# On the Trigonometric Description of the Michelson-Morley Experiment 

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#### Abstract

One formula with two trigonometric corrections describing the round trip of the beams in the Michelson-Morley experiment is presented. The first trigonometric correction describes the round trip path of those beams, while the second trigonometric correction describes the trigonometric geometric mean of the two-way speed of those beams. This formula gives the null fringe shift result for the first order experiments (Fizeau experiment, Hoek experiment), the null fringe shift result for the second order experiment (Michelson-Morley experiment), and predicts a measurable fringe shift result for the fourth order experiment. This trigonometric model can be tested experimentaly by the advanced LIGO (Laser Interferometer Gravitational-Waves Observatory) technology with three arms separated by the angle $\pi / 4$ and the longitudinal arm directed to the CMB rest frame in the direction to the constellation Crater (known in the Greek mythology as the Cup of the god Apollo). This proposed fourth order experiment can be named as the advanced LIFE (Laser Interferometer Fringe Enigma) experiment. The published predictions before the arrival of experimental data from the advanced LIFE experiment can estimate the power of our models.


Keywords: fringe shift observation, the first order experiment, the second order experiment, the fourth order experiment, advanced LIGO technology, advanced LIFE experiment, the predictive power of our models.

## 1. Introduction

The conundrum of the Michelson-Morley experiment continues to attract attention of many researchers. Since its publication in 1887 generations of researchers proposed different solutions of this null experiment. There were published thousands contributions to this topic. E.g., the entrance gate to the most cited papers via Wikipedia -„Michelson-Morley Experiment", the entrance gate to alternative proposals via „Projekt G.O. Mueller".
We will present one formula with two trigonometric corrections in order to model the real path of the beams through the arms and their average two-way speed. This formula gives the null fringe shift result for the first order experiments (Fizeau experiment, Hoek experiment), the null fringe shift result for the second order experiment (Michelson-Morley experiment), and predicts a measurable fringe shift result for the fourth order experiment. This trigonometric model can be tested experimentally by the advanced LIGO technology (Laser Interferometer Gravitational-Waves Observatory). We propose to construct three arms with the LIGO technology separated by the angle $\pi / 4$ and with the main longitudinal arm oriented towards the CMB (Cosmic Microwave Background) rest frame in the direction to the constellation Crater (known in the Greek mythology as the Cup of the god Apollo). This proposed fourth order experiment can be named as the advanced LIFE (Laser Interferometer Fringe Enigma) experiment. At this moment there are no available experimental data on the fourth order experiment. We all are in the same starting position and we can propose the fringe shift result for this advanced LIFE experiment. The published predictions before the arrival of experimental data from the advanced LIFE experiment can estimate the power of our models.

## 2. Which "Mean" is the Correct Mean for the M-M- Experiment?

The ancient Greeks defined ten distinct "means" for the evaluation of two positive numbers. Pythagoras introduced three means: the arithmetic, the geometric, and the harmonic. Later Eudoxus added three more "means". The last four Greek means were added by Myonides and Euphranor. Our knowledge about these old Greece means comes mainly from reviews of Nicomachus and Pappus.

From these ancient ten means only the arithmetic (A), geometric (G) and harmonic (H) survived till today. During the last four centuries the three iterated means were developed: $A G(x, y)$ mean (the arithmetic-geometric mean), $\mathrm{AH}(\mathrm{x}, \mathrm{y}$, ) mean (the arithmetic-harmonic mean) that converges to the geometric mean, and $\mathrm{GH}(\mathrm{x}, \mathrm{y})$ mean (the geometric-harmonic mean).
We have the following inequality involving three Pythagoreans means (A, G, H) and three iterated means (HG, AH, AG):
$\min (\mathrm{x}, \mathrm{y}) \leq \mathrm{H}(\mathrm{x}, \mathrm{y}) \leq \mathrm{HG}(\mathrm{x}, \mathrm{y}) \leq \mathrm{G}(\mathrm{x}, \mathrm{y})=,\mathrm{AH}(\mathrm{x}, \mathrm{y},) \leq \mathrm{AG}(\mathrm{x}, \mathrm{y},) \leq \mathrm{A}(\mathrm{x}, \mathrm{y},) \leq \max (\mathrm{x}, \mathrm{y})$.
The application of the average speed of motion depends on the circumstances. The arithmetic mean should be applied for two different speeds acting for the same time. The harmonic mean should be applied for two different speeds acting over the same distance. Now, we have a dilemma - what is the correct mean for the average speed if both times and lengths differ as we observe in the M-M experiment in the longitudinal arm? One interesting idea proposed Sergey Vikhrov (June 21 2016) - let us try to use the geometric mean for the two-way speed in the longitudinal arm.


Figure 1 The arithmetic, geometric, harmonic and reciprocal geometric means (inspired by Sergey Vikhrov June 212016 )

It was very inspirative idea that we have further developed - we should come from the geometric mean of speeds in the longitudinal arm to the arithmetic mean of speeds in the transverse arm - the first proposed trigonometric geometric correction. On the other side we have to use a second trigonometric correction that transforms the Pythagorean path in the transverse arm into the arithmetic mean of beam lengths in the longitudinal arm. Moreover, the rotation of the interferometer plays a very important role. In the first and second order experiments we see no fringe shift. How should we modify our experimental arrangement in order to see any reproducible fringe shift?
This newly proposed formula can be used for the fringe shift estimation in the first, second and fourth order experiments.

## 3. Formula Describing Time for the Round-Trip of the Beam in Michelson Interferometer

We propose one formula for the calculation of the time $\mathrm{T}_{\mathrm{TGM}}$ (trigonometric geometric mean) for the round-trip of the beam in the Michelson interferometer:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{TGM}}=2 \frac{L}{\sqrt{1-\frac{v^{2}}{c^{2}} \sin ^{2} \varepsilon}} \times \frac{1}{c \sqrt{1-\frac{v^{2}}{c^{2}} \cos ^{2} \varepsilon}} \tag{1}
\end{equation*}
$$

L is the length of the arm of the Michelson interferometer,
$\mathrm{c}=299972458 \mathrm{~ms}^{-1}$,
v is the speed of the instrument towards the CMB rest frame,
$\varepsilon$ is the angle between the direction towards to the constellation of Crater and the arm of the Michelson interferometer.

For the longitudinal arm and $\varepsilon=0$ we will get for the round-trip time $\mathrm{T}_{\mathrm{L}}$ :

$$
\begin{equation*}
\mathrm{T}_{\mathrm{L}}=2 \frac{L}{1} \times \frac{1}{c \sqrt{1-\frac{v^{2}}{c^{2}}}} \tag{2}
\end{equation*}
$$

For the transverse arm and $\varepsilon=\pi / 2$ we will get for the round trip time $\mathrm{T}_{\mathrm{T}}$ :

$$
\begin{equation*}
\mathrm{T}_{\mathrm{T}}=2 \frac{L}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \times \frac{1}{c} \tag{3}
\end{equation*}
$$

Generally, the proposed formula modifies both the path of the beam and the average round-trip speed in the dependence on the angle $\varepsilon$.
Let us rotate the Michelson interferometer and calculate the length difference $\Delta$ after the arrival of two beams through both perpendicular arms: $\Delta=\left(\mathrm{T}_{\mathrm{TGM1}}-\mathrm{T}_{\mathrm{TGM} 2}\right)^{*} \mathrm{c}$ for any $\varepsilon$ :

$$
\begin{equation*}
\Delta=2 \frac{L}{\sqrt{1-\frac{v^{2}}{c^{2}} \sin ^{2} \varepsilon}} \times \frac{1}{c \sqrt{1-\frac{v^{2}}{c^{2}} \cos ^{2} \varepsilon}}-2 \frac{L}{\sqrt{1-\frac{v^{2}}{c^{2}} \sin ^{2}\left(\frac{\pi}{2}-\varepsilon\right)}} \times \frac{1}{c \sqrt{1-\frac{v^{2}}{c^{2}} \cos ^{2}\left(\frac{\pi}{2}-\varepsilon\right)}} \tag{4}
\end{equation*}
$$

In any position of the Michelson interferometer we will get $\Delta=0$ because of the properties of used trigonometric functions: $\sin ^{2}(\pi / 2-\varepsilon)=\cos ^{2}(\varepsilon)$ and $\cos ^{2}(\pi / 2-\varepsilon)=\sin ^{2}(\varepsilon)$. It seems that Nature cleverly used the properties of these trigonometric functions to obtain the null fringe shift result.

## 3. Advanced LIFE Experiment (Laser Interferometer Fringe Enigma) - Experimentum Crusis

In order to prove that this concept brings something new we have to propose a new experiment in order to compare the predictive power of this model with the properties of Nature.

We propose to construct three arms interferometer with arms separated by the angle $\pi / 4$ and with the main longitudinal arm oriented towards the CMB (Cosmic Microwave Background) rest frame in the direction to the constellation Crater (known in the Greek mythology as the Cup of the god Apollo). This proposed fourth order experiment can be named as the advanced LIFE (Laser Interferometer Fringe Enigma) experiment. The experimental data that will come in the future from this experiment will play an important role in the estimation of the predictive power of different models.
Let us describe the length difference after the arrival of two beams through the longitudinal arm and the middle arm as $\Delta_{1}$ and the length difference after the arrival of two beams through the middle arm and the transverse arm as $\Delta_{2}$ :

$$
\begin{align*}
& \Delta_{1}=2 \frac{L}{\sqrt{1-\frac{v^{2}}{c^{2}} \sin ^{2} \varepsilon}} \times \frac{1}{c \sqrt{1-\frac{v^{2}}{c^{2}} \cos ^{2} \varepsilon}}-2 \frac{L}{\sqrt{1-\frac{v^{2}}{c^{2}} \sin ^{2}\left(\frac{\pi}{4}+\varepsilon\right)}} \times \frac{1}{c \sqrt{1-\frac{v^{2}}{c^{2}} \cos ^{2}\left(\frac{\pi}{4}+\varepsilon\right)}}  \tag{5}\\
& \Delta_{2}=2 \frac{L}{\sqrt{1-\frac{v^{2}}{c^{2}} \sin ^{2}\left(\frac{\pi}{4}+\varepsilon\right)}} \times \frac{1}{c \sqrt{1-\frac{v^{2}}{c^{2}} \cos ^{2}\left(\frac{\pi}{4}+\varepsilon\right)}}-2 \frac{L}{\sqrt{1-\frac{v^{2}}{c^{2}} \sin ^{2}\left(\frac{\pi}{2}-\varepsilon\right)}} \times \frac{1}{c \sqrt{1-\frac{v^{2}}{c^{2}} \cos ^{2}\left(\frac{\pi}{2}-\varepsilon\right)}} \tag{6}
\end{align*}
$$

For the case when the longitudinal arm is directed to the CMB rest frame $(\varepsilon=0)$ we can find the fringe shift n as the difference between $\Delta_{1}$ and $\Delta_{2}$ divided by the laser wavelength $\lambda$ :

$$
\begin{equation*}
n=\frac{\Delta_{1}-\Delta_{2}}{\lambda}=\frac{4 L}{\lambda}\left(\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}-\frac{1}{\left(1-\frac{1}{2} \frac{v^{2}}{c^{2}}\right)}\right) \cong \frac{4 L}{\lambda} \frac{1}{8} \frac{v^{4}}{c^{4}} \tag{7}
\end{equation*}
$$

For the expected fringe shift value we have derived a fourth order formula. Our generation has only one experimental device available for such an imperceptible fringe shift - LIGO technology. LIGO interferometer with the arm length 4 km but with the effective beam path 1120 km and the laser wavelength 1064 nm can be used for this experiment. If we will enter these LIGO values in the expected fringe shift value together with the interferometer speed towards to the CMB rest frame $\left(\mathrm{v}=370 \mathrm{kms}^{-1}\right)$ we will get $\mathrm{n} \approx 1.22$.

## 6. Conclusions

1) One formula with two trigonometric corrections describing the round trip time of beams in the Michelson-Morley experiment was presented.
2) The formula was tested for any position of arms of the Michelson interferometer - the null fringe shift result was obtained.
3) One new fourth order experiment using the advanced LIGO technology (Laser Interferometer Gravitational-Waves Observatory) was proposed under the name the advanced LIFE experiment (Laser Interferometer Fringe Enigma).
4) The expected fringe shift formula for this fourth order experiment was presented.
5) The future experimental data from the advanced LIFE experiment can evaluate the predictive power of our model.

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