



Study on the Logical Ideas in Chinese Ancient Mathematics from “Liu Hui’s Commentary of the Chiu Chang Suan Shu” (Research of the Relations between Calculation and Proof, Arithmetic and Logic)

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Abstract

“Liu Hui’s Commentary of the Chiu Chang Suan Shu” is the best commentary for the most important basic book in Chinese ancient mathematics, the Chiu Chang Suan Shu, and it contains large numbers of logical reasoning and logical ideas. Abundant logical ideas and logical contents in Chinese ancient mathematics were contained in Liu Hui’s Commentary. Based on analyzing the items in Liu Hui’s Commentary and relative ancient literatures, abundant logic contents in the Commentary and the logical contents in ancient mathematics were demonstrated from three angles including concept, logical method and symbolic logic in the article. The relations between calculation and proof, arithmetic and logic in ancient mathematics were also expounded in the article, and the conclusions showed that there was proof logic or arithmetic logic existing in the ancient times of China.

Keywords: Chiu Chang Suan Shu, Chinese mathematics, Logical ideas, Arithmetic

“Liu Hui’s Commentary of the Chiu Chang Suan Shu” is the best commentary for the most important basic book in Chinese ancient mathematics, the Chiu Chang Suan Shu, and it contains large numbers of logical reasoning and logical ideas, so the conclusions that Chinese ancient mathematics had large numbers of logical contents and embodied abundant logical ideas can be proved, just as Wang Dianji said, “in Chinese ancient logical literatures, there were many scientific historical materials to discuss concept, judgment and reasoning, which only need to be collected (Wang, 1961)”.

1. Concept

Concept is the base of the logic. Chinese ancient mathematicians often directly or indirectly defined the mathematical concepts, and used refined words and simple mode to open out the connotation of concept. Qian Baocong said, “The mathematical noun without specific definition will certainly influence the development of theoretical research”. Under the principle of “Shen Bian Ming Fen”, Liu Hui specifically defined a series of mathematical concepts. And his commentary of the Chiu Chang Suan Shu directly specifically defined tens of mathematical concepts (except for a few concept of them (such as “power”) which were created by Liu Hui, most of them didn’t be defined but used in the Chiu Chang Suan Shu, such as Lv, Cui Fen, Lie Cui, cube, Li Yuan, evolution, extraction of cubic root, Yang Ma, Xie Du, Bie XV, Fang Cheng, Zheng Fu, Gou, Gu and Xian) at least (seen in Table 1).

2. Logical methods of deduction and induction

(1) “Liu Hui’s Commentary of the Chiu Chang Suan Shu” firstly “reasons by words and explains by figures”. “Words” are logical rules, and “figures” are visual images. “Reasons by words” means to analyze the references of the formulas, propositions and operation rules by the relation, judgment and reasoning. “Explains by figures” is to use figures to visually express the operation structure and arithmetic process of mathematics.

Chinese ancient mathematics had strict demonstration and emphasized logic, deduction and reasoning. “Every problem in Liu Hui’s Commentary of the Chiu Chang Suan Shu can be decomposed by some judgments connected from

beginning to end, and if we carefully analyze the associations among these judgments, we can find that these judgments have composed many reasoning which compose a proof. Therefore, the commentaries in Liu Hui's Commentary of the Chiu Chang Suan Shu have possessed the structure of proof. For most commentaries, the reasoning in them are deducted reasoning, and most proofs are also deducted proofs (Wu, 1987)." Liu Hui's Commentary of the Chiu Chang Suan Shu used 219 words of "Gu", and except for three of them in the critical interpretation, others all denoted the causality. In ancient books except for Mohist Cannon, the using quantity of "Gu" to denote the causality in Liu Hui's Commentary of the Chiu Chang Suan Shu was most. Just as Mr. Guo Shuchun said, "most of 'Gu' in Liu Hui's Commentary were used in reasoning, i.e. connecting 'Lei', 'Gu' and 'Li', which was 'Tui'..., Liu Hui's 'reasons by words' is 'Tui' in fact".

The existing "Liu Hui's Commentary of the Chiu Chang Suan Shu" only includes words, but the mark of figure still can be seen in the commentary, and the using attention of the figure is to open out the logical references that the formula or operation rule came into existence, and to fully embody the logical idea of "using figure to explain". In addition, Yang Hui had the opinion that "when the problem can not be solved clearly, the figure should be used to prove it" to reason by figures. The beginning of the "Si Yuan Yu Jian" written by Zhu Shijie in Yuan Dynasty also had "Si Yuan Zi Cheng Tu Jie", which indicated the headstream that Chinese ancient mathematics used figures to reason.

(2) The reduction was the character of the logic in the arithmetic theory in Chinese ancient mathematics. A series of similar problems were listed in the Chiu Chang Suan Shu which pointed out the general solution of these problems. But this kind of solution was positively reduced, such as "Yue Fen Shu", "Ke Fen Shu", "Jin You Shu", "Fang Cheng Shu" and "Zheng Fu Shu". The "Kai Fang Zuo Fa Ben Yuan Tu" early in Jia Xian's "Huang Di Jiu Zhang Suan Shu Suan Fa Xi Cao" was reduced from examples. Zhu Zai in the Ming Dynasty gave the general conversion method between novenary decimal fraction and denary fraction, which was obvious result of reduction. Qin Jiu Shao's "Da Yan Zong Shu Shu" was most the prominent representation of the reduction method.

3. The budding of the constructed and mechanized symbolic logic idea

Comparing with Euclid's Elements of Euclid, Chinese ancient mathematics has its own characters, i.e. the combination of the structured property and the mechanical property. 246 problems in the Chiu Chang Suan Shu were structured by the mathematical mode, and each type of problem had a general solution. The contents and system of "Ce Yuan Hai Jing" were also structured, and the volume 1 of "Shi Bie Za Ji" in the 13 volumes contained the definitions, theorems and formulas to solve the problems, and the solutions from volume 2 to volume 13 all can be deduced by the "Tian Yuan Shu" based on that, and as the creed of the book, "Zhu Za Ge Mu" pointed out the basic principle of the whole book. By starting from definitions, theorems and formulas, other propositions could be deduced, and the whole "Shi Bie Za Lun" was the theoretical base to solve problem and reason for the whole book. It is obvious that it was a perfect deduction system and a basic self-sufficient structured mathematical system. In addition, the book of "Shu Xue" also had strong structured and mechanical characters, and its many algorithms had same or similar principles with the program design of computer. For example, if Qin Jiuzhao's arithmetic is used to form the "flow chart" by imitating the symbol and figure process of program design, it would be similar with the arithmetic design of computer.

4. Relations of calculation and proof, arithmetic and logic in Chinese ancient mathematics

Proof is a kind of logical method, and strict math proof is the representative in the proof as the strictness and symbol of the logical proof. Calculation is a kind of programmed computation, and it is the core concept in the math and computer science. This kind of programmed computation is a kind of mathematical proof.

The arithmetic tendency in Chinese ancient mathematics could be regarded as one kind of general logic, and the author thought it was a kind of demonstration logic, not simply a kind of Aristotle deduction logic, because the algorithms in traditional math still had deducted calculation, and Mr. Guo Shuchun also point out that and concretely explained that by the literatures in the Liu Hui's Commentary of the Chiu Chang Suan Shu, but the structured frame and the systematic deduction system with the moods of the syllogism were still deficient, which might be related with Chinese ancient thinking mode and language expression, and the characters of Chinese ancient mathematics. Aristotle logic was based on Euclidian geometry, and took the deduction proof and axiom system as its characters. But the situation in China was different. On the one hand, Chinese cultural background and language environment were different. On the other hand, though China had the budding of logic, but the logics such as Mohism logic still lacked in certain system and formalized characters. From the view of speculative philosophy, Chinese ancient books had the contents about definition, judgment and reasoning, but because of different language expression and mathematical base, they were still different to Aristotle logic. However, Chinese ancient mathematics still has following remarkable characters.

(1) Chinese ancient mathematics conceived abundant logical ideas such as the arithmetic tendency. The mathematical mechanization made by the academician Wu Wenjun is a structured explanation to the ancient mathematics, and it uses mechanical proof to research the abundant contents in ancient mathematics, and open out the mechanical and programmed character of the ancient mathematics which is similar with the present computer proof. And the computer

proof is a kind of logic proof in fact.

(2) Though the logic of Mohism still could not leave the “practical” aspect (more people thought that Chinese ancient mathematics was a kind of “practical math”) and the influences of the polity, culture and language expression, but it still contained abundant logical contents, which came down to the commonness and particularity of logic. For this problem, Wang Dianji’s “History of Chinese Logic Thoughts” made more expatiations. The logical contents were also contained in Chinese ancient mathematics, but they need to be further dug and studied, and they were in various different books and literatures and lacked in a set of complete system. Even the book with most logic contents, Mohist Canon also adulterated many contents such as polity, ethic and natural science, and it was not the logical monograph like Aristotle’s Organon. The “Yin Ming” of ancient India, it was also called Buddha logic, used the three parts of a syllogism to proofing and explaining, and the representative “Yin Ming Ru Zheng Li Lun” and the late “Yin Ming Zheng Li Men Lun” were closed to the syllogism, but same with the Mohism logic, it was a kind of argument logic, and it was not better than proof, and it only pursued win in the talk argument, and it was only used as the tool to explain the Buddha theory and Mohism ethic and political ideas, which was still different with Aristotle’s logic that only pursued the argument without topics and attentions, and was general learning. Therefore, Chinese logic and “Yin Ming” was a kind of argument logic, and Aristotle’s logic was more a reasoning logic. In another word, the former emphasized deduction and induction and the induction occupied more, but the latter more emphasized the deduction. In ancient Greece, the rhetoric method and the argument method were developed, both the Socrates’ spiritual midwifery and Plato’s idea all reflected this kind of argument thinking, which was related with the political and cultural environment at that time, and from this view, Chinese logic and Indian logic all were similar with this aspect. The commonness of the culture was also reflected in this aspect obviously. But Aristotle developed the reasoning system of deduction by the natural language, but that was deficient for China and India. So China didn’t lack in logical ideas and mathematical proof, but lacked in the systematical work. Because of the time background and the language limitation, Chinese Mohist logic had not been developed as mature logic system, which obviously influenced the mathematics. In fact, math had close relation with logic. And logic also influenced the math, which could be found in Liu Hui’s Commentary of the Chiu Chang Suan Shu. In addition Aristotle still had another kind of argument logic which had been ignored for a long time, and it was more close to Chinese Mohist logic and “Indian classical logic”.

(3) The arithmetic in traditional mathematics could be a kind of deductive logic proof, which is obvious. This arithmetic thing doesn’t mean simple thing entering into logic only from the logical meaning. First, the arithmetic embodies the logical content, so it is logical method, and important logic method. It is also regarded as the existence of a kind of arithmetic logic, which is obviously different with the arithmetic logic in the present computer theory, but both of them still have commonness. Second, the arithmetic problem is different with the logic problem. The so-called logic still needs to be defined, and for the generalized logic, the arithmetic problem certainly is the logic problem, but for the special logic (Aristotle’s logic), the arithmetic should be further studied. Third, if the arithmetic is the logic problem, whether the formal logic or the modern logic (sigh logic and symbolic logic) it belongs to? It is obvious that the arithmetic more belongs to the formal logic and the characters of modern logic such as formalization and axiom are different with Aristotle’s logic.

For the relation between arithmetic and logic in Chinese ancient math, the problem of calculation and proof was important and disputable, and there are many works need to be done. Many ancient mathematical literatures need to be understood seriously, and many logical contents in them need to be further studied. Traditional mathematics is extensive and profound, and abundant logical contents could certainly be dug through further researches, which will be very meaningful and valuable for the research about the Mohism logic.

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Table 1. Specific definitions of mathematic concepts in the Chiu Chang Suan Shu

No.	Liu Hui's Commentary of the Chiu Chang Suan Shu	Comparing with other ancient literatures
1	"To multiply length and width is called power."	"To multiply both numbers is called power (Zhu, Shijie in Yuan Dynasty)."
2	"To multiply denominator and numerator is called Qi, and to multiply various denominators is called Tong."	
3	"The comparison of fractions is called Lv."	
4	"Lv is to compare with itself, and if the fraction exists, it can be countable, and if the fraction can be reduced, reduce the fraction. And the comparison of denominator and numerator is Lv (seen in the chapter of Fang Tian in Liu Hui's Commentary of the Chiu Chang Suan Shu)."	
5	"Cui Fen is the meanings of grading."	
6	"Grading is a sort of idea of Lv. (seen in the chapter of Cui Fen in Liu Hui's Commentary of the Chiu Chang Suan Shu)"	
7	"(Evolution) is one face of exponentiation."	"Ping is the same height", "Zhi is the parameter", "Zhong is the same length", and "Fang is the four sides of the pillar", and "End is the thing without depth and in front of the object" (Mohist Canon)
8	"(Extraction of cubic root) is to seek one face of the object, and the object with height and depth is called cube."	
9	"3D round is Wan (seen in the chapter of Shao Guang in Liu Hui's Commentary of the Chiu Chang Suan Shu)."	"Yuan is the set of the points which have same distance to the centre of a circle" (Mohist Canon)
10	"The shape of Yang Ma is one part of the square cone."	
11	"To Xie Jie the cube of the object will obtain two Xie Dus."	
12	"For Xie Jie and Xie Du, one is Yang Ma, and the other is Bie XV (seen in the chapter of Shang Gong in Liu Hui's Commentary of the Chiu Chang Suan Shu)."	

13	“Various objects respectively list their fractions and numerators. Supposed that each row is the rate, and two objects are calculated by two programmable computations, and three objects are calculated by three programmable computations, and in this way, list the number of the rows, which is called Fang Cheng.”	The mathematicians who defined “Fang Cheng” included Liu Hui, Yang Hui, Cheng Dawei, Li Ji, Li Yan and Mei Wending.
14	“The gain and loss in the computation are contrary quantities, so the positive number and the negative number should be used to name them (seen in the chapter of Fang Cheng in Liu Hui’s Commentary of the Chiu Chang Suan Shu).”	
15	“(Gou Gu) is used to calculate the height, depth and length of the object. The short face is called Gou, the long face is called Gu, and the face intersecting above faces is called Xian. Gou is shorter than Gu, and Gu is shorter than Xian (seen in the chapter of Gou Gu in Liu Hui’s Commentary of the Chiu Chang Suan Shu).”	
16	“Complex fractions are different to be used, for example, $\frac{2}{4}$ can be $\frac{4}{8}$, or $\frac{1}{2}$ by the reduction.”	
17	“The deserved number through the subtraction of the combination will be reduced, which can be solved by the fraction method, so the deserved number can be obtained (seen in the chapter of Fang Tian in Liu Hui’s Commentary of the Chiu Chang Suan Shu).”	“Ping Fen means that various factions are different, and to make them accordant, reduce excessive part and increase reduced part, which is called Ping Fen (explained by Li Chunfeng in Tang Dynasty)”
18	Supposed that the diameter is three Chi, and if it is reduced with the circumference, the diameter is 1250, and the circumference is 3927, that is the rate of both numbers.	“Yuan”, “Mou He Fang Gai”, “Qi”, and “Yang Ma” (Liu Hui and Zu Chongzhi)