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Quantitative Analysis in Economics Based on Wavelet Transform: A New Approach

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Abstract

Recently, a number of new techniques to analyze complex, non-linear and non-stationary economic and financial data have been introduced. One of the techniques that may substitute conventional approaches based on a Fourier transform (FT) is wavelet transform (WT). On the other hand, despite the fact that wavelets have a huge potential enabling accurate representation of relationships between economic variables in the time-scale space, their use in economics is still rather limited with apparent reasons. In this paper, we will examine the use of the wavelets for the analysis of complex economic events and introduce the so-called truncated wavelets and an additional metric that may be valuable for processing of real economic and financial data. The presented approach may also contribute to the enhancement of our understanding of economic phenomena. The results are illustrated on a real example.

Keywords: wavelets, variations of GDP, Heisenberg's uncertainty, Gabor transform, Fourier transform, consumer price index, Wavelet transform

1. Introduction

Economists have usually been attempted to explain and forecast variations of financial and economic data using macroeconomic fundamentals. Most of these techniques are based on the quantitative spectral approach using a "stationary signals" assumption applied to real situations. These approaches usually employ a Fourier transform that translates a time-dependent signal f(t) into its representation in the frequency domain F(ω):

$$F(\omega) = \int_{-\infty}^{+\infty} f(t) \exp(-i\omega t)\, dt \qquad (1)$$

One can see that F(ω) is a linear operator and represents a superposition of periodic harmonics (sinωt, cosωt) such that each harmonic does not change its characteristics (period and frequency) over time. This is a severe limitation that does not allow a wide use of a Fourier transform to handle realistic economic and financial phenomena.

Indeed, variations of GDP and stock indices are among myriads of examples when this assumption is no longer valid. In fact, conventional spectral methods have proven to be inadequate to describe the evolutionary nature of realistic time series data in general. The Fourier transform does not allow the frequency content of signal f(t) to change over time and therefore one faces problems reproducing signals that have time-varying features using the Fourier transform. In other words, (1) can tell us how much of each frequency exists in the signal but it does not tell us when in time these frequency components exist.

An analogy is given by human speech: each word of which involves a distinct set of frequencies that last specifically within an exact period of time only.

Let us represent a Fourier transform in a more generalized way:

$$F(\omega,\tau) = \int_{-\infty}^{+\infty} f(t) W(t-\tau) \exp(-i\omega t)\, dt, \qquad (2)$$

where W is some filter. We wrote (2) as an attempt to introduce an additional time domain τ and map function f(t) onto a 2-dimentional plane (ω, τ). Actually, (2) represents a Gabor transform that has recently been used for the analysis of non-stationary phenomena. First of all, we will make the following useful observation. All spectral methods including the Gabor and Fourier representations follow Heisenberg's uncertainty. This uncertainty can be written as follows:

$$\Delta\omega\Delta t \sim 1 \tag{3}$$

The Heisenberg uncertainty states that one cannot represent a signal accurately both in the time and the frequency domains simultaneously. In fact, if we start with a continuous wave (single harmonic) in the time domain, its frequency display shows a single spike corresponding to the frequency of the signal. The significance of this result is that one needs to have the signal continuously on in the time domain to abolish uncertainty in determining its frequency! This means, as an example, that for constructing a perfect Dirac-type spike, we need to employ the information over the whole frequency axes.

The analog of this situation is well-known in physics: to have a well-defined energy, a physical state must last a long time. We cannot precisely know the particle's energy and the exact time the particle obtains this particular energy. The Heisenberg principle poses a very serious constraint on the spectral analysis, especially when the signal is no longer continuous (a usual situation in economics). Besides a Gabor transform introduce a great deal of uncertainty: a width of the window $W(t-\tau)$.

Let us represent filter function W(t) in the Gaussian form:

$$\mathrm{W}(\mathrm{t},\tau)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(t-\tau)^2}{2\sigma^2}\right), \tag{4}$$

where τ is some fixed reference time and σ is analogues to variance in the Gaussian statistics of random variables and describes the window (confidence interval) in which much of energy is located. Substituting W in (2) by its expression (4) yields:

$$\mathrm{F}(\omega,\tau)=\int_{-\infty}^{+\infty} f(t)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(t-\tau)^2}{2\sigma^2}\right)\exp(-i\omega t)\,\mathrm{dt} \tag{5}$$

We can use different parameters of the filter function. Let us consider small values of variance. For small values of variance, the Gaussian function may be approximated by the Dirac delta-function spike and (5) reduces to:

$$\mathrm{F}(\omega,\tau) \rightarrow f(\tau)\exp(-i\omega\tau) \tag{6}$$

One can see that according to the Heisenberg principle, we need a continuous representation of the signal in the frequency domain to achieve a high resolution in the time domain. Since, this is not the case in real situation, a Gabor transform assumes an empirically chosen window (not small and not large) and reduces to the Fourier transforms of a signal in time stripes in which the signal's mean (expected value) does not change much within the chosen window. This corresponds to the situation when we fix $\sigma.A$ Gabor transform fails in practice for narrow windows due to the lack of continuous data. The problem with the narrow window Gabor transform is that it uses constant length windows of small widths. These fixed length windows give the uniform partition of the time space and the Gabor transform reduces to the sums of band-limited Fourier transforms. Surprisingly, being aware of this limitation, the Gabor transform has extensively been used for the analysis of economic and financial data.

Another problem with the Gabor transform is that when even a wide range of frequencies is available, the fixed time window (σ is $const$) tends to contain a large number of high frequencies and a few low frequencies which results in an overpopulation of high frequency components and a lower content of low frequency components. Hence, as the signal is examined under a fixed time-frequency window with constant intervals in the time and frequency domains, the Gabor transform does not allow an adequate resolution for all frequencies. This is one of the major drawbacks of the Gabor transform that will be resolved using a wavelet transform.

What we wish is to have a reliable mathematical tool that will decompose complex financial and economic data into different scales at each fixed time. To achive this, we will not fix σ as in the Gabor transform but allow it to vary. Having this decomposition we will no longer depend on the Hiesenberg principle inherently embedded in all spectral methods. In this case, we will simultaneously see not only long-term variations, but also short-scale wiggles at each given time. This will be achieved via the implementation of the so-called wavelet transform (WT).

The wavelet transform uses local base functions that can be stretched and translated with a flexible resolution in both frequency and time domains. In the case of wavelet transform WT, the time resolution is adjusted to the frequency with the window width narrowing when focusing on high frequencies.

2. Wavelet Transform

Wavelet analysis is a transform when both time and frequency domains are taken into account simultaneously. A continuous wavelet transform maps an original time series, which is a function of just one variable into a function of two variables: time and scale, providing a great volume of information.

The pioneering work of Ramsey and Lampart (1998a and 1998b) and Ramsey (2002) was followed by Aguiar-Conraria, L. and Soares, M. J. (2011a) and Aguiar-Conraria, L. and Soares, M. J (2011b, Rua, A., and Nunes, L. C. (2012), Rua, A., and Silva Lopes, A. (2012), Crowley and Mayes (2008) among others. For comprehensive review papers, the authors send the reader to Adisson, P. (2002), Crowley, P. (2007), Percival, D. and Walden, A. (2000). De Melo (2011) showed the use of wavelet transform and other modern mathematical methods applied to risk analysis.

The continuous wavelet transform can be presented as:

$$\mathrm{WT}[f(a,\tau)]=\int_{-\infty}^{+\infty} f(t)\varphi(t,\tau,a)^{*}dt \tag{7}$$

where asterisk * stands for complex conjugate. $\varphi(t,\tau,a)$ are basis functions. Usually, these basis functions are derived from the so-called mother wavelet $\varphi(t)$ and are defined as:

$$\varphi(t,\tau,a)=\frac{1}{\sqrt{a}}\ \varphi\left(\frac{t-\tau}{a}\right), \tag{8}$$

where τ determines the time position and a is the scale parameter. Let us introduce the following basis functions of a Gaussian wavelet:

$$\varphi(\mathrm{t}-\tau,\sigma,\omega)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(t-\tau)^2}{2\sigma^2}\right)\mathrm{R}\left[\omega,\frac{t-\tau}{\sigma}\right], \tag{9}$$

where $\mathrm{R}(\omega,\frac{t-\tau}{\sigma})$ is the resolution function. If

$$\mathrm{R}\left(\omega,\frac{t-\tau}{\sigma}\right)=\exp[\mathrm{i}\omega(t-\tau)] \tag{10}$$

is the monochromatic wave, then this wavelet reduces to the well-known Morlet wavelet basis functions that describe monochromatic oscillations within the Gaussian envelope. In the Morlet wavelet the ratio of frequency over scale that supplies the image with maximum resolution is fixed. In this wavelet, variance σ has the meaning of scale parameter "a" in (8) and τ is the reference point at the time axes. Unlike the Gabor transform, σ may vary, one can see that low scales (small values of σ) capture rapidly changing details, that is, high frequencies, whereas higher scales (large values of σ) capture slowly changing features, that is, low frequencies.

Now we can formulate an important feature of the wavelet transform. First of all, we can associate WT with the Fourier transform. Secondly, we see that instead of dealing with the time/frequency plane, we decomposed the signal into the time/scale plane that is free of the Heisenberg restriction. Instead we tune frequency within each scale gaining the optimal resolution within each scale. These remarkable properties of the wavelet transform, we will use to consider a realistic example.

Example 1. Analysis of the GDP data

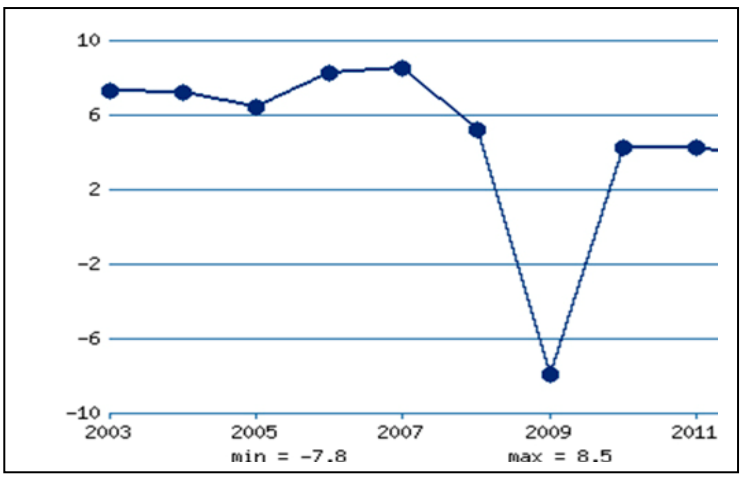


Figure 1. Plot of a Russian GDP data with time

Applying (7) with (9) to the curve depicted on Figure 1, we obtain:

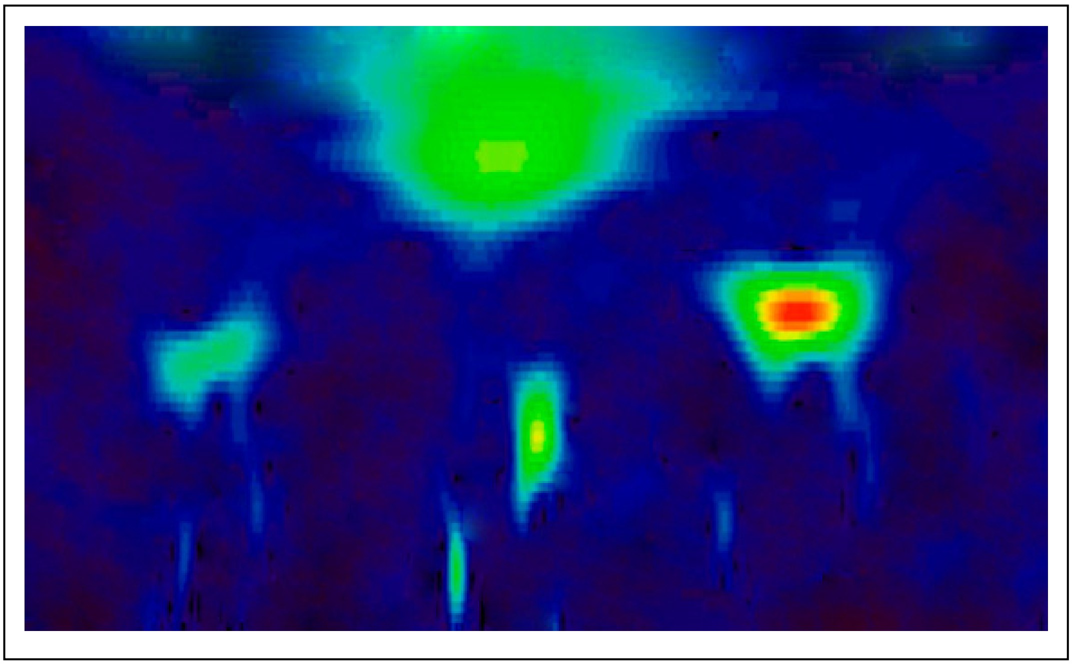


Figure 2. Wavelet transform of the curve presented in Figure 1.

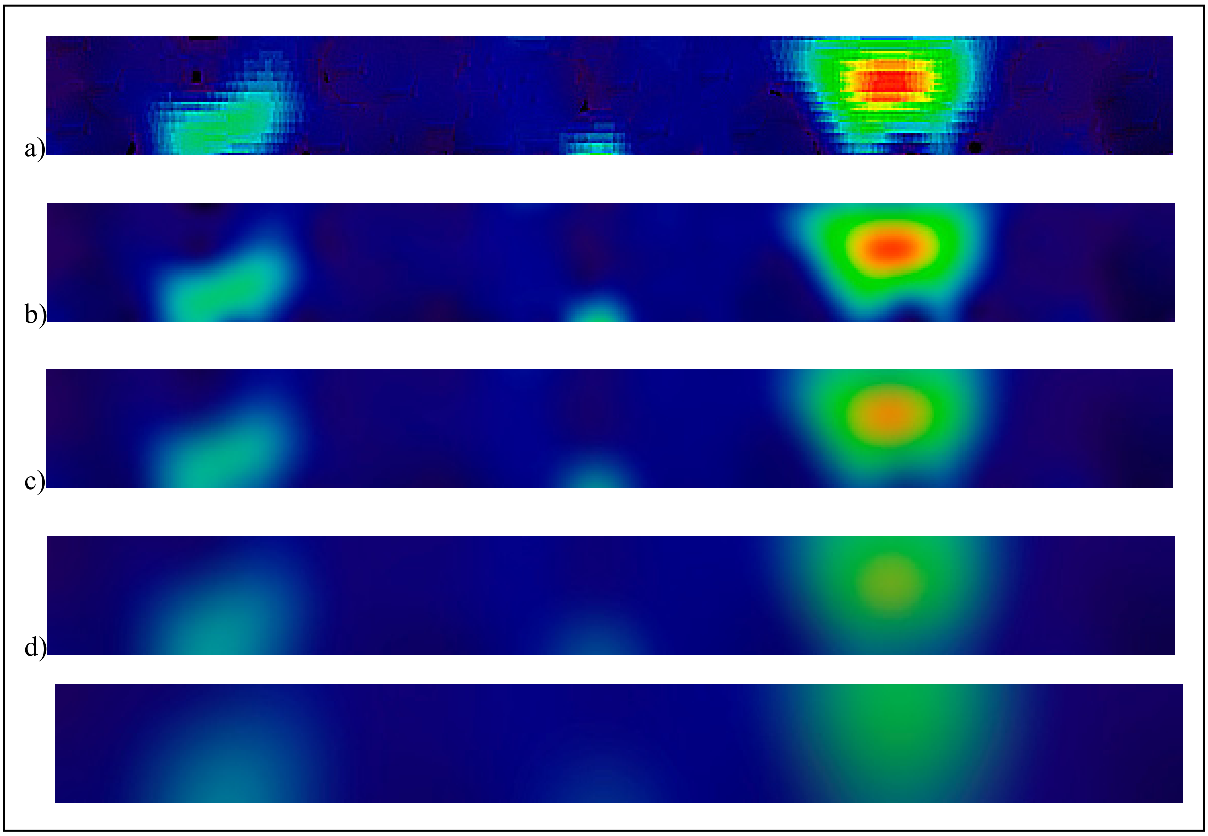


Figure 3. Wavelet decomposition with varying frequencies at a fixed scale (a) $\omega = 8$, b) $\omega = 6$, c) $\omega = 4$, d) $\omega = 2$

For the calculations of the wavelet transform we used the following resolution function:

$$\mathrm{R}\left(\omega, \frac{t-\tau}{\sigma}\right) = \sqrt{2\pi}\,\sigma\exp[-\sigma^2\omega^2/2)]\exp[\mathrm{i}\omega(t-\tau)]\,] \tag{11}$$

We found that this representation of the R-function tunes the scale value to the range of frequencies choosing them to obtain maximum resolution possible. This we clearly see at Figure 3.

3. Wavelet Energy Density

Let us now define the wavelet energy density- a measure that calculates energy over fixed values of time and scale. To this end, we will represent the wavelet energy density as:

$$E\ (\tau,\sigma) = |WT[f(t)]|^2 \tag{12}$$

(14) manifests that for bursts, much of energy is concentrated at large scales and rapidly vanishes at smaller scales. This we see at Figure 2. WT maps the GDP curve in a blip that is well-seen at the larger scales of the 2-D plane. It may be further tuned up by adjusting frequency within the Gaussian envelope (Figure 3a), or can be smoothed for further processing (Figure 3c,d).

It is known that GDP is the best measure of the overall condition of the economy because it includes the output of all sectors. Along with, other indicators have been used. We can try to correlate the blip of Figure 2 with other indicators and to analyze what the blip on GDP corresponds to.

Inflation is another indicator defined as the rate of increase in the general price level of goods and services. The consumer price index (CPI) is used as a measure of inflation. The CPI measures changes in the prices paid for goods and services by urban consumers for the specified month. The CPI is essentially a measure of individuals' cost of living changes and provides a gauge of the inflation rate related to purchasing those goods and services. Two other frequently watched inflation measures are the producer price index, which measures prices producers pay for inputs, and the GDP deflator, the series used to adjust GDP for changes in the overall price level over time. Analysts watch trends in these series, as well as interest rate spreads, the yield curve, and measures and surveys of inflation expectations to measure both the level of inflation and inflation expectations in the economy. We will consider inflation data for the time span (2003-2013) presented by the following Table.

Table 1. Monthly inflation data in Russia.

2013	0,97	0,56	0,34	0,51	0,66	0,42	0,82	0,14	0,21	0,57	0,56	0,58	5,91
2012	0,50	0,37	0,58	0,31	0,52	0,89	1,23	0,10	0,55	0,46	0,34	0,54	6,58
2011	2,37	0,78	0,62	0,43	0,48	0,23	-0,01	-0,24	-0,04	0,48	0,42	0,44	6,10
2010	1,64	0,86	0,63	0,29	0,50	0,39	0,36	0,55	0,84	0,50	0,81	1,08	8,78
2009	2,37	1,65	1,31	0,69	0,57	0,60	0,63	0,00	-0,03	0,00	0,29	0,41	8,80
2008	2,31	1,20	1,20	1,42	1,35	0,97	0,51	0,36	0,80	0,91	0,83	0,69	13,28
2007	1,68	1,11	0,59	0,57	0,63	0,95	0,87	0,09	0,79	1,64	1,23	1,13	11,87
2006	2,43	1,66	0,82	0,35	0,48	0,28	0,67	0,19	0,09	0,28	0,63	0,79	9,00
2005	2,62	1,23	1,34	1,12	0,80	0,64	0,46	-0,14	0,25	0,55	0,74	0,82	10,91
2004	1,75	0,99	0,75	0,99	0,74	0,78	0,92	0,42	0,43	1,14	1,11	1,14	11,74
2003	2,40	1,63	1,05	1,02	0,80	0,80	0,71	-0,41	0,34	1,00	0,96	1,10	11,99

The inflation curve is given by the following chart.

Figure 4. Inflation curve with years

In fact, the indicators range from labor market conditions to industrial production, from monetary policy indicators and interest rates to fiscal policy, from regional and domestic to international indicators, from oil prices to stock market indices. Reflecting the complexity of the economy, government agencies review these charts and tables, as well as the results of econometric models, when they evaluate the economic health of the nation. What is surprising though, that powerful mathematical tools that are efficiently used in other fields of science and technology have not found yet the right path in economy. For the sake of simplicity (other indicators we will leave for further analysis), we will consider the inflation data and compare them with the GDP variations.

We will now introduce a variable that will be useful for the analysis of relationships between different indicators. Suppose we have two time-dependent functions (in our case: GDP and inflation variations with years). We perform the Wavelet transform to both functions and will compute WT (GDP) and WT (INF) respectively. Both functions depend on time and scale. We determine the cross-correlation function as:

$$\Phi(GDP, INF)(t,\sigma) = \int_{=-\infty}^{+\infty} WT(GDP)[\tau+t,\sigma]\, WT(INF)[\tau,\sigma]^* d\tau \quad (13)$$

Here, we should make an important remark. Many published papers on economics and econometrics come up with confusing and often erroneous results by the following reason. When a time series is non-stationary (GDP or price curves, say), the limitations of methods that calculate autocorrelation and cross-correlation that assume stationarity are evident. In fact, suppose a time series GDP(t) has a large upward trend as we see on the plot of the GDP EU data. Then a large value of GDP(t)) is more likely to be followed by a large value of GDP(t+δt)). This implies large GDP autocorrelations, not because large autocorrelations actually exist, but because the autocorrelation function is being used for a non-stationary time series violating the stationarity yielding erroneous results.

The same situation one can observe while computing cross-correlation of the signals. To this end, one can come to the following result: a market index time series will strongly cross-correlate with any other time series that has a large upward trend, though there is no true cross-correlation at all. This is a very serious observation and should be taken care of accordingly.

We will now introduce another function that correctly represents the coherence metric of time series. We see that coherence measures (autocorrelation and cross-correlation) cannot be applied to non-stationary signals. The main reason for this is their trends that may contribute to false quantities of coherencies. On the other hand, we know that the trends are represented by large scales. We will then remove large scales from the wavelet transform and will consider the truncated values of the WT.

$$\Phi(GDP, INF)(t,\sigma) = \int_{0}^{+\infty} ZWT(GDP)[\tau+t,\sigma]\, ZWT(INF)[\tau,\sigma]^* d\tau, \quad (14)$$

where Z is a truncation operator

$$Z = \begin{cases} 1 \; if & 0 < \sigma < \sigma_0 \\ 0 & otherwise \end{cases} \quad (15)$$

In this case, σ_0 is a fixed scale that represents the trend of a time series. Now $ZWT(GDP)$ represents a stationary signal as well as $ZWT(INF)$ and we can apply the covariance analysis to analyze the coherency of these signals. What we see from (14) is that the wavelet transform acts as an ideal detrending operator that converts non-stationary signals into stationary ones.

We will introduce the so-called cross-correlation measure that has a meaning of cross-energy spectrum distributed over the time-lags

$$E_\Phi = max_\sigma[\Phi(GDP, INF)(t,\sigma)]^2 \quad (16)$$

With the truncation operator in the wavelet transform plane can form the autocorrelation function that is a wavelet power spectrum:

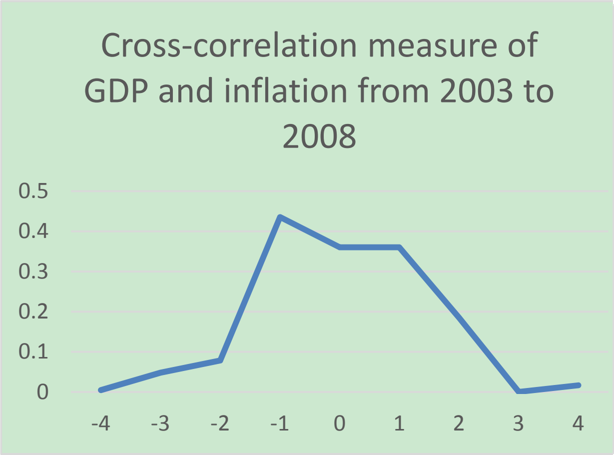
$$PS\,(\sigma) = \int_0^T |ZWT(f)[\tau,\sigma]|^2 \; d\tau, \quad (17)$$

where T defines the upper limit on the time axes. We can also from now the cross power spectrum CPS that appears to be valuable for the concurrent analysis of two signals f and g.

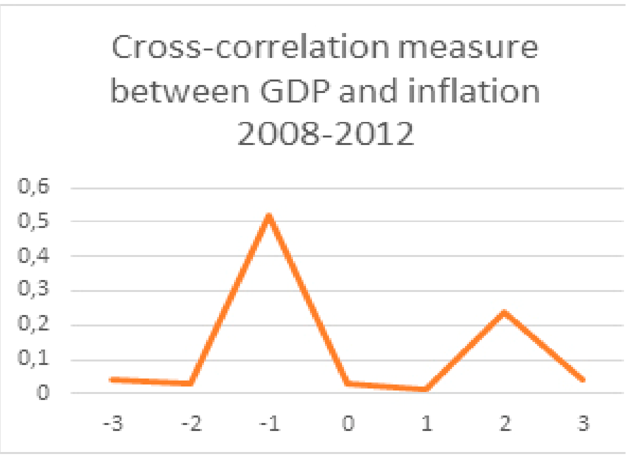
$$WCPS\,(\sigma) = \int_0^T ZWT(f)[\tau,\sigma] ZWT(g)[\tau,\sigma]^* \; d\tau, \quad (18)$$

Let us consider again two functions: the first one is the GDP plot (Figure 1) and the inflation curve for the same period of time (Figure 4). We will calculate the cross power spectrum of two signals.

This yields



a) 2003-2008



b) 2008-2011

Figure 5. Cross-correlation measure of the wavelet transform defined by (16)

One can see a very good correlation of signals. In the first period (figure a), variations of GDP and inflation match at zero lags, maximizing the cross-power spectrum, while drop in GDP and inflation are seen with the time lag with much smaller values of the cross-power spectrum.

4. Conclusion

As it was mentioned in the paper, the use of wavelet transform in economics and finance is much smaller compared to other fields of science and technology. One of the reasons for this is a common practice to use conventional methods based on a Fourier or Gabor transform, though it is evident that they cannot be applied to real data due to their highly non-stationary characteristics. Another reason is that, the such commonly used in economics estimators as covariance analysis of the available indicators such as GDP, money supply, consumer price index, producer price index, etc. fails again as the cross-correlation analysis fails again applied to wavelet transform.

In this paper, we considered a generalized wavelet that in a particular case of unit amplitudes reduces to the Morlet wavelet. The introduced wavelet appears to be handy for the wavelet processing as it enables even very small details.

Secondly, we have introduce the truncated wavelet transform ZWT that allows to remove large scales and thus trends. The truncated wavelet transform may be very practical and may evoke interest within the economists as they transform non-stationary signals and thus a common cross-correlation analysis used by economists will make sense.

We introduced a convenient cross-correlation measure that is illustrated on realistic signals. This analysis shows some remarkable properties of the truncated wavelets making them powerful tools to be used in economics for the cross-analysis of highly non-stationary, realistic problems.

References

Adisson, P. (2002). *The illustrated wavelet transform handbook.* The Institute of Physics, London.

Aguiar-Conraria, L., & Soares, M. J. (2011a). Business Cycle Synchronization and the Euro: a Wavelet Analysis. *Journal of Macroeconomics, 33*(3), 477-489. http://dx.doi.org/10.1016/j.jmacro.2011.02.005

Aguiar-Conraria, L., & Soares, M. J. (2011b). Oil and the macroeconomy: using wavelets to analyze old issues. *Empirical Economics, 40*(3), 645 - 655. http://dx.doi.org/10.1007/s00181-010-0371-x

Bezrukova, T., & Petrov, P. (2012). Determination of relevance of use of the mechanism of strategic controlling in a control system of the industrial organization. *Economy in the industry, 2*, 61-72.

Bezrukova, T., & Petrov, P. (2013). Formation of the concept of strategic controlling at the industrial enterprises. *Management accounting, 2*, 31-40.

Bezrukova, T., Morkovina, S., Bezrukov, B., & Popkova, E. (2013). Methodological Approach to the Identification of Predictive Models of Socio-Economic Processes for Investment and Innovative Development of Enterprises. *World Applied Sciences Journal, 26*(1), 20-27.

Crowley, P. (2007). A guide to wavelets for economists. *Journal of Economic Surveys, 21*, 207-264. http://dx.doi.org/10.1111/j.1467-6419.2006.00502.x

Crowley, P., & Mayes, D. (2008). How Fused is the Euro Area Core? An Evaluation of Growth Cycle Comovement and Synchronization Using Wavelet Analysis. *Journal of Business Cycle Measurement and Analysis, 4*(1), 63-95.

De Melo, F. (2011). Mathematical aspects of risk analysis. *Technical report,* № 5-EBRD, 1-65. European bank for reconstruction and development.

Percival, D., & Walden, A. (2000). *Wavelet methods for time series analysis*. Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511841040

Ramsey, J. (2002). Wavelets in Economics and Finance: Past and Future. *Studies in Nonlinear Dynamics& Econometrics, 6*(3), article 1. http://dx.doi.org/10.2202/1558-3708.1090

Ramsey, J., & Lampart, C. (1998a). Decomposition of economic relationships by time scale using wavelets. *Macroeconomic dynamics, 2*(1), 49-71.

Ramsey, J., & Lampart, C. (1998b). The decomposition of economic relationships by time scale using wavelets: expenditure and income. *Studies in Nonlinear Dynamics and Econometrics, 3*(1), 23-42. http://dx.doi.org/10.2202/1558-3708.1039

Rua, A., & Nunes, L. C. (2012). A wavelet-based assessment of market risk: The emerging markets case. *Quarterly Review of Economics and Finance, 52*, 84-92. http://dx.doi.org/10.1016/j.qref.2011.12.001

Rua, A., & Silva Lopes, A. (2012). *Cohesion within the euro area and the U. S.: a wavelet-based view.*