

A Comparative Study on Modeling Methods for Switched Reluctance Machines

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Abstract

Switched reluctance motor (SRM) has many specific advantages and has been gradually used in all kinds of applications. However, because it's inherently multivariable, strong-coupling and highly nonlinear, there are lots of difficulties to build its model. In this paper, many modeling methods include numerical, analytical and intelligent approaches are studied and compared in terms of complexity and accuracy. The static flux linkage and torque characteristics obtained by each method are given. The advantages and disadvantages of each method are summarized as well.

Keywords: Switched reluctance motor (SRM), Modeling methods, Finite element analysis (FEA), Fourier decomposition (FD), Back propagation neural network (BPNN)

1. Introduction

The SRM has the definite advantages of simple magnetless and rugged construction, simple control, ability of extremely high speed operation, relatively wide constant power capability, minimal effects of temperature variations offset, low manufacturing cost and ability of hazard-free operation (W. Xu *et al.*, 2009). These distinct advantages make the SRM well suited to kinds of applications, such as more/all electric aircrafts (S. Song, 2009) and electric vehicles (X. D. Xue *et al.*, 2008).

In contrast to traditional motors (such as induction motors), the SRM is intended to operate in deep magnetic saturation to increase the output power density and the energy conversion factor. Therefore, because of saturation effects and the variation of magnetic reluctance, all the relevant characteristics (i.e., flux linkage, torque) of the machines are highly nonlinear functions of both rotor position and phase current (R. T. Naayagi *et al.*, 2005). These nonlinearities make the modeling process of the SRM relatively difficult.

Due to saturation effects in the SRM, establishing the accurate nonlinear mapping relationship of the flux linkage with respect to phase current and rotor position is the basis of performance calculation (J. Xiu *et al.*, 2007). For years, many modeling methods have been proposed by worldwide researchers (O. Ichinokura *et al.*, 2003; S. Song *et al.*, 2007; Z. Lin *et al.*, 2007). Generally speaking, these methods can be divided into three categories: numerical, analytical and intelligent approaches. They have advantages and disadvantages of their own.

In this paper, three typical methods belong to different categories are studied respectively and their performances are compared in terms of complexity and accuracy. These three methods are based on finite element analysis (FEA), Fourier decomposition (FD) and back propagation neural network (BPNN).

2. Numerical methods

Among numerous numerical methods, FEA is the most frequently used approach (K. N. Srinivas *et al.*, 2005). There are many softwares that can be used to analyze and calculate the electromagnetic field of the motor accurately in two-dimension (2-D) and three-dimension (3-D) ways. Before getting the static characteristics and dynamic performances of the motor, following steps need to be completed firstly.

- (1) Build the model of the motor according to its physical dimension and material.
- (2) Build the model of the power converter according to its topology.
- (3) Set the terminal and boundary conditions.
- (4) Add driving sources.
- (5) Generate the finite element mesh.

Fig. 1 shows the 2-D FEA model of a four-phase 8/6-pole SRM. Fig. 2 shows the model of an asymmetric half bridge power converter, where S1~S8 are voltage-controlled switches, they represent the power switches in the converter. W1~W8 are current-controlled switches, they are used to implement current chopping control. Fig. 3

shows the finite element mesh of the SRM.

After the completion of above steps, the electromagnetic characteristics of the motor can be obtained. Fig. 4 shows the magnetic field distribution inside the motor at one position. Fig. 5 shows the static flux linkage and torque characteristics of the SRM obtained by FEA. It should be noted that the FEA only can get some discrete points (represented by circles in the figure) and the curves are obtained by the “plot” function of the MATLAB to indicate the variation trend of the data.

3. Analytical methods

In analytical methods, the nonlinear relationships of the phase inductance and torque to phase current and rotor position are represented by several analytical expressions. Among numerous analytical methods, the linear and quasi-linear models are well-known. In this paper, a method called FDM is studied. It's based on Fourier decomposition (FD) (J. Hur *et al.*, 2003).

It is well known that the phase inductance curve of the SRM is a periodic function, so it can be expanded in Fourier series as shown in Eq. (1).

$$L(\theta, i) = L_0(i) + \sum_{m=1}^n [L_m(i) \cos(mN_r \theta)] \quad (1)$$

where $L(\theta, i)$ is the phase inductance, $L_0(i)$ and $L_m(i)$ are coefficients, N_r is the number of rotor poles, θ is the angular position of the rotor, $m = [1, n]$ is the harmonic coefficient, n is the maximum harmonic order.

The modeling accuracy is strongly affected by n . In this paper, n is equal to 1. In this case Eq. (1) can be rewritten as

$$L(\theta, i) = L_0(i) + L_1(i) \cos(N_r \theta) \quad (2)$$

To calculate the two undetermined coefficients in Eq. (2), the inductance values at two different rotor positions are needed. In this paper, 0° and 30° (see Fig. 5(a)), namely unaligned and aligned position, are selected. Substituting the inductance values at these two positions into Eq. (2), following equations can be obtained.

$$\begin{cases} L(0^\circ, i) = L_{\min} = L_0(i) + L_1(i) \\ L(30^\circ, i) = L_{\max} = L_0(i) - L_1(i) \end{cases} \quad (3)$$

where L_{\min} and L_{\max} are inductance values at unaligned and aligned positions, respectively.

Those two undetermined coefficients in Eq. (2) can be easily obtained from Eq. (3), as

$$\begin{cases} L_0(i) = (L_{\min} + L_{\max})/2 \\ L_1(i) = (L_{\min} - L_{\max})/2 \end{cases} \quad (4)$$

It is well known that L_{\min} can be seen as a constant, while L_{\max} changes along with the phase current nonlinearly. The analytical expression between L_{\max} and phase current i can be obtained by polynomial fitting, as

$$L_{\max}(i) = \sum_{m=0}^n a_m i^m \quad (5)$$

where a_m are polynomial coefficients that can be obtained by fitting.

The data of phase inductance at aligned position with different phase currents needed for polynomial fitting can be easily obtained from FEA results (see Fig. 5(a)) by following equation.

$$L(i_0, \theta_0) = \frac{\psi(i_0, \theta_0)}{i_0} \quad (6)$$

where $L(i_0, \theta_0)$ and $\psi(i_0, \theta_0)$ are phase inductance and flux linkage at θ_0 with phase current equal to i_0 , respectively.

The analytical expressions of those two undetermined coefficients in Eq. (2) can be obtained by substituting Eq. (5) and the value of L_{\min} into Eq. (4). Finally, the phase inductance can be expressed as

$$L(\theta, i) = \left(L_{\min} + \sum_{m=0}^n a_m i^m \right) / 2 + \left(L_{\min} - \sum_{m=0}^n a_m i^m \right) \cos(N_r \theta) / 2 \quad (7)$$

Based on Eq. (7), the torque can be expressed as

$$T = \partial W'_{mag} / \partial \theta \Big|_{i=c} = \partial \int_0^i L(\theta, i) di / \partial \theta = -\frac{N_r i^2}{4} \left(L_{\min} - \sum_{m=0}^n \frac{2a_m}{m+2} i^m \right) \sin(N_r \theta) \quad (8)$$

where T is the torque, W'_{mag} is the magnetic co-energy.

On the basis of Eq. (6) ~ (8), the static flux linkage and torque characteristics of the SRM can be obtained as shown in Fig. 6.

4. Intelligent methods

There are many intelligent methods that can be used to build the model of SRM, such as artificial neural network (ANN) and fuzzy inference system (FIS). ANN has excellent capacity of nonlinear mapping, adaptive learning and parallel processing. In this paper, the back propagation neural network (BPNN) based on Levenberg-Marquardt (LM) optimization algorithm (Y. Cai, 2006; J. Sun *et al.*, 2005) are used to implement the nonlinear mapping relationship of the flux linkage and torque to phase current and rotor position, namely $i(\psi, \theta)$ and $T(i, \theta)$. This method has strong robustness and adaptive ability and the user does not need have any professional knowledge about the SRM.

The data needed to train the BPNN can be obtained by FEA (see Fig. 5). To verify the approximation and generalization ability of the trained BPNN, the data in Fig. 5 are divided into two parts. In Fig. 5(a), the data with even rotor position values are used to train $i(\psi, \theta)$. In Fig. 5(b), the data with even phase current values are used to train $T(i, \theta)$.

In this paper, the BPNN used to train the $i(\psi, \theta)$ has two inputs (ψ and θ), one output (i) and two hidden layers, both hidden layers have ten neurons. The BPNN used to train the $T(i, \theta)$ also has two inputs (i and θ), one output (T) and two hidden layers, both hidden layers have eight neurons.

Fig. 7 shows the variation of the error along with the epochs during training. It can be seen that the convergence speeds of both BPNN are fast.

Fig. 8 shows the comparison between the simulation results of BPNN and the data from FEA. All the FEA data in this figure are used to train the BPNN. It can be seen that both trained BPNN have strong approximation ability.

Fig. 9 shows the comparison between the simulation results of BPNN and the data from FEA. All the FEA data in this figure are **NOT** used in the training of the BPNN. It can be seen that both trained BPNN have strong generalization ability.

5. Comparison of the modeling methods

The numerical methods can get the static characteristics and dynamic performances of the SRM accurately, but quite complex and time-consuming. Furthermore, it's relatively difficult to implement the control algorithms by FEA softwares. The modeling and computational time of analytical methods is much shorter than that required by numerical methods. However, because of the introduction of many hypotheses and simplifications, the modeling accuracy of analytical methods is worse than that of numerical methods (see Fig. 6). The intelligent methods can find the reasonable balance between modeling time and accuracy.

According to the above-mentioned advantages and disadvantages of each modeling method, the numerical methods are commonly used to verify the performances of the designed motors in detail. The analytical methods are commonly used to briefly study the operating characteristics of the motor and quickly evaluate the performances of different control algorithms. The intelligent methods can be used to some applications in which repetitive computations are required, such as design optimization and dynamic performance analysis.

6. Conclusion

In this paper, three categories of modeling methods, namely numerical, analytical and intelligent approaches, are studied and compared in terms of complexity and accuracy. Three typical methods respectively based on FEA, FD and BPNN are given as examples. The simulation results show that they have advantages and disadvantages of their own. The numerical methods can get the static characteristics and dynamic performances of the SRM accurately, but quite complex and time-consuming. The modeling and computational time of analytical methods is much shorter than that required by numerical methods. However, its modeling accuracy is worse. The intelligent methods can find the reasonable balance between modeling time and accuracy. For different applications, the modeling methods should be selected carefully according to the features of the application and the advantages and disadvantages of the modeling methods.

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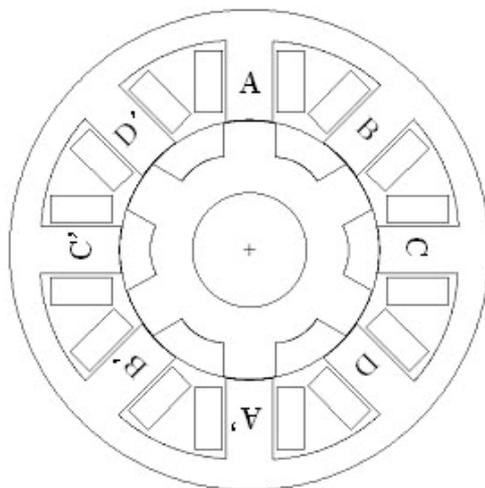


Figure 1. The 2-D FEA model of a four-phase 8/6-pole SRM

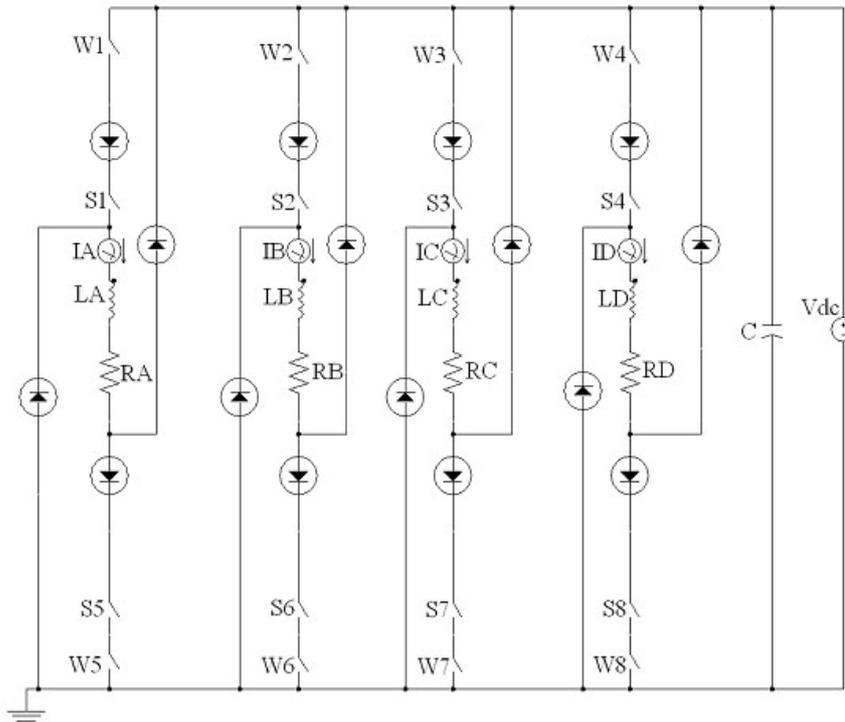


Figure 2. Model of an asymmetric half bridge power converter

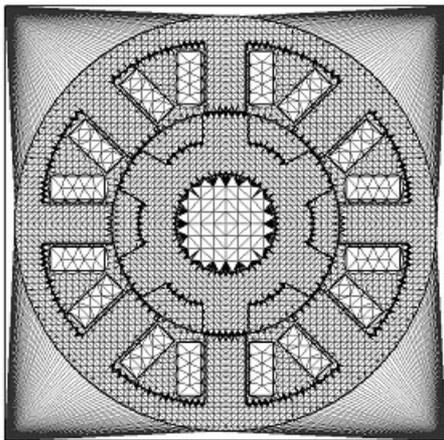


Figure 3. Finite element mesh of the SRM

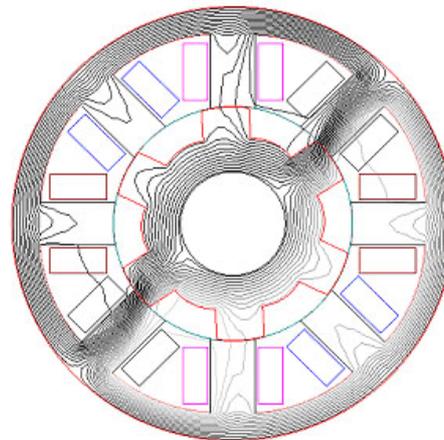
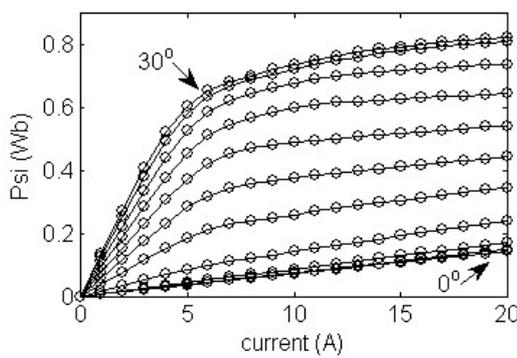
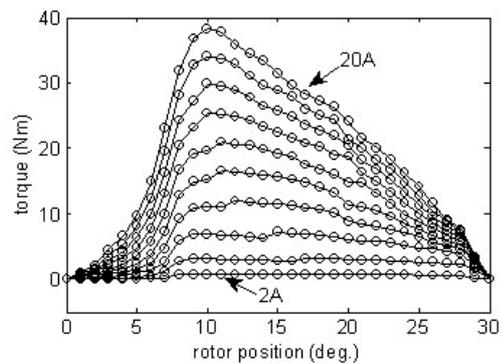


Figure 4. The magnetic field distribution inside the motor



(a) Flux linkage characteristics



(b) Torque characteristics

Figure 5. Flux linkage and torque characteristics obtained by FEA

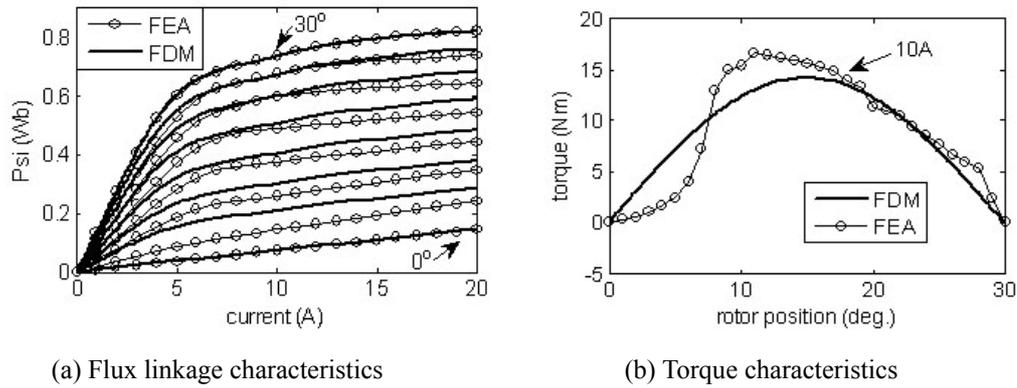


Figure 6. Comparison between FEA and FDM

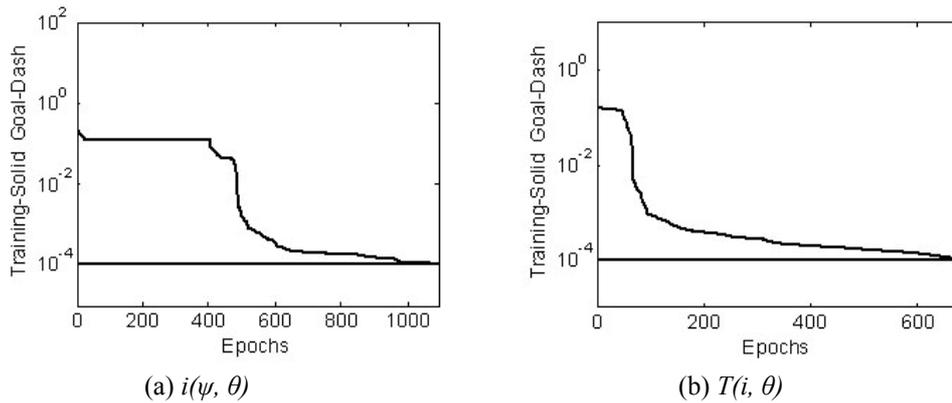


Figure 7. Variation of the error along with the epochs

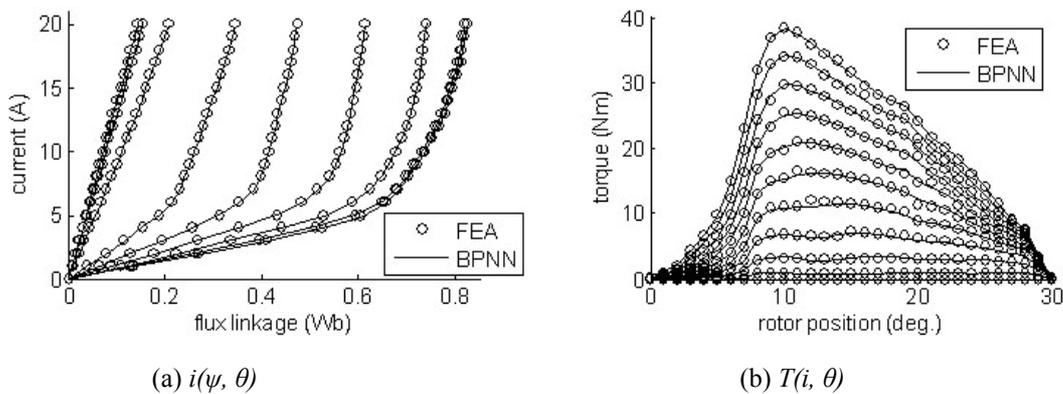


Figure 8. Verification of the approximation ability

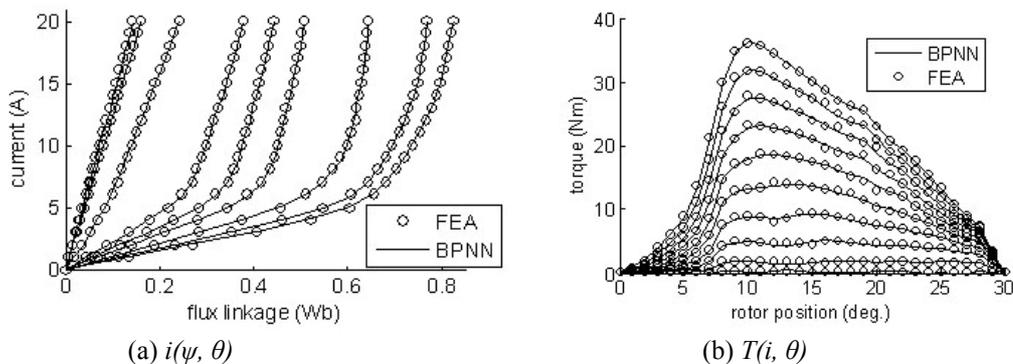


Figure 9. Verification of the generalization ability