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# Limiting Velocities of Primary, Obscure and Normal Particles: Self-Annihilating Obscure Particle as an Example of Dark Matter Particle

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**Abstract**

From recently established bicubic equation, three particle limiting velocities are derived, primary, $c_{1,}$obscure, $c_2$ and normal, $c_{3,}$that in principle may belong to a single particle. The values of limiting velocities are governed by the congruent particle parameter, $z = 3\sqrt{3}mv^2/2E$, with $m, v$ and $E$ being, respectively, particle mass, velocity and energy, generally satisfying $-1 \leq z \leq 1$, and here just $0 \leq z \leq 1$.

While $c_3$ is practically the same in value as $v$, $c_1$ and $c_2$ can depart from $v$ as $z$ changes from 1 to 0, since $c_{1,}c_2$ and $c_3$,are, in forms, explicitly different from each other, which offers the chance to look at possible new forms of matter, such as dark matter. For instance, one finds that $c_3$ could be slightly different from $c$, the velocity of light, for the 2010 Crab Nebula Flare PeV electron energy region and for the OPERA 17 GeV muon neutrino velocity experiments, while at the same time, although not measurable in these experiments, calculated $c_1$ and $|c_2|$, are numerically about $10^5$ times larger than $c_3$.

There is a belief that an exemplary particle of small velocity, $v = 10^{-3}c$ ,and small energy, $E = 1eV$ , but as yet of not known mass, should belong to the dark matter class. Once knowing $z$ the value of the mass is fixed with $3\sqrt{3}m(z)v^2 = 2Ez$ ,and its maximum value $m(1)$ is at $z = 1$, $m(1) = 2E/\left(v^2 3\sqrt{3}\right)$.This mass value defines the test particle, with which one calulates primary, obscure and normal particle rest energies at $z = 1$. Snce at $z = 1$ theory predicts $c_1^2(1) = (3/2)v^2, -c_2^2(1) = 3v^2, c_3^2(1) = (3/2)v^2$, the rest energies are $m(1)c_1^2(1) = m(1)c_3^2(1) = 0.58eV$ and $m(1)\left(-c_2^2(1)\right) = 1.15eV$. The primary and normal particles, with positive kinetic energies self-creation process increase their energies from $0.58eV$ to desired$1eV$. The obscure particle, with negative kinetic energy self-annihilation process decreases its energy of $1.15eV$ to desired $1eV$. This makes the obscure (imaginary $c_2$) particle as a good candidate for a dark matter particle,since as it is believed that a trapped dark matter particle with self-annihilation properties helps keeping the equilibrium between capture and annihilation rates in the sun.

**Keywords:** Bicubic Equation, Lmiting velocities, Dark Matter particles

## 1. Introduction

The three solutions of the recently established particle limiting velocity bicubic equation (Soln, 2014, 2015) sets a particle into three possible categories: a primary particle with primary limiting velocity $c_1$, an obscure particle with the obscure imaginary limiting velocity $c_2$ and a normal particle with the normal limiting velocity $c_3$. Each of the limiting velocities depends on real particle parameters, mass $m$, ordinary velocity $v$ and energy $E$ through the dimensionless congruent particle parameter $z = 3\sqrt{3}mv^2/2E$ , with generally, $-1 \leq z \leq 1$, but here sufficiently that $0 \leq z \leq 1$ . In terms of this parameter, the three particle limiting velocity solutions satisfy the zero limiting velocity squares sum rule, $c_1^2 + c_2^2 + c_3^2 = 0$, which is possible because $-c_2^2 \geq 0$.

By establishing the three particular numerical congruent parameter $z$ identities, one derives the Lorentz-like expressions for energies and momenta for the primary ($c_1$), obscure ($c_{2)}$ and normal ($c_{3)}$ particle. The Lorentz-like $\gamma$ factors show singularities for the primary and normal energy expressions, respectively at $v^2 = c_1^2$ and $v^2 = c_3^2$, while for the obscure energy expression a singularity occurs at unphysical imaginary velocity $v$, $v^2 = -|c_2|^2$.

Furthermore, numerically one finds out that when the congruent parameter $z$ varies from 1 to 0, $c_1^2/v^2$ and $-c_2^2/v^2$ increase, while $c_3^2/v^2$ stays close to 1. This indicates that $c_3$ and $v$ are practically equal to each other with specialty that at $z = 0$, $c_3 \simeq v \simeq c$ , while $c_1$and $c_2$ are variable limiting velocities with increasing absolute values as $z \longrightarrow 0$. Also, concerning the question of Lorentz invariance (LI) or Lorentz violation (LV), one finds that the primary particle with usually the limiting velocity being much larger than the velocity of light, $c_1 \gg c$, is most likely LV, while the obscure particle with imaginary limiting velocity $c_2$, usually satisfying, $|c_2| \gg c$, is LV by definition. That means that the normal particle with normal limiting velocity $c_3$ supports either LI or LV when its value, respectively is either $c_3 = c$ or $c_3 \neq c$. This kind of questions are addressed in the 17 $GeV$ OPERA velocity neutrino experiment (Strauss, 2014) and in the $PeV$ energy region of neutrino and electron velocities observations from the 2010 Crab Nebula Flare (Stecker, 2014) and the Ice Cube PeV events (Aartsen et al., 2013; Bezrukov & Gorbunov, 2015).

Recently Fan, Reece, and Wang, (2010), M. Cirelli et al. (2013) and F. Bezrukov et al. Bezrukov and Gorbunov (2015) assumed that the small value particle velocity of $v = 10^{-3}c$ and small energy $E \simeq 1eV$ would belong to the dark matter class. With fixed $v = 10^{-3}c$ and $E \simeq 1eV$ the maximum mass for this test particle is at the congruent parameter $z = 1$ , $m(1) = 2E/\left(v^2 3\sqrt{3}\right), m(1)\,v^2 = 0.38eV$. This, with squares of limiting velocities at $z = 1$,yield the primary, obscure and normal particle rest energies as, $m(1)c_1^2(1) = m(1)(3/2)\,v^2 = 0.58eV, m(1)\left(-c_2^2(1)\right) = m(1)3v,^2 = 1.15eV, m(1)c_3^2(1) = m(1)(3/2)\,v^2 = 0.58eV$ .The self-creation energy processes of primary and normal particle changes $0.58eV$ to $1eV$ in energy, while the self-annihilation energy process of obscure particle decreases.$1.15eV$ to $1eV$ in energy.This fact makes it a good dark matter particle since in Adrian-Martnez et al. (2016) a particle with such a property helps keeping the equilibrium between capture and annihilation rate in the sun, for example. At these low energy the absolute values of $c_1$, $c_2$ and $c_3$ are all bellow $c$, the velocity of light.

In Section 2 the necessary details from the bicubic equation particle limiting velocity solutions are reviewed and assorted. Here, through the inversion of limiting velocity solutions, the three limiting velocity congruent functions, primary congruent function, $s_1$, obscure congruent function, $s_2$, normal congruent function, $s_3$, are introduced. These, in addition to $z$-congruent function, are used to prove important inequalities of $c_1$, $c_2$ and $c_3$ with respect to energy $E$. Also, in the limit of small ordinary particle velocity $v$,the limiting velocities in terms of particle energy $E$ are established. Associated with three limited velocity forms, $c_1$, $c_2$ and $c_3$, here, within the inverse trigonometric function formalism, three identities for the dimensionless congruent variable $z = 3\sqrt{3}mv^2/2E$, are presented. With their help, three particle energy expressions with Lorentz-like factors, are derived associated with the primary, $c_1$, obscure, $c_2$ and normal, $c_3$, limiting particle velocity forms.

Numerical evaluations and physical interpretations of results are done in Section 3. The three newly derived particle energy expressions for the primary, obscure and normal particle with respect to $c_1$, $c_2$ and $c_3$ limiting velocities, are here analyzed. The normal particle energy expression enlighten the results of the analysis in Soln (2014, 2015) directly from the solution of limiting velocity $c_3$ for 17 $GeV$ OPERA velocity neutrino measurement (Strauss, 2014) as well as in the $PeV$ energy region of neutrino and electron velocities observations from the 2010 Crab Nebula Flare (Stecker, 2014) and the Ice Cube PeV events (Aartsen et al., 2013; Bezrukov & Gorbunov, 2015). Here,from the energy point of view, one looks for possible dark matter qualities among the primary, obscure and normal particles. It appears, rather naturally, that the obscure particle with $c_2$ limiting velocity shows the biggest promise followed by the primary particle with $c_1$ limiting velocity and the normal particle with $c_3$ limiting velocity with a possibility of being a dark matter particle. Here also the specific assumption from (Fan, Reece, & Wang, 2010; Cirelli, Del Nobile, & Panci, 2013; Bezrukov & Gorbunov, 2015) that the low velocity $v = 10^{-3}c$ and small energy $E \simeq 1eV$ should be the characteristic of a particular dark matter particle. With the congruent parameter value $z = 1$ one obtans maximum primary, obscure and normal mass energy values. The positive kinetic energies contributions of primary and normal particles performed the self-creation process by increasing $E(c_{1,3})$ from $0.58eV$ to $1eV$ , while the obscure particle energy having the negative kinetic energy contribution performed the self-annihilation process by decreasing $E(c_2)$ from $1.15eV$ to $1eV$ . Of hand, the obscure (imaginary $c_2$) particle with self-annihilation process is most acceptable as a dark matter particle. This particularly so (Adrian-Martnez et al., 2016) because when trapped in massive astrophysical object, they, with self-annihilation property,help reaching equilibrium between capture and annihilation rates in this object.

Section 4 is devoted to summarizing of the main results and giving an overview of further problems and possibilities.

## 2. Elements of the Limiting Velocity Bicubic Equation Solutions

As shown in Soln (2014, 2015) the velocity of light in the Einstein's kinematics is taking the place of particle limiting velocity. Hence combining the particle mass shell condition $\overrightarrow{p}^2c^2 - E^2 + m^2c^4 = 0$ with the momentum $\overrightarrow{p} = E\overrightarrow{v}c^{-2}$ one ends up for $c$, now identified as the particle limiting velocity, with the bicubic equation (Soln, 2014, 2015),

$$m^2\left(c^2\right)^3 - E^2c^2 + E^2v^2 = 0 \tag{1}$$

where, $m$, $v$ and $E$ are formally particle mass, velocity and energy. From (1) one calculates $c^2$, the square of particle limiting velocity. The three solutions, according to Soln (2014, 2015), for the primary, $c_1$, obscure, $c_2$, and normal, $c_3$, limiting velocities can be written as,

$$\begin{aligned}
z &= \frac{3\sqrt{3}mv^2}{2E}, -1 \le z \le 1, \\
D &= \frac{1}{4}\left(\frac{3\sqrt{3}}{2z}\right)^4\left[1 - \frac{4}{27}\left(\frac{3\sqrt{3}}{2z}\right)^2\right] < 0, && (2.0)\\
\frac{c_1^2}{v^2} &= \frac{3}{z}\sin\left(\frac{\pi}{3} - \frac{1}{3}\sin^{-1}(z)\right) > 0, && (2.1)\\
\frac{c_2^2}{v^2} &= -\frac{3}{z}\cos\left(\frac{1}{3}\sin^{-1}(z) - \frac{\pi}{6}\right) < 0, && (2.2)\\
\frac{c_3^2}{v^2} &= \frac{3}{z}\sin\left(\frac{1}{3}\sin^{-1}(z)\right) > 0. && (2.3)
\end{aligned}$$

where solutions for $c_i^2$, $i = 1, 2, 3$, are given per $v^2$ so that the role of the congruent parameter $z$ is better seen. Here one talks about the same particle,say electron,which has different limiting velocity solutions (2.1, 2, 3). This particle of mass,$m$, velocity, $v$, has energy $E$ of fixed numerical value which, however,is differentiated by $c_1, c_2$ and $c_3$ into different forms. Even with these different $E$ forms, directly from solutions (2.1, 2, 3) one can show the zero sum rule for squares of limiting velocities,

$$\sum_{i=1,2}^{3} c_i^2 = 0 \tag{2.4}$$

Although the same $m$, $E$, and $v$ yield solution (2.1, 2, 3) their effects will depend on their functional dependences and how they relate to $c_1$, $c_2$ and $c_3$.

Introducing the shorthand,

$$s_i = \frac{\sqrt{3}mc_i^2}{2E} = \frac{zc_i^2}{3v^2}, i = 1, 2, 3;\ s_{1,3} > 0, s_2 < 0 \tag{3}$$

and inverting solutions (2) one obtains,

$$\begin{aligned}
(2.1): &\quad z = \sin\left(\pi - 3\sin^{-1}(s_1)\right) && (4.1)\\
(2.2): &\quad z = \sin\left(\frac{\pi}{2} + 3\cos^{-1}(-s_2)\right) && (4.2)\\
(2.3): &\quad z = \sin\left(3\sin^{-1}(s_3)\right) && (4.3)
\end{aligned}$$

where $z's$ in relations (4) are the same numerically as indicated in front of them, and $s_{1,2,3}$ are, respectively, primary, obscure and normal congruent parameters satisfying

$$|z| = \left|\frac{3\sqrt{3}}{2E}mv^2\right| \le 1, \left|s_{1,3}\right| = \left|\frac{\sqrt{3}mc_{1,3}^2}{2E}\right| \le 1, |-s_2| = \left|\frac{\sqrt{3}m\left(-c_2^2\right)}{2E}\right| \le 1 \tag{5}$$

These relations are all together due to the properties of trigonometric and inverse trigonometric functions.

The zero squares sum rule (2.4) is a beautiful test whether the description is going in the right direction. For instance it is directly evident in the ordinary energy regime, $E \gtrapprox mv^2$, from the Taylor series solutions of (2) for limiting velocities, as shown in Soln (2014, 2015) which here are rewritten first in terms of congruent parameter $z = 3\sqrt{3}mv^2/2E \ll 1$, and then with few first terms from the series

$$\frac{c_1^2}{v^2} = \frac{3\sqrt{3}}{2z} - \frac{1}{2} + O(z) : \; E \simeq mc_1^2 + \frac{mv^2}{2} - mv^2 O(z), \tag{6.1}$$

$$\frac{c_2^2}{v^2} = -\frac{3\sqrt{3}}{2z} - \frac{1}{2} + O(z) : \; E \simeq m\left(-c_2^2\right) - \frac{mv^2}{2} + mv^2 O(z), \tag{6.2}$$

$$\frac{c_3^2}{v^2} = 1 + O(z) . : \; c_3^2 \simeq (1 + O(z))v^2, \tag{6.3}$$

$$E = E : \; c_1^2 + c_2^2 + v^2 = v^2 O(z); \; c_1^2 + c_2^2 + c_3^2 = v^2 O(z) \tag{6.4}$$

These simple examples demonstrate that the same $E$ comes in different forms because of the presence of limiting velocities. Perturbatively from these relations one notices that $c_1^2 < E/m$, $E/m < -c_2^2$ and $c_3^2 \geq v^2$.

From the congruent parameter $z$ appearing in in the bicubic limiting velocity solutions (2), one has general expressions for energies by utilizing from (2.0) $E = 3\sqrt{3}mv^2/2z$. This can be specialized for each form of limiting velocity by specifying $\left(v^2/z\right)' s$ from relations (2) to obtain for each limiting velocity form, $c_1$, $c_2$ and $c_3$, the different expressions for $E$ but with the same numerical value.

$$(2.1) : \quad E = \frac{\sqrt{3}3mv^2}{2z} = \frac{\sqrt{3}mc_1^2}{2\sin\left(\frac{1}{3}\left(\pi - \sin^{-1}(z)\right)\right)}, \tag{7.1}$$

$$(2.2) : \quad E = \frac{\sqrt{3}3mv^2}{2z} = \frac{\sqrt{3}m\left(-c_2^2\right)}{2\cos\left(\frac{1}{3}\sin^{-1}(z) - \frac{\pi}{6}\right)}, \tag{7.2}$$

$$(2.3) : \quad E = \frac{\sqrt{3}3mv^2}{2z} = \frac{\sqrt{3}mc_3^2}{2\sin\left(\frac{1}{3}\sin^{-1}(z)\right)}. \tag{7.3}$$

Different expressions (7) for the same value $E$ are the starting points for putting $E$ into three expressions, respectively for, primary, obscure and normal particle, all of them containing the Lorentz-like factors. To carry out this project one starts with three numerical identities for $z$ with restricted values,

$$z = 3\sqrt{3}mv^2/2E, \; -1 \leq z \leq 1, \tag{8.0}$$

$$z = 3\sin\left(\frac{\pi}{3} - \frac{1}{3}\sin^{-1}(z)\right) - 4\sin^3\left(\frac{\pi}{3} - \frac{1}{3}\sin^{-1}(z)\right), \tag{8.1}$$

$$z = -3\cos\left(\frac{1}{3}\sin^{-1}(z) - \frac{\pi}{6}\right) + 4\cos^3\left(\frac{1}{3}\sin^{-1}(z) - \frac{\pi}{6}\right), \tag{8.2}$$

$$z = 3\sin\left(\frac{1}{3}\sin^{-1}(z)\right) - 4\sin^3\left(\frac{1}{3}\sin^{-1}(z)\right). \tag{8.3}$$

Next one connects relations (8) to respective energies in (7) by inserting the $z's$ from (8) into the $v^2/c_i^2, i = 1, 2, 3$ from limiting velocity solutions (2) to obtain,

$$(2.1): \quad 1-\frac{v^2}{c_1^2} = 1-\frac{(8.1)z}{3\sin\left(\frac{\pi}{3}-\frac{1}{3}\sin^{-1}(z)\right)} = \frac{4}{3}\sin^2\left(\frac{\pi}{3}-\frac{1}{3}\sin^{-1}(z)\right),$$

$$: \quad \frac{2}{\sqrt{3}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\sin^{-1}(z)\right) = \left(1-\frac{v^2}{c_1^2}\right)^{\frac{1}{2}}; \tag{9.1}$$

$$(2,2): \quad 1+\frac{v^2}{\left(-c_2^2\right)} = 1+\frac{(8.2)z}{3\cos\left(\frac{1}{3}\sin^{-1}(z)-\frac{\pi}{6}\right)} = \frac{4}{3}\cos^2\left(\frac{1}{3}\sin^{-1}(z)-\frac{\pi}{6}\right),$$

$$: \quad \frac{2}{\sqrt{3}}\cos\left(\frac{1}{3}\sin^{-1}(z)-\frac{\pi}{6}\right) = \left(1-\frac{v^2}{c_2^2}\right)^{\frac{1}{2}}; \tag{9.2}$$

$$(2.3): \quad 1-\frac{v^2}{c_3^2} = 1-\frac{(8.3)z}{3\sin\left(\frac{1}{3}\sin^{-1}(z)\right)} = \frac{4}{3}\sin^2\left(\frac{1}{3}\sin^{-1}(z)\right),$$

$$: \quad \frac{2}{\sqrt{3}}\sin\left(\frac{1}{3}\sin^{-1}(z)\right) = \left(1-\frac{v^2}{c_3^2}\right)^{\frac{1}{2}} \tag{9.3}$$

These yield, after substituting the denominators in relations (7) with derived corresponding expressions in relations (9):

$$(7.1)E = \quad mc_1^2\left(1-\frac{v^2}{c_1^2}\right)^{-\frac{1}{2}}, \ \overrightarrow{p} = \frac{E\overrightarrow{v}}{c_1^2} = m\overrightarrow{v}\left(1-\frac{v^2}{c_1^2}\right)^{-\frac{1}{2}}, \tag{10.1}$$

$$(7.2)E = \quad m\left(-c_2^2\right)\left(1-\frac{v^2}{c_2^2}\right)^{-\frac{1}{2}}, \overrightarrow{p} = \frac{E\overrightarrow{v}}{\left(-c_2^2\right)} = m\overrightarrow{v}\left(1-\frac{v^2}{c_2^2}\right)^{-\frac{1}{2}} \tag{10.2}$$

$$(7.3)E = \quad mc_3^2\left(1-\frac{v^2}{c_3^2}\right)^{-\frac{1}{2}}, \ \overrightarrow{p} = \frac{E\overrightarrow{v}}{c_3^2} = m\overrightarrow{v}\left(1-\frac{v^2}{c_3^2}\right)^{-\frac{1}{2}} \tag{10.3}$$

$$\overrightarrow{p}^2c_i^2 - E^2 + m^2c_i^4 = \quad 0, i = 1,2,3 \tag{10.4}$$

where in (10.4) one summarizes from (10.1, 2, 3) the unified particle mass shell like form for all particle kinds, primary, obscure and normal with respective limiting velocities $c_1, c_2$,and $c_3$. This was enabled by the congruent parameter $z$ through relations (8) and (9).

## 3. Numerical Procedures for Limiting Velocities Utilizing the Congruent Parameter z

Already in deriving particle energies and linear momenta with Lorentz-like factors (10.1, 2, 3, 4) the dimensionless congruent parameter $z = 3\sqrt{3}mv^2/2E$ , $-1 \le z \le 1$, was proven to be indispensable. As it is seen from relations (10.1,2,3) the Lorentz-like energy expressions with limiting velocities $c_1, c_2$,and $c_3$ have similar forms. Also from relations (9) and (10), the real difference between primary, obscure and normal particles come, respectively, from $c_{1,2,3}^2/v^2$ which one plots as functions of dimensionless congruent parameter $z$ according to (2.1), (2.2) and (2.3) for $0 \le z \le 1$. From the $z$ expression, the same value of $z$ imply the same respective values of $m, v$ and $E$. Then the relative values of limiting velocities are obtained by looking, for example, at $(c_i^2/v^2)/(c_j^2/v^2) = c_i^2/c_j^2; i, j = 1, 2, 3$, etc. Hence, from relations (2.1, 2, 3) one evaluates and plots starting with $z = 1$ and in small steps decreases to $z = 0$. Roughly speaking smaller the $z$, larger the energy $E$ and vice versa.

As in the Table 1 the congruent parameter $z$ varies from 1 to 0, one notices that the ratios of limiting to ordinary velocity squares also change: $c_1^2/v^2$ and $c_2^2/v^2$ quite a lot, wile $c_3^2/v^2$ only slightly. The $c_3^2/v^2$ only for $1 \ge z \gtrapprox 10^{-1}$ is slightly larger than unity, while for $0 < z \le 10^{-1}$ it is practically equal to unity. That means that in that region $v^2 \simeq c_3^2$ where it is likely that $c_3 = O(c)$. Concerning $c_1^2/v^2$ and $c_2^2/v^2$, the picture is quite different; both of them increase steadily in absolute values as $z \to 0$. Namely, the indication is that limiting velocities $c_1$and $c_2$ are, unlike $c_3$,rather strongly variable limiting velocities with increasing absolute values as $z \to 0$.The question is how much these behaviors of $c_1$ and $c_2$ qualify the respective primary and obscure particles, to be dark matter particles. The interesting things to notice from the Table are the magnitudes from $z = 1$ to $z = 0$ of primary, obscure normal particle velocities, respectively:$2/3 \geq v^2/c_1^2 \to 0$ ; $1/3 \geq v^2/(-c_2^2) \to 0$; $2/3 \leq v^2/c_3^2 \to 1$.

Table 1. Ratios of limiting to ordinary velocity squares as assigned by z

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $z$ : | 1 | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 | 0.2 |
| $c_1^2/v^2$ : | 1.5 | 2.08 | 2.52 | 3.04 | 3.70 | 4.60 | 5, 92 | 8.11 | 12.46 |
| $c_2^2/v^2$ : | −3 | −3.30 | −3.66 | −4.14 | −4.76 | −5.64 | −6.95 | −9.12 | −13.46 |
| $c_3^2/v^2$ : | 1.5 | 1.22 | 1.14 | 1.09 | 1.07 | 1.04 | 1.03 | 1.01 | 1.006 |

| | | | | | |
|---|---|---|---|---|---|
| $10^{-1}$ | $10^{-2}$ | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ | $10^{-6}$ |
| 25.47 | 259.3 | 2, 597 | 25, 980 | 259, 807 | $2,598 \times 10^3$ |
| −26.59 | −260.3 | −2, 598 | −25, 981 | −259, 809 | $-2.598 \times 10^3 - 1$ |
| 1.001 | 1 | 1 | 1 | 1 | 1 |

While $c_1^2, c_2^2$ and $c_3^2$ were calculated from the bicubic equation solutions in terms of $z$, one can verify these solutions by expressing $z$ in terms of $c_1^2, c_2^2$ and $c_3^2$.This one does by substituting $E$ in the exression (8.0) for $z$ with the desired $E$ from the Lorentz-like energy expressions (10.1,2,3) to obtain not only the corresponding $z$ but also the corresponding mass-energy, where $m(z)$ indicates to be the mass as appearing in the expression of $z$ in (2.0)

$$(10.1): \quad z = \frac{3\sqrt{3}v^2}{2c_1^2}\left(1-\frac{v^2}{c_1^2}\right)^{\frac{1}{2}};\ m(z)v^2 = \frac{v^2}{c_1^2}\left(1-\frac{v^2}{c_1^2}\right)^{\frac{1}{2}} E; \tag{11.1}$$

$$(10.2): \quad z = \frac{3\sqrt{3}v^2}{2\left(-c_2^2\right)}\left(1-\frac{v^2}{c_2^2}\right)^{\frac{1}{2}}; m(z)v^2 = \frac{v^2}{(-c_2^2)}\left(1-\frac{v^2}{c_2^2}\right)^{\frac{1}{2}} E; \tag{11.2}$$

$$(10.3): \quad z = \frac{3\sqrt{3}v^2}{2c_3^2};\left(1-\frac{v^2}{c_3^2}\right)^{\frac{1}{2}};\ m(z)v^2 = \frac{v^2}{c_3^2}\left(1-\frac{v^2}{c_3^2}\right)^{\frac{1}{2}} E. \tag{11.3}$$

With these relations one easily verifies that the calculated values for limiting velocities in the Table are correct and, at the same time, that the formal expression for each $E$ in (10.1,2,3) is correct, although numerically each of them has the same value. In act, with the help from the Table, one verifies that respectively, $z, v, m$ and $E$ all have the same values in (10.1,2,3) and (11.1,2,3).

Next, one briefly treats the limiting velocities for OPERA muon neutrino velocity experiment (Strauss, 2014) and the Crab Nebula Flare 2010 (Stecker, 2014) electron velocity observation . With respective data, the congruent parameter $z$ values are also given from which, after the comparisons with the Table, the values of limiting velocities are indicated

$$\begin{aligned} OPERA: &\quad E(\nu_\mu) = 17GeV,\ m(\nu_\mu)c^2 = 0.076eV, \\ \left(1-1.8\times10^{-6}\right)c \leq &\quad v \leq \left(1+2.3\times10^{-6}\right)c, \\ z \simeq &\quad 1.16\times10^{-11} : \frac{c_3^2}{v^2} \simeq 1, c_3 \simeq v \simeq c; \end{aligned} \tag{12.1}$$

$$\begin{aligned} CRAB\ \ NEBULA: &\quad E(e) \simeq 5.1PeV,\ m(e)c^2 = 0.51MeV, \\ v = &\quad (1+\delta)c, -8\times10^{-17} < \delta \leq 5\times10^{-21}, \\ z \simeq &\quad 2.6\times10^{-10} : \frac{c_3^2}{v^2} \simeq 1, c_3 \simeq v \simeq c \end{aligned} \tag{12.2}$$

In both cases the valid assumptions are that $mv^2 \ll E$ and $mc_3^2 \ll E$ so that one can write, respectively from (2.3) and (10.3), for both cases,

$$(2.3):\quad \frac{c_3^2}{v^2} \simeq 1+\left(\frac{mv^2}{E}\right)^2;(10.3):\frac{c_3^2}{v^2}=\frac{1}{1-\left(\frac{mc_3^2}{E}\right)^2}\simeq 1+\left(\frac{mc_3^2}{E}\right)^2:$$

$$mv^2 \ll E, mc_3^2 \ll E : c_3 \simeq v \simeq c. \tag{13}$$

These samples show how useful is the congruent parameter $z$. Both examples in (12) and (13) have the congruent parameter satisfying $z \ll 1$ and $c_3 \simeq v \simeq c$ so that the Einstein's rest energy value is $mc^2 \simeq mc_3^2 \simeq mv^2$. As the Table shows, the situation for $c_3^2/v^2$ would change somewhat for $0.3 < z \leq 1$, where $c_3 \neq v$ and likely $c_3 \neq c$ particularly if $v \simeq c$. The further usefulness of the congruent parameter $z$ for results in (12) and (13) is evident by comparing them with "exact" calculations Soln (2014, 2015) ,respectively of the neutrino OPERA velocity experiment (Strauss, 2014) and the electron Crab Nebula Flare of 2010 observation (Stecker, 2014).

$$\frac{c_3^2}{v^2} \simeq 1+2\times 10^{-23},\ \frac{c_3^2}{v^2} \simeq 1+2\times 10^{-20} \tag{14.1,2}$$

:

As the Table indicates primary, $c_1$, and obscure imaginary, $c_2$, limiting velocities behave dramatically different from the normal, $c_3$ , limiting velocity as the congruent parameter $z$ changes from 1 to 0. Discussions of primary, $c_1$, and obscure imaginary, $c_2$, limiting velocities for the neutrino OPERA velocity experiment (Strauss, 2014)) and the electron Crab Nebula Flare of 2010 observation (Stecker, 2014) can be found in Soln (2014, 2015); From these discussions, one deduces that the primary $c_1$, and the absolute value of obscure imaginary, $c_2$, limiting velocities are numerically about $10^5$ times larger than the normal $c_3$ , limiting velocity and that they support the LV. While such large values for $c_1$and $|c_2|$ may be legitimate for hypothetical neutrino and electron processes from Strauss (2014), Stecker (2014), unfortunately, observationaly they are not yet within the reach.

Recently Fan, Reece, and Wang, (2010), Cirelli, Del Nobile, and Panci, (2013) and Bezrukov and Gorbunov (2015) assumed that a small velocity, $v \simeq 10^{-3}c$, and low energy, $E \simeq 1eV$, particle would fit into the dark matter description. From relation (2.0) this test dark matter paricleis has the maximum mass at $z = 1$ , $m(1) = 2E/\left(v^2 3\sqrt{3}\right)$. With $E$ and $v$ fixed, this maximum test mass $m(1)$, because of rather low energy, here does not enter into limiting velocities evaluations. Next, expressing $m(z)v^2$ in terms of $E$ and $z$ values and connecting, from the Table $v^2$ with $c_i^2 = 0, i = 1, 2, 3$ , one deduces in steps the following relations:

$$\begin{aligned} m(z)v^2 &= \frac{2}{3\sqrt{3}}Ez,\ v=10^{-3}c, E=1eV, z=1 : m(1)v^2 \simeq 0.38eV,\\ c_{1,3}^2 &= \frac{3}{2}v^2, \left(-c_2^2\right) = 3v^2;\\ m(1)c_{1,3}^2 &= \frac{3}{2}\times 0.38eV \simeq 0.58eV, m(1)\left(-c_2^2\right) = 3\times 0.38eV \simeq 1.15eV \end{aligned} \tag{15}$$

From relations (10) the $E's$ of $1eV$ can be now written in the primary, obscure and normal forms,

$$(10.1,3):\quad E(c_{1,3}) = m(1)c_{1,3}^2\left[1+\left(\left(1-\frac{v^2}{c_{1,3}^2}\right)^{-\frac{1}{2}}-1\right)\right] \simeq m(1)c_{1,3}^2(1+0.73) \simeq 1eV,$$

$$(10.2):\quad E(c_2) = m(1)\left(-c_2^2\right)\left[1+\left(\left(1+\frac{v^2}{\left(-c_2^2\right)}\right)^{-\frac{1}{2}}-1\right)\right] \tag{16.1.3}$$

$$= m(1)\left(-c_2^2\right)\left[1-\left(\frac{v^2}{2\left(-c_2^2\right)}-\frac{3v^4}{8\left(-c_2^2\right)^2}\right)\right] \simeq m\left(-c_2^2\right)\left(1-\frac{1}{8}\right) \simeq 1eV. \tag{16.2}$$

The obscure particle energy having the negative kinetic energy contribution of $-(1/8)\times m\left(-c_2^2\right)$ performed the self-annihilation process by decreasing $E(c_2)$ from $1.15eV$ in (15) to $1eV$ in (16.2). Contrary to the obscure particle, the positive kinetic energies contributions of primary and normal particles performed the self-creation process by increasing $E(c_{1,3})$ from $0.58eV$ in (15) to $1eV$ in (16.1, 3). The self-annihilation process property of the obscure particle makes it a very good candidate to be a dark matter particle. Namely, in S. Adrian-Martinez et al. (2016) it is argued that dark matter particles trapped in massive astrophysical object such as the Sun, could self-annihilate, reaching equilibrium between capture and annihilation rates over the age of the solar system (Adrian-Martnez et al., 2016). There is no doubt that the fact tat $c_2^2 < 0$ is connected to the self-annihilation properties of the obscure particle. One may entertain the idea that choosing $z \ll 1$, say $z = 10^{-6}$, would bring $c_3$to $c$; that is normal particle closer to LI. But, from the Table, one has at $z = 10^{-6}$ that $c_3 = v = 10^{-3}c$, that is still LV. Now, the self-creation energy processes of primary and normal particles do not reject them from roles of a dark matter particles. Just perhaps less attractive than the obscure particle.

## 4. Final Remarks

The three solutions of the particle limiting velocity bicubic equation,although interconnected through the formalism, are independent enough to give three separate and quite detailed descriptions of primary, obscure and normal particles with respective primary,$c_1$, imaginary obscure, $c_2$, and normal, $c_3$, limiting velocities. The well known limiting velocity experiments, the OPERA neutrino velocity experiment (Strauss, 2014) and the 2010 Crab Nebula Flare (Stecker, 2014) and the Ice Cube PeV (Aartsen et al., 2013; Bezrukov & Gorbunov, 2015) electron events are very well described with normal particles (Soln, 2014, 2015).

As far as the dark matter is concerned, it does not give too many clues about itself. Here, one would like to know if and when the dark matter particle could be associated with primary, obscure or even a normal particle. The example of the self-annihilating obscure particle as a dark matter particle, associated with the energy $E \simeq 1eV$ and velocity $v \simeq 10^{-3}c$ from (Fan, Reece, & Wang, 2010; Cirelli, Del Nobile, & Panci, 2013; Bezrukov & Gorbunov, 2015) points to the fact that persistence and patience can point to the eventual appropriate descriptions of all kind of particles. To this end, the vastly expanded formalism of bicubic equation limiting velocity formalism that utilizes the congruent dimensionless parameter $z$, could be very helpful in the endeavor to treat the dark matter particles on the same level as usual particles. Particularly useful in this pursuit should be the Lorentz like expressions for energies and momenta for all three kind of particles, primary, obscure and normal.

One cannot but to notice that, in relations (15) and (16), the absolute values of limiting velocities $c_{1,2,3}$ are very ”nonrelativistic” in the examples of $v \simeq 10^{-3}c$, and low energy, $E \simeq 1eV$, particle put forward in (Fan, Reece, & Wang, 2010; Cirelli, Del Nobile, & Panci, 2013; Bezrukov & Gorbunov, 2015) and with the assigned congruent parameter $z = 1$; while on the other hand, on the examples of neutrino OPERA velocity experiment (Strauss, 2014) and the electron Crab Nebula Flare of 2010 observation (Stecker, 2014), relations, the absolute values of limiting velocities $c_{1,2,3}$ are very ”relativistic” with very large $E's$ and relativistic $v's$ and pushing the congruent parameter to $z \ll 1$. So, it appears that regardless if $z \simeq 1$ or $z \ll 1$ when $v's$ are non relativistic and $E's$ are rather small, the normal particle will have nonrelativistic $c_3 < c$ which correspond to examples from (Fan, Reece, & Wang, 2010; Cirelli, Del Nobile, & Panci, 2013; Bezrukov & Gorbunov, 2015).

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