# Analyses of Anaerobic Batch Digestion of Municipal Solid Waste in the Production of Biogas Using Mathematical Models

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#### **Abstract**

The process dynamics of anaerobic digestion of municipal solid waste (MSW) in a batch bioreactor for the production of biogas has been analysed. An anaerobic batch digester was designed for the treatment of MSW in Port Harcourt metropolis, Nigeria, while at the same time generate biogas as a useful by-product. In the course of the design, the biochemical behaviour of the MSW in batch processing was investigated and analysed. Mathematical models were developed to describe the behaviour of the waste using material balance analysis. The models were validated by the formulation of a Microsoft Visual Basic Version 6.0 programme to simulate the digestion process for a fractional conversion of 0.2-0.8 and Total solids (TS) concentration of 4-30%. The results were analysed using Microsoft Chart Editor and showed that the fractional conversion has various levels of effect on other process parameters like the mean cell residence time, substrate and microbial concentrations and the volume of biogas/methane produced.

Keywords: anaerobic, batch, digestion, municipal solid waste, biogas, models

## 1. Introduction

Municipal Solid Waste (MSW) load in Port Harcourt metropolis, Nigeria increases at a hyper-geometric rate. In 2006 the per capita MSW generation was 1.11kg/person/day (Igoni, Ayotamuno, Ogaji and Probert, 2007). This computation was with a recorded 2003 population figure of 1,356,000 persons. Considering a proportionate progression of waste load and population, by 2016, after about a decade, the waste load may be estimated as 1,947,134.25kg at a projected population of 1,754,175 persons, using a 2% annual incremental factor.

In spite of the huge quantity of MSW generated in Port Harcourt, there has not been any satisfactory approach to manage the waste effectively and efficiently. The management of the waste is, at best, merely the collection and disposal of the waste, which are also improperly undertaken. There are no designated waste receptacles, so wastes are deposited mainly along roads, either by the side or on the median. There are also no engineered landfills for disposal, so borrow pits are used. This improper management of the waste results in huge deposits of refuse along streets and major roads in the city, causing obstruction to human and vehicular traffic and blocking of drain channels, leading to flooding. They elicit offensive odours, destroy road infrastructure and cause pollution of ground water, especially when the waste is not treated before disposal.

The study and development of management strategies for the control and handling of the large quantity of MSW generated in Port Harcourt metropolis has dominated research space for quite some time now (Ayotamuno and Gobo, 2002; Igoni et al, 2007). The large amount of waste generated consistently overwhelms the capacity and capability of the Rivers State Waste Management Agency, the authority in charge of waste management in the area, even in its pedestrian bid to collect and dispose the waste, thus keeping the city continually filthy. This may have led Ayotamuno and Gobo (2002) to observe that the city of Port Harcourt has been turned from its hitherto generally acclaimed status of a 'garden city' to a new and debasing nomenclature of 'garbage city'.

In the entire waste management process in Port Harcourt, there is no treatment component to process the waste to reduce its volume and pollution potential and make it amenable to easy handling and disposal. The treatment of the MSW would tremendously mitigate some of the identified hazards of its poor management. Best environmental practicable option in waste treatment are those that, in addition to achieving the primary goal of treating the waste,

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also convert the waste into useful end products. Igoni et al (2007) investigated the composition of MSW in Port Harcourt and found that the waste consists mainly of organic materials, up to 69.3%. This was indicative of the amenability of the waste to biological decomposition. In Igoni (2006) the behaviour of the MSW subjected to anaerobic degradation in a batch process was investigated. In this paper, mathematical models have been developed for the analysis of the behaviour of the waste in anaerobic batch processing producing biogas. These models would be relevant in the development and characterization of physical systems for the anaerobic digestion of the MSW.

### 2. Theory of Anaerobic Processing

### 2.1 Basic Concept of Anaerobic Digestion

When organic materials decompose, they do so in the presence or absence of air. The decomposition that occurs in the absence of air is referred to as anaerobic decomposition. Several authors have defined anaerobic digestion from different perspectives. Sincero and Sincero, 1999 say it is "a biological decomposition of organic waste done in the absence of air", while the Oregon State Department of Energy, OSDE, (2002) present it as "a biochemical process in which particular kinds of bacteria digest biomass in an oxygen-free environment". Several different types of bacteria work together to break down complex organic wastes in stages, resulting in the production of biogas. This is why Chawla (1985) says anaerobic digestion is "a bioreactor in which organic matter is progressively degraded in the absence of oxygen by a process known as methanogenesis". Anaerobic digestion has been widely used for the treatment of industrial, agricultural and municipal waters and sludge, which accounts for why Hobson, Bousfield and Summers (1981) define it as a method of stabilizing, and thus reducing pollution from the sewage sludge produced in several treatment works; and Reynolds and Richards (1996) say it is the "biological oxidation of degradable organic sludge by microbes under anaerobic conditions". However, Kiely (1998) explains that recently anaerobic digestion is also being applied to the treatment of municipal solid waste and thus offers a more holistic definition when he says that anaerobic digestion is "the use of microbial organisms in the absence of oxygen, for the stabilization of organic materials by conversion to methane and inorganic products, including carbon dioxide". So, anaerobic digestion evolved originally and primarily as a waste treatment process, with biogas generated only as a "waste product"

## 2.2 Microbiology of the Anaerobic Process

The essential components of the anaerobic digestion (AD) process are the organic waste and the bacteria, interacting in an airtight enclosure called anaerobic digester. The organic waste constitutes the 'food source' for the bacteria, which convert it into the various end products and by-products. On the other hand, the bacteria involved in the process are usually facultative anaerobes, described as obligate anaerobes during methanogenesis.

Anaerobic decomposition is a complex process, occurring in three basic stages because of the activities of the variety of microorganisms. Initially, a set of microorganisms converts organic material to a form that a second set of organisms utilizes to form organic acids. Then in the final stage, methanogenic anaerobic bacteria utilize these acids to complete the decomposition process and give off biogas.

Reynolds and Richards (1996) enumerate the three stages involved in anaerobic digestion as i) liquefaction of solids; ii) digestion of soluble solids, and iii) gas production. In describing these three stages Kiely (1998) noted that four different trophic microbiological groups (bacteria) are recognized in AD, and that it is the cumulative effect of all these groups that ensures process continuity and stability. He then explains the three stages thus:

- Hydrolysis: the breakdown of high molecular compounds by hydrolytic and fermentative bacteria to low molecular compounds, as in lipids to fatty acids, polysaccharides to monosaccharides, proteins to amino acids, etc.
- Acidogenesis: where the lower molecular components of fatty acids, amino acids and monosaccharides are converted by acetogenic (acid forming) bacteria to lower molecular intermediate compounds such as propionate, butyrate, formate, methanol and acetate; and
- Methanogenesis: this is the final stage of methane production from hydrogen by hydrogenophillic methanogens, and from acetate by aceticlastic methanogens.

To prosecute these stages the complex organic substrates such as carbohydrates, proteins, fats and lipids will be hydrolyzed into simpler soluble products, which are further converted into acetic acid, hydrogen and carbon dioxide. Kiely (1998) cites Gujer and Zender (1983), as enumerating seven sub-processes of the anaerobic process thus:

- Hydrolyses of complex particulate organic matter
- Fermentation of amino acid and sugars

- Anaerobic oxidation of long chain fatty acids and alcohol
- Anaerobic oxidation of intermediary products
- Acetate production from CO<sub>2</sub> and H<sub>2</sub>
- Methane production by hydrogenophillic methanogenes using CO<sub>2</sub> and H<sub>2</sub>O.

These sub-processes are diagramatized in the flowchart in Figure 1.

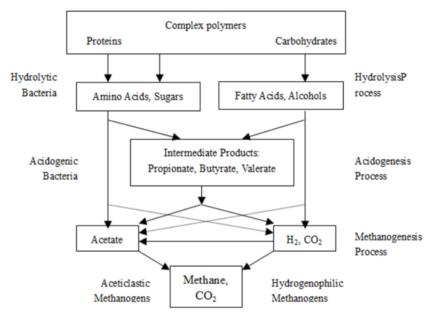


Figure 1. Stages in methane production from organic waste

Note. Adapted from Kiely (1998).

Eckenfelder (2000) further explained that during hydrolysis, there is no reduction of the chemical oxygen demand (COD); but during the conversion of the monomers to volatile fatty acids (VFAs) there is minimal reduction of COD. Eventually when the organic acids are broken down to CH<sub>4</sub> and CO<sub>2</sub>, there is considerable reduction of COD. A schematic of the carbon and hydrogen flow in the anaerobic digestion process is shown in Figure 2.

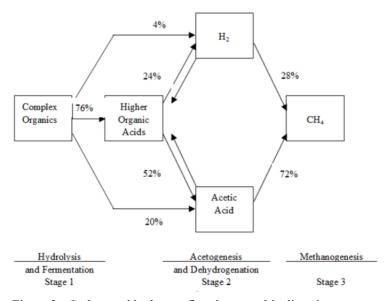


Figure 2. Carbon and hydrogen flow in anaerobic digestion process

Note. Adapted from Eckenfelder (2000).

The breakdown of carbohydrates, nitrogenous compounds and fats can simply be expressed using chemical formula as follows:

$$C_6H_{12}O_6 + 2H_2O \longrightarrow 2C_2H_4O_2 + 2CO_2 + 4H_2$$
 (1)

From the acetic acid and hydrogen products of the above reaction, methane would be produced thus.

$$2C_2H_4O_2 \longrightarrow 2CH_4 + 2CO_2$$
 (2)

$$4H_2 + CO_2 \longrightarrow CH_4 + 2H_2O$$
 (3)

When these expressions are combined, the generalized equation for the anaerobic digestion process is obtained as follows:

## 3. Methodology

## 3.1 Anaerobic Digestion Process Rate Equation

For anaerobic digestion processes, particularly for mixed cultures the biomass degradation rather than the number of organism is of essence. If 'X' represents the mixed population of microorganisms utilizing the organic waste, then the rate of increase in biomass, which is proportional to the initial biomass concentration, is normally modeled as a first-order process (Kiely, 1998, Sincero and Sincero, 1999). The rate equation expressing this first order relationship is of the form:

$$r_x = d[X]/dt = \mu[X] \tag{5}$$

where:

 $r_x = d[X]/dt$  - growth rate of biomass, mg/l/day

X - concentration of biomass, mg/l

μ - specific growth rate of the mixed population, days<sup>-1</sup>

= mass of cells produced / mass of cells present per unit time

T - time, days

By first-order kinetics, as had been stated, if  $X_0$  represents the biomass at time, t = 0, then

$$\int_{x_0}^{x} (d[X]/[X]) = \mu \int_0^t dt$$
 (6a)

integrating:

$$In\left[X\right] = In\left[X_{o}\right] + \mu t \tag{6b}$$

$$In([X]/[X_o]) = \mu t \tag{6c}$$

$$[X] = [X_o] \operatorname{Exp}(\mu t) \tag{6d}$$

However, a growth rate that follows this expression, called the exponential growth rate may not always be the case, particularly for the batch culture where environmental conditions change during its lifetime.

Introducing a conversion parameter called the fractional conversion ( $\alpha$ ), which refers to the 'fraction of the reactant converted to the product' (Levenspiel, 1999), such that if  $X_o$  be the initial concentration of the reactant, and X is the concentration of the reactants at any point in time, t, then the conversion of the reactants in a constant-volume system will be:

$$\alpha = (X_o - X)/X_o = 1 - X/X_o \tag{7}$$

and

$$d\alpha = dX / X_{\alpha} \tag{8}$$

Considering the rate equation in terms of the fractional conversion gives

$$d\alpha / dt = \mu(1 - \alpha) \tag{9}$$

Which upon rearrangement and integration becomes:

$$\int_0^\alpha [d \propto /(1 - \propto)] = \mu \int_0^t dt \tag{10a}$$

or 
$$-\ln(1-\alpha) = \mu t \tag{10b}$$

and a plot of  $\ln(1-\alpha)$  or  $\ln(X/X_o)$  against time is expected to produce a straight line through the origin.

#### 3.2 Bacteria Growth Pattern in Batch Culture

Monod (1949) was the first to identify that in pure cultures,  $\mu$  is a function of or limited by the concentration(s) of a limiting substrate and then developed the empirical equation:

$$\mu = \mu_{\text{max}} \{ [S] / (K_s + [S]) \}$$
 (11)

Where:

 $\mu_{\text{max}}$  - maximum growth rate, days<sup>-1</sup>

S - concentration of limiting substrate, mg/l

 $K_s$  = half saturation constant (i.e concentration of S when  $\mu = \mu_{max}/2$ , mg/l

Therefore, substituting for ' $\mu$ ' in equation (10b) relates the fractional conversion to both the maximum growth rate of biomass and the substrate concentration thus:

$$-\ln(1-\alpha) = \mu_{\max}\{[S]/(K_s + [S])\}t$$
 (12)

Figure 3 describes the growth pattern of bacteria in batch culture. It shows that after an initial lag period for the bacteria to adapt to their new environment, there is an exponential increase in the number of viable cells, which is facilitated by the availability of excess organic matter, and limited only by the ability or otherwise of the microorganism to process the substrate. On the other hand, the declining growth phase arises from a shortage of substrate; and this continues until the number of viable bacteria becomes stationary, when the rate of reproduction is equal to the rate of death.

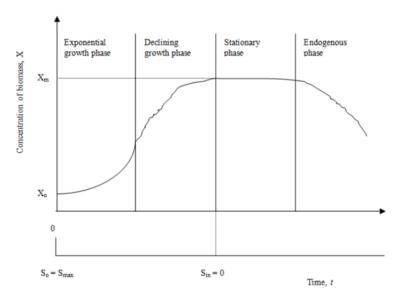


Figure 3. Schematic of microbial growth pattern in batch cultures

Note. Adapted from Viessman and Hammer (1999).

Considering the dynamics of death of microbes in process operation as a decay of microbial population  $k_d[X]$ , where  $k_d$  is the rate of decay or the endogenous decay coefficient, then the model for the rate of increase of the mixed population of microbes [X] becomes

$$d[X]/dt = \mu_{\max}\{[S]/(K_s + [S])\}[X] - k_d[X]$$
(13)

Relating this to the rate equation, then

$$r_{x} = \mu_{\max}\{[S]/(K_{s} + [S])\}[X] - k_{d}[X]$$
(14)

#### 3.3 Substrate Kinetics

Substrate kinetics is founded on the premise that as organisms grow, substrates are consumed; so that the rate of decrease in substrate concentration is proportional to the rate of increase of the concentration of the organism (Reynolds and Richards, 1996). So, in terms of the net growth rate of biomass, equation (12) becomes

$$-\ln(1-\alpha) = \left[\mu_{\max}\{[S]/(K_s + [S])\} - k_d\right]t \tag{15}$$

Now, assuming that all substrate could be converted into biomass, the rate of decrease of the substrate will be described as:

$$-d[S]/dt = d[X]/dt \tag{16}$$

But practically this idealization is not feasible due to inefficiencies in the conversion process. However, introducing proportionality constant, U, gives the following relationship, which is of a more practical relevance.

$$-d[S]/dt = U\mu_{\max} \{ [S]/(K_S + [S]) \} [X] - k_d[X]$$
(17)

or  $-d[S]/dt = \mu_{\text{max}}(1/Y)\{[S]/(K_s + [S])\}[X] - k_d[X]$  (18)

where: U - specific substrate utilization rate

Y - specific yield of organisms, mg/l

These formulations are depicted graphically as shown in Figure 4.

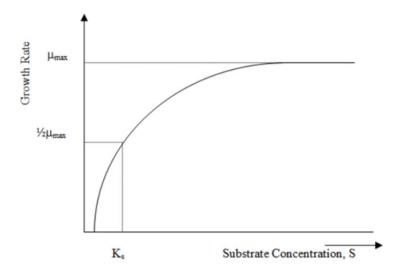


Figure 4. Relationship between the growth constant,  $\mu$ , and the substrate concentration, S *Note*. Adapted from Reynolds and Richards, 1996.

#### 3.4 Material Balance of the Anaerobic Digestion Process

The general approach to the analysis of anaerobic digestion process is the development of material balance expressions during the processes. Several literatures (Tchobanoglous, Burton and Stensel, 2003; Agunwamba, 2001; Kiely, 1998; Reynolds and Richards, 1996; and Tchobanoglous and Burton, 1991) state the general form of a material balance expression as follows:

This is further simplified as:

Accumulation = Inflow + Net growth - Outflow 
$$(19b)$$

And considering the anaerobic digestion process, this expression can be symbolically represented as:

$$(d[X]/dt)V_r = Q[X_o] + V_r \mu_{net} - Q[X]$$
(20)

where: d[X]/dt - rate of change of microorganism concentration in the reactor measured in terms of mass (Mixed Liquor Volatile Suspended Solids), mass MLVSS/unit volume.time

V<sub>r</sub> - volume of reactor

Q - flow rate, volume/time

X<sub>o</sub> - concentration of microorganisms in influent, mass MLVSS/unit volume

X - concentration of microorganism in reactor, mass MLVSS/unit volume

 $\mu_{net}$  - net rate of microorganisms growth, mass MLVSS/unit volume time

Since 
$$\mu_{\text{net}} = \mu_{\text{max}}\{[S]/(K_s + [S])\}[X] - k_d[X]$$
 (21)

then, 
$$(d[X]/dt)V_r = Q[X_a] + V_r(\mu_{\text{max}}\{[S]/(K_s + [S])\}[X] - k_d[X]) - Q[X]$$
 (22a)

or 
$$(d[X]/dt)V_r = Q([X_o] - [X]) + V_r(\mu_{\max}\{[S]/(K_s + [S])\}[X] - k_d[X])$$
 (22b)

This represents the general model for the anaerobic digestion process.

#### 4. Results and Discussion

#### 4.1 Development of the Batch Processing Models

Applying the general form of the material balance expression to a batch process where there is no flow (i.e. Q = 0), the first term of the right hand side of equation (22b) becomes zero;

i.e. 
$$Q([X_0] - [X]) = 0$$
 (23)

# (a) Material balance for mass of microorganism

$$\therefore (d[X]/dt)V_{bd} = [\mu_m[S][X]/(K_s + [S])]V_{bd} - k_d[X]V_{bd}$$
(24a)

or (eliminating V<sub>bd</sub>)

$$d[X]/dt = \mu_m[S][X]/(K_s + [S]) - k_d[X]$$
(24b)

And this represents the mass balance for the mass of microorganisms in the batch reactor.

## (b) Material balance for total substrate utilization

The material balance for the total substrate utilization in a batch process is equally given as;

$$(d[S]/dt)V_{bd} = -\{k[S][X]/(K_s + [S])\}V_{bd}$$
(25a)

or (eliminating V<sub>bd</sub>)

$$d[S]/dt = -k[S][X]/(K_S + [S])$$
(25b)

where: k - maximum rate of substrate utilization per unit mass of cells produced (mass/mass, time)

and 
$$k = \mu_m / Y \tag{26}$$

The solution of equation (25b) is as follows:

$$dt/d[S] = -(K_s + [S])/k[S][X]$$
(27a)

$$dt/d[S] = -(K_s/k[S][X]) - (1/k[X])$$
(27b)

$$dt = -(K_s dS / k[S][X]) - (d[S] / k[X])$$
(27c)

$$\int_{t=0}^{t=t} dt = \int_{S_o}^{S_e} (K_s dS / k[S][X]) - \int_{S_o}^{S_e} (dS / k[X])$$
(27d)

so that 
$$t = (K_s / k[X]) \ln(S_o / S_e) + ([S_o] - [S_e] / k[X])$$
 (28)

This equation (28) expresses the time, t, required to achieve a given fractional conversion ( $\alpha$ ), which is also called the time for batch digestion, obtained from a computer solution of the equation using Simpson's numerical approximation, thus:

Let 'N' represent the range of integration, such that N = 10

then interval 
$$h = (S_e - S_o)/N$$
 (29a)

such that 
$$S(N) = S_o + (N \times h)$$
 (29b)

$$f(N) = -\{ [K_s + S(N)]/(k[X]S(N) - k_d[K_s - S(N)]) \}$$
(29c)

$$S(0) = f(0) + f(10)$$

$$S(1) = f(1) + f(3) + f(5) + f(7) + f(9)$$

$$S(2) = f(2) + f(4) + f(6) + f(8)$$
(29d)

$$t = (h/3)[S(0) + 4S(1) + 2S(2)]$$
(30)

so that

The percentage stabilization, which describes the efficiency of removal of biodegradable waste load is defined as:

$$E = (So - Se)100/So$$
 (31)

#### 4.2 Validation of Models

The models were validated by simulating the anaerobic batch processing with a computer programme using the Microsoft Visual Basic Version 6.0 programming language. The simulation was done through a range of fractional conversion factors of 0.2-0.8 and percentage total solids concentrations of 4–30. The simulation results are presented in Tables 1-5. The simulation considered the effect of the fractional conversion on the time required for digestion; volumes of methane and biogas; effluent substrate and microbial concentrations; and effluent substrate stabilization. The resulting curves from the above relationships were further analysed mathematically using the Microsoft Chart Editor. The resulting curves are shown in Figures 5-10.

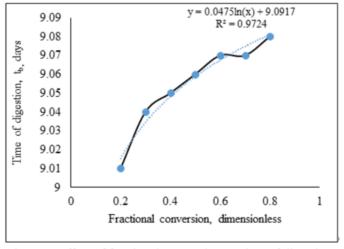


Figure 5. Effect of fractional conversion on time of digestion

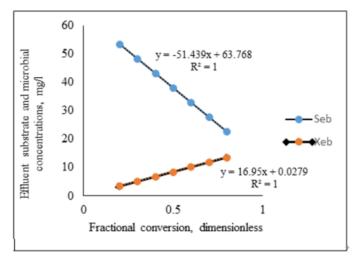


Figure 6. Effect of fractional conversion on effluent substrate and microbial concentration

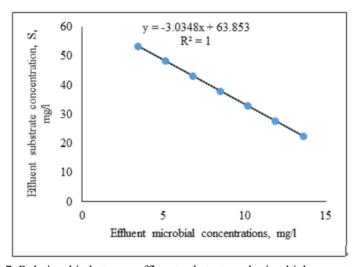


Figure 7. Relationship between effluent substrate and microbial concentrations

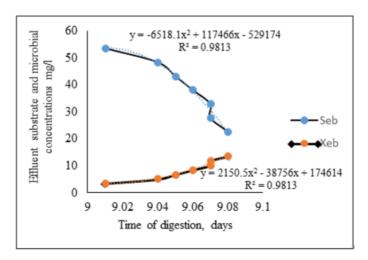


Figure 8: Effect of time of digestion on effluent substrate and microbial concentrations

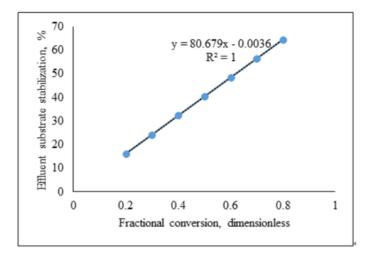


Figure 9: Effect of fractional conversion on effluent substrate stabilization

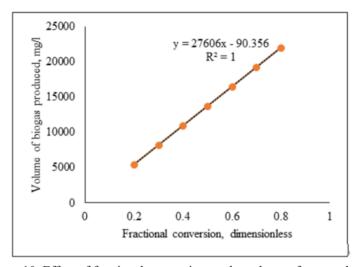


Figure 10. Effect of fractional conversion on the volume of gas produced

Figure 5 shows that the best approximation for the dependence of the time of digestion on fractional conversion is a logarithmic function, with the following mathematical relationship:

$$t_{\rm b} = 0.0475 \ln(\alpha) + 9.0917 \tag{32}$$

From this relationship, after an initial conversion requiring considerable time, successive conversions would require less and less times. This is easily understandable because from Figure 6, while the effluent substrate concentration has a decreasing linear relationship with fractional conversion, that of the effluent microbial concentration is an increasing linear function. The following relationships are respectively established.

$$S_e = -51.439\alpha + 63.768 \tag{33}$$

$$X_c = 16.95\alpha + 0.0279 \tag{34}$$

The linear relationship between effluent substrate concentration and effluent microbial concentration is shown in Figure 7. From equations (32)-(34), an increase in fractional conversion results in increased microbial growth, a decreased substrate concentration and a marginal increase in time of digestion. This is also depicted in Figure 8, relating the time of digestion with effluent substrate and microbial concentrations, respectively resulting in the following second order polynomial functions.

$$S_e = -6518.1t_b^2 + 117466t_b - 529174 \tag{35}$$

$$X_e = 2150.5t_b^2 - 38756t_b + 174614 \tag{36}$$

These findings corroborate the position of Reynolds and Richards (1996) when they said that, in substrate

kinetics, the rate of decrease in substrate concentration is proportional to the rate of increase of microbial growth. Levenspiel (1999) also states that the relationship between time of digestion and fractional conversion is a logarithmic one.

Figure 9 shows that the relationship between the effluent substrate stabilization and fractional conversion is a linear function of the fractional conversion, such that the progression of the conversion process results in a more effective stabilization of the ensuing substrate. The mathematical relationship is as in equation (37).

$$E_b = 80.679\alpha - 0.0036 \tag{37}$$

The effect of fractional conversion on the volume of biogas produced is shown in Figure 10. The more substrate converted, the more gas produced, represented by the linear function in equation (38).

$$V_b = 27606\alpha - 90.356 \tag{38}$$

#### 5. Conclusion

As the population of Port Harcourt metropolis, Nigeria increases at a rapid rate, the management of municipal solid waste can no longer continue on the usual pedestrian level of mere collection and disposal in very inappropriate manners. There is increasing need to treat the waste to achieve an environmentally friendly material for disposal. The analysis of the behaviour of the waste in anaerobic batch processing has provided a springboard for further development of treatment systems for MSW in the city. The various models used in the analysis would be useful in large scale design and development of anaerobic batch bioreactors for effective processing and stabilization of MSW in the generation of biogas.

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# Appendix

Simulation results for the batch anaerobic processing

Table 1. Summary of Batch Digester parameters at 10% TS (i.e. TS = 31186.84, and VS = 20770.43)

α	Se	Xe	$t_d$	$E_b$	$V_{m}$	$V_t$
0.2	5.70	0.393	8.45	16.13	351.92	586.53
0.3	5.15	0.574	8.69	24.20	554.40	924.00
0.4	4.60	0.754	8.82	32.27	758.43	1264.05
0.5	4.05	0.935	8.90	40.18	963.58	1605.97
0.6	3.507	1.115	8.96	48.40	1169.85	1949.75
0.7	2.958	1.296	9.01	56.47	1377.45	2295.75
0.8	2.41	1.476	9.05	64.54	1586.85	2644.75

Table 2. Summary of Batch Digester parameters at 15% TS (i.e. TS = 54603.04, and VS = 36365.62)

α	Se	Xe	$t_{\rm d}$	E <sub>b</sub>	V <sub>m</sub>	$V_{t}$
0.2	12.957	0.853	8.79	16.13	760.98	1268.29
0.3	11.710	1.263	8.91	24.20	1166.72	1944.53
0.4	10.46	1.674	8.97	32.27	1573.36	2622.27
0.5	9.22	2.084	9.00	40.28	1980.79	3301.31
0.6	7.97	2.495	9.03	48.40	2389.08	3981.80
0.7	6.725	2.905	9.05	56.47	2798.47	4664.12
0.8	5.479	3.315	9.07	64.54	3209.38	5348.96

Table 3: Summary of Batch Digester parameters at 20% TS (i.e. TS = 83234.43, and VS = 55434.13)

α	Se	Xe	$t_{\rm d}$	$E_b$	$V_{\rm m}$	$V_{t}$
0.2	23.27	1.507	8.92	16.13	1346.85	2244.75
0.3	21.04	2.244	8.98	24.20	2044.72	3407.86
0.4	18.80	2.981	9.01	32.27	2743.26	4572.10
0.5	16.56	3.718	9.03	40.28	3442.46	5737.43
0.6	14.32	4.456	9.04	48.40	4142.44	6904.06
0.7	12.08	5.193	9.07	56.47	4843.42	8072.37
0.8	9.84	5.93	9.08	64.54	5545.81	9243.01

Table 4: Summary of Batch Digester parameters at 25% TS (i.e. TS = 117081.02, and VS = 77975.96)

α	Se	Xe	td	Eb	Vm	Vt
0.2	36.75	2.36	8.97	16.13	2159.94	3599.89
0.3	33.22	3.52	9.02	24.20	3264.13	5440.22
0.4	29.68	4.69	9.04	32.27	4368.91	7281.52
0.5	26.14	5.85	9.05	40.34	5474.30	9123.83
0.6	22.61	7.02	9.06	48.40	6580.43	10967.38
0.7	19.07	8.18	9.07	56.47	7687.52	12812.54
0.8	15.54	9.35	9.08	64.54	8795.98	14659.97

Table 5: Summary of Batch Digester parameters at 30% TS (i.e. TS = 156142.79, and VS = 103991.10)

α	Se	Xe	td	Eb	Vm	Vt
0.2	53.48	3.42	9.01	16.13	3260.32	5433.87
0.3	48.34	5.11	9.04	24.20	4914.98	8191.63
0.4	43.19	6.81	9.05	32.27	6570.18	10950.29
0.5	38.05	8.50	9.06	40.34	8225.96	13709.94
0.6	32.90	10.20	9.07	48.40	9882.48	16470.80
0.7	27.76	11.89	9.07	56.47	11539.97	19233.28
0.8	22.62	13.59	9.08	64.54	13198.81	21998.02

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