q_N}{d_N}\right) + v_2(f_R(\rho_N, \rho_R) - d_R) + v_3(f_N(\rho_N, \rho_R) - d_N) + \sum_{i\in\Phi} v_{4i}(d_R - q_{ci}) + v_5\prod_{i\in\Phi}(d_R - q_{ci}) + v_6(p - d_R - d_N)$$

where $v_1, v_2, v_3, v_{4i}, v_5, v_6$ are Lagrange multipliers. Optimal conditions can then be derived using the following equations:

$$\frac{\partial L_M^*}{\partial y} y^* = 0$$
$$\frac{\partial L_M^*}{\partial y} \le 0$$
$$v_2^*(f_R(\rho_N^*, \rho_R^*) - d_R^*) = 0$$
$$v_3^*(f_N(\rho_N^*, \rho_R^*) - d_N^*) = 0$$
$$v_{4i}^*(d_R^* - q_{ci}^*) = 0$$
$$v_6^*(p^* - d_R^* - d_N^*) = 0$$
$$v_2^*, v_3^*, v_{4i}^*, v_6^* \ge 0$$

and (2)~(7), where y denotes $\rho_N, \rho_R, d_N, d_R, Q_N, Q_R, Q_{ci}$ or q_{ci}.

Detailed optimal conditions are derived as follows:

$$\left(d_N^* + v_2^*\frac{\partial f_R(\rho_N^*,\rho_R^*)}{\partial \rho_N} + v_3^*\frac{\partial f_N(\rho_N^*,\rho_R^*)}{\partial \rho_N}\right)\rho_N^* = 0 \quad (9)$$

$$d_N^* + v_2^*\frac{\partial f_R(\rho_N^*,\rho_R^*)}{\partial \rho_N} + v_3^*\frac{\partial f_N(\rho_N^*,\rho_R^*)}{\partial \rho_N} \le 0 \quad (10)$$

$$\left(d_R^* + v_2^*\frac{\partial f_R(\rho_N^*,\rho_R^*)}{\partial \rho_R} + v_3^*\frac{\partial f_N(\rho_N^*,\rho_R^*)}{\partial \rho_R}\right)\rho_R^* = 0 \quad (11)$$

$$d_R^* + v_2^*\frac{\partial f_R(\rho_N^*,\rho_R^*)}{\partial \rho_R} + v_3^*\frac{\partial f_N(\rho_N^*,\rho_R^*)}{\partial \rho_R} \le 0 \quad (12)$$

$$\left(\rho_R^* + \frac{Q_R^*}{2p}h - \frac{K}{Q_R^*} - c_m - \frac{1}{2p}\sum_{i\in\Phi}(Q_{ci}^* h_{ci}) - \sum_{i\in\Phi}\rho_{ci}^{new} - \rho_c^{new} - v_1^* Q_R^* {d_R^*}^{-2} - v_2^* + \sum_{i\in\Phi} v_{4i}^* + v_5^*\frac{\partial\left(\prod_{i\in\Phi}(d_R^* - q_{ci}^*)\right)}{\partial d_R} - v_6^*\right) d_R^* = 0 \quad (13)$$

$$\rho_R^* + \frac{Q_R^*}{2p}h - \frac{K}{Q_R^*} - c_m - \frac{1}{2p}\sum_{i\in\Phi}(Q_{ci}^* h_{ci}) - \sum_{i\in\Phi}\rho_{ci}^{new} - \rho_c^{new} - v_1^* Q_R^* {d_R^*}^{-2} - v_2^* + \sum_{i\in\Phi} v_{4i}^* + v_5^*\frac{\partial\left(\prod_{i\in\Phi}(d_R^* - q_{ci}^*)\right)}{\partial d_R} - v_6^* \le 0 \quad (14)$$

$$\left(-\frac{p - d_N^*}{2p}h + d_N^* K {Q_N^*}^{-2} - \frac{v_1^*}{d_N^*}\right) Q_N^* = 0 \quad (15)$$

$$-\frac{p - d_N^*}{2p}h + d_N^* K {Q_N^*}^{-2} - \frac{v_1^*}{d_N^*} \le 0 \quad (16)$$

$$\left(-\frac{p - d_R^*}{2p}h + d_R^* K {Q_R^*}^{-2} + \frac{v_1^*}{d_R^*}\right) Q_R^* = 0 \quad (17)$$

$$-\frac{p - d_R^*}{2p}h + d_R^* K {Q_R^*}^{-2} + \frac{v_1^*}{d_R^*} \le 0 \quad (18)$$

$$\left(-\frac{d_R^*}{2p}h_{ci} + c_o q_{ci}^* {Q_{ci}^*}^{-2}\right) Q_{ci}^* = 0 \quad \forall i \in \Phi \quad (19)$$

$$-\frac{d_R^*}{2p}h_{ci} + c_o q_{ci}^* {Q_{ci}^*}^{-2} \le 0 \quad \forall i \in \Phi \quad (20)$$

$$\left(-\frac{c_o}{Q_{ci}^*} - \rho_{ci} + \rho_{ci}^{new} - v_{4i}^* + v_5^*\frac{\partial\left(\prod_{j\in\Phi}\left(d_R^* - q_{cj}^*\right)\right)}{\partial q_{ci}}\right) q_{ci}^* = 0 \quad \forall i \in \Phi \quad (21)$$

$$-\frac{c_o}{Q_{ci}^*} - \rho_{ci} + \rho_{ci}^{new} - v_{4i}^* + v_5^*\frac{\partial\left(\prod_{j\in\Phi}\left(d_R^* - q_{cj}^*\right)\right)}{\partial q_{ci}} \le 0 \quad \forall i \in \Phi \quad (22)$$

$$\left(\rho_N^* + \frac{Q_N^*}{2p}h - \frac{K}{Q_N^*} - c_m - \rho_c^{new} + v_1^* Q_N^* {d_N^*}^{-2} - v_3^* - v_6^*\right) d_N^* = 0 \quad (23)$$

$$\rho_N^* + \frac{Q_N^*}{2p}h - \frac{K}{Q_N^*} - c_m - \rho_c^{new} + v_1^* Q_N^* {d_N^*}^{-2} - v_3^* - v_6^* \leq 0 \tag{24}$$

$$v_2^*(f_R(\rho_N^*, \rho_R^*) - d_R^*) = 0 \tag{25}$$

$$v_3^*(f_N(\rho_N^*, \rho_R^*) - d_N^*) = 0 \tag{26}$$

$$v_{4i}^*(d_R^* - q_{ci}^*) = 0 \quad \forall i \in \Phi \tag{27}$$

$$v_6^*(p^* - d_R^* - d_N^*) = 0 \tag{28}$$

$$v_2^*, v_3^*, v_{4i}^*, v_6^* \geq 0 \quad \forall i \in \Phi \tag{29}$$

(2)~(8)

3.2.3 Profit-Maximizing Model of the Collector

The profit-maximizing model of the collector can be formulated as follows:

$$\max \ \pi_C = \sum_{i\in\Phi}(\rho_{ci} - c_{ci}^m)q_{ci} - \frac{d_R}{2p}\sum_{i\in\Phi} Q_{ci}h_{ci} - x \tag{30}$$

$$\text{s.t.} \quad q_{ci} \leq \beta_i \mathrm{g}(x) d_N \quad \forall i \in \Phi \tag{31}$$

$$q_{ci}, x \geq 0 \quad \forall i \in \Phi \tag{32}$$

Eq. (30) is the objective function, which maximizes the profit of the collector. The costs in Eq. (30) include collecting and refurbishing cost, inventory holding cost, and return-facilitating costs. Eq. (31) requires that the selling quantity of refurbished components be less than or equal to the available quantity of components that the collector refurbished. Eq. (32) is a nonnegative constraint.

For deriving the optimal conditions, a Lagrange function is defined as

$$L_C = \pi_C + \sum_{i\in\Phi} u_i(\beta_i \mathrm{g}(x) d_N - q_{ci})$$

Optimal conditions can then be derived as follows:

$$(\rho_{ci} - c_{ci}^m - u_i^*)q_{ci}^* = 0 \quad \forall i \in \Phi \tag{33}$$

$$\rho_{ci} - c_{ci}^m - u_i^* \leq 0 \quad \forall i \in \Phi \tag{34}$$

$$\left(-1 + \sum_{i\in\Phi} u_i^* \beta_i \frac{d\mathrm{g}(x^*)}{dx} d_N^*\right) x^* = 0 \tag{35}$$

$$-1 + \sum_{i\in\Phi} u_i^* \beta_i \frac{d\mathrm{g}(x^*)}{dx} d_N^* \leq 0 \tag{36}$$

$$u_i^*(\beta_i \mathrm{g}(x^*) d_N^* - q_{ci}^*) = 0 \quad \forall i \in \Phi \tag{37}$$

$$u_i^* \geq 0 \quad \forall i \in \Phi \tag{38}$$

(27)~(28)

3.2.4 Equilibrium Model

When the system arrives at equilibrium, all optimal conditions of the manufacturer and the collector must be hold, including (2)-(25), and (27)-(34). Eqs. (20), (21), (29), and (30) can be combined into:

$$\left(-\frac{c_o}{Q_{ci}^*} + \rho_{ci}^{new} - c_{ci}^m - v_{4i}^* + v_5 \frac{\partial\left(\prod_{j\in\Phi}\left(d_R^* - q_{cj}^*\right)\right)}{\partial q_{ci}} - u_i^*\right) q_{ci}^* = 0 \quad \forall i \in \Phi \tag{39}$$

$$-\frac{c_o}{Q_{ci}^*} + \rho_{ci}^{new} - c_{ci}^m - v_{4i}^* + v_5 \frac{\partial\left(\prod_{j\in\Phi}\left(d_R^* - q_{cj}^*\right)\right)}{\partial q_{ci}} - u_i^* \leq 0 \quad \forall i \in \Phi \tag{40}$$

For the purpose of simplicity, let

$$F_1^* = -d_N^* - v_2^* \frac{\partial f_R(\rho_N^*, \rho_R^*)}{\partial \rho_N} - v_3^* \frac{\partial f_N(\rho_N^*, \rho_R^*)}{\partial \rho_N} \tag{41}$$

$$F_2^* = -d_R^* - v_2^* \frac{\partial f_R(\rho_N^*, \rho_R^*)}{\partial \rho_R} - v_3^* \frac{\partial f_N(\rho_N^*, \rho_R^*)}{\partial \rho_R} \tag{42}$$

$$F_3^* = -\rho_R^* - \frac{Q_R^*}{2p}h + \frac{K}{Q_R^*} + c_m + \frac{1}{2p}\sum_{i\in\Phi}(Q_{ci}^* h_{ci}) + \sum_{i\in\Phi}\rho_{ci}^{new} + \rho_c^{new} + v_1^* Q_R^* {d_R^*}^{-2} + v_2^* - \sum_{i\in\Phi} v_{4i}^* - v_5 \frac{\partial(\prod_{i\in\Phi}(d_R^* - q_{ci}^*))}{\partial d_R} + v_6^* \tag{43}$$

$$F_4^* = \frac{p - d_N^*}{2p}h - d_N^* K {Q_N^*}^{-2} + \frac{v_1^*}{d_N^*} \tag{44}$$

$$F_5^* = \frac{p-d_R^*}{2p}h - d_R^* K {Q_R^*}^{-2} - \frac{v_1^*}{d_R^*} \quad (45)$$

$$F_{6i}^* = \frac{d_R^*}{2p}h_{ci} - c_o q_{ci}^* {Q_{ci}^*}^{-2} \quad (46)$$

$$F_{7i}^* = \frac{c_o}{Q_{ci}^*} - \rho_{ci}^{new} + c_{ci}^m + v_{4i}^* - v_5 \frac{\partial\left(\prod_{j\in\Phi}\left(d_R^* - q_{cj}^*\right)\right)}{\partial q_{ci}} + u_i^* \quad (47)$$

$$F_8^* = -\rho_N^* - \frac{Q_N^*}{2p}h + \frac{K}{Q_N^*} + c_m + \rho_c^{new} - v_1^* Q_N^* {d_N^*}^{-2} + v_3^* + v_6^* \quad (48)$$

$$F_9^* = 1 - \sum_{i\in\Phi} u_i^* \beta_i \frac{d\mathrm{g}(x^*)}{dx} d_N^* \quad (49)$$

$$F_{10}^* = \frac{Q_N^*}{d_N^*} - \frac{Q_R^*}{d_R^*} \quad (50)$$

$$F_{11}^* = f_R(\rho_N^*, \rho_R^*) - d_R^* \quad (51)$$

$$F_{12}^* = f_N(\rho_N^*, \rho_R^*) - d_N^* \quad (52)$$

$$F_{13i}^* = d_R^* - q_{ci}^* \quad (53)$$

$$F_{14}^* = \prod_{i\in\Phi}(d_R^* - q_{ci}^*) \quad (54)$$

$$F_{15}^* = p^* - d_R^* - d_N^* \quad (55)$$

$$F_{16i}^* = \beta_i \mathrm{g}(x^*) d_N^* - q_{ci}^* \quad (56)$$

Then system equilibrium can be formulated as the following variational inequality model:

$$F_1^*(\rho_N - \rho_N^*) + F_2^*(\rho_R - \rho_R^*) + F_3^*(d_R - d_R^*) + F_4^*(Q_N - Q_N^*) + F_5^*(Q_R - Q_R^*) + \sum_{i\in\Phi} F_{6i}^*(Q_{ci} - Q_{ci}^*) + \sum_{i\in\Phi} F_{7i}^*(q_{ci} - q_{ci}^*) + F_8^*(d_N - d_N^*) + F_9^*(x - x^*) + F_{10}^*(v_1 - v_1^*) + F_{11}^*(v_2 - v_2^*) + F_{12}^*(v_3 - v_3^*) + \sum_{i\in\Phi} F_{13i}^*(v_{4i} - v_{4i}^*) + \sum_{i\in\Phi} F_{14}^*(v_5 - v_5^*) + F_{15}^*(v_6 - v_6^*) + \sum_{i\in\Phi} F_{15i}^*(u_i - u_i^*) \geq 0 \quad (57)$$

where the feasible region is subject to the following nonnegative constraints.

$$\rho_N, \rho_R, d_R, Q_N, Q_R, Q_{ci}, q_{ci}, x, v_2, v_{4i}, u_i \geq 0 \quad (58)$$

4. Solution Algorithm

To solve a variational inequality model, an iterative solution algorithm is often adopted, which is introduced in the following.

Step 1: Find an initial solution $\boldsymbol{X}^0$. Let $k = 1$.

Step 2: Compute $\boldsymbol{X}^k$ by solving the subproblem:

$\sum_i F_i\left(X_i^k, X_{j\neq i}^{k-1}\right)\left(X_i - X_i^k\right) \geq 0 \quad \forall \quad X_i$

Step 3: if $|\boldsymbol{X}^k - \boldsymbol{X}^{k-1}| \leq \epsilon$ for some tolerance $\epsilon > 0$, then stop; otherwise, set $k = k+1$ and go to Step 2.

Note that the subproblem in Step 2 can be transformed into a mathematical programming model, and therefore the solution can be computed using any appropriate mathematical programming algorithm.

5. Numerical Examples

Two refurbishable components are considered. The relevant parameters are set in Table 1:

Table 1. Parameter setting

(a)

	h_{ci}	c_o	ρ_{ci}^{new}	c_{ci}^m	β_i
Component 1	8	20	100	5	0.3~0.9
Component 2	6	20	80	5	0.7

(b)

	h	p	K	c_m	ρ_c^{new}
Products	20	12000	250	50	800

Demand functions are assumed as:

$$f_N = -\rho_N + 0.2\rho_R + 10000$$
$$f_R = -\rho_R + 0.2\rho_N + 6000$$

Return ratio is assumed as:

$$g(x) = 0.8 - \frac{0.6}{0.001x + 1}$$

Based on the above information, the proposed model and solution algorithm are tested, and the results are illustrated in Figure 4.

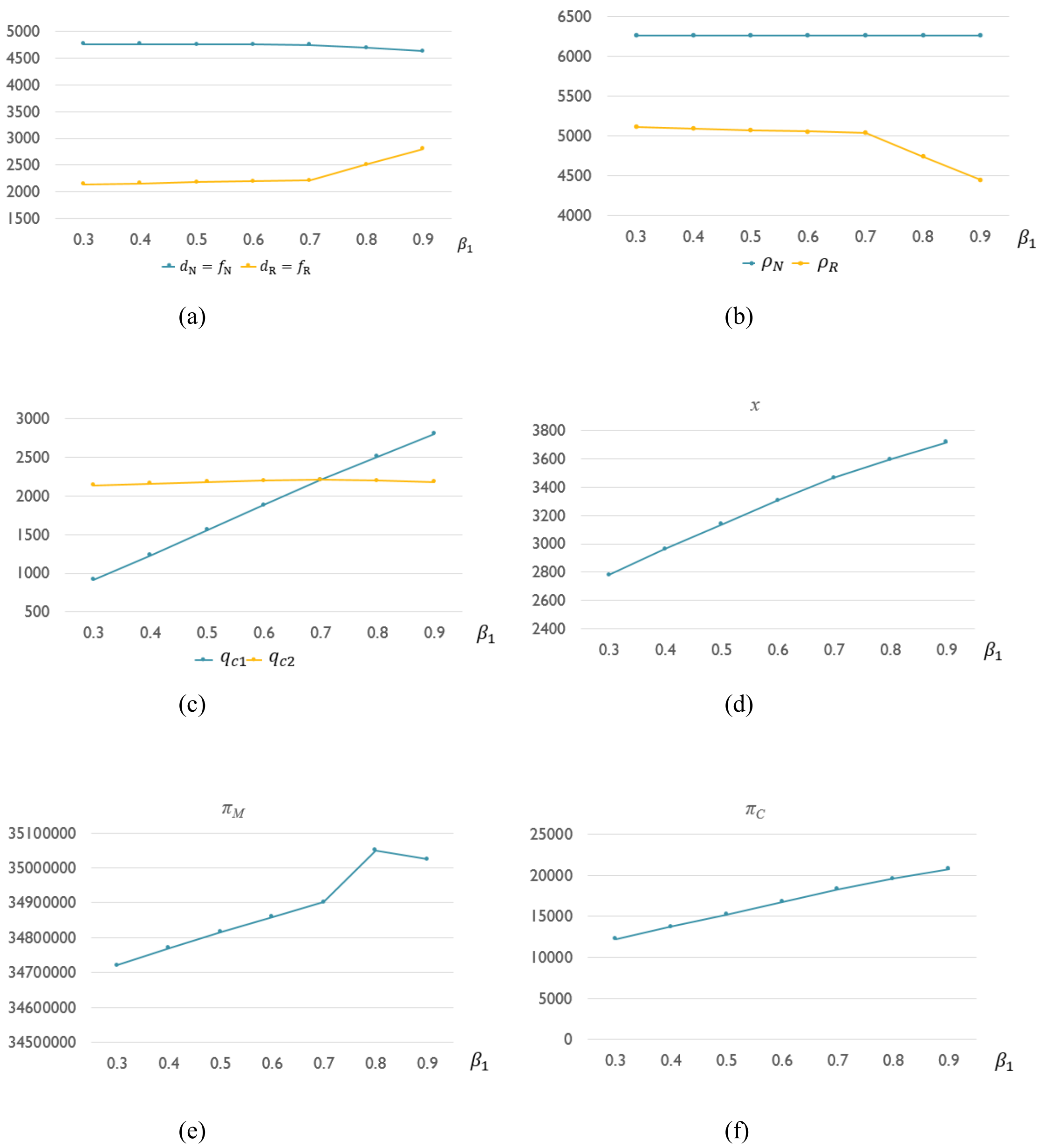


Figure 4. Test results

In Figure 4(a), it is observed that the supply/demand quantity of remanufactured products will increase as β_1 increases. However, if β_1 is less than β_2, which is set as 0.7, the increase is not obvious. The reason is that a remanufactured product may use one or two refurbished components and the quantity is mainly affected by the larger one of β_1 and β_2. Similarly, In Figure 4(b), the price of remanufactured products will decrease as β_1 increases. However, if β_1 is less than β_2, the decrease is not obvious.

Figure 4(c) shows that the higher availability of returned component is, the higher transaction quantity of refurbished components is. In fact, q_{c1} is equal to $\beta_1 g(x) d_N$ in all the test instances, implying that no surplus refurbished components are produced. Figure 4(d) indicates that the collector is willing to invest more return-facilitating costs to increase returned components if β_1 is higher. In Figures 4(e) and (f), it is observed that the higher β_1 generally produces higher profits of the manufacturer and the collector.

6. Conclusions

This study investigates the optimal inventory and pricing policies in a decentralized production/remanufacturing system with multiple refurbished components. The inventory levels are well illustrated and the holding costs are calculated. The proposed model can describe the equilibrium of the system well, and the algorithm can solve the model efficiently. The availability of returned components plays an important role in this study. Numerical examples show its influences on the quantities and prices of the products and recycled components, as well as on the profits. In addition to the results and discussions in Section 5, we emphasize that in a deterministic problem the manufacturer and the collector can change the retail prices and return-facilitating costs to influence the product demands and the quantity of returned end-of-use products, such that no surplus or shortage of any products and components will occur. Since different returned components have different availability, it results that a remanufactured product may contain one or several refurbished components. Considering different prices for different remanufactured products may be a good direction for future research.

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