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From Mathematical Induction to Discrete Time

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Abstract

Proof by induction involves a chain of implications in which the stages are well ordered. A chain of cause and effect in nature also involves a chain of implications. For this chain to "imply" or bring about its effects in a logical sense, it also has to be organized into a well ordering of stages (which are the points or quanta of time). This means that time must be quantized rather than continuous. An argument from relativity implies that space is quantized as a consequence.

Keywords: induction, ordering, quantization, relativity, spacetime

1. Introduction

The distinction between continuous and discrete time depends upon the kinds of ordering points of time have in these two cases. Continuous time points are totally ordered, meaning that for every two, one is earlier than the other. Discrete time points are well ordered, meaning that for any set of points, one is earliest, which is a stricter condition than total ordering. In a well ordering, the "next" point after a given point is always well defined. In the total ordering that applies, that of the real numbers, the "next" point is never well defined!

2. Induction and Causality

In proof by induction, a series of implications is shown which, when combined, completes the argument. But the proof only goes through if the ordering of the stages of the induction is a well ordering. This ordering allows taking the stages one-by-one from the least throughout.

An earlier event does not cause a later event except through a chain of cause and effect encompassing the intervening time. For instance, instead of "A plane ran out of fuel, therefore it crashed", the sequence is better approximated by "A plane ran out of fuel, therefore it descended to 20,000 feet, 19,000 feet, ... and finally crashed". The stages in the chain of cause and effect that led to the plane crash are like the stages of a proof by induction, and like with the proof, the final result is not shown unless each stage is shown one-by-one according to a well ordering of the stages. If the points of time were merely totally ordered like the real numbers, the plane running out of fuel would not be logically required by a chain of implications to crash.

For instance, the statement $t_1 < t_2 < t_3 < t_4 \ldots < t_f$, where the terms t_1 etc. are points (or quanta) of time, encompassing all the times between a cause and a later effect, does not exist if time is continuous and ordered like the standard order of the real numbers. But if time is well ordered, for instance, ordered like the natural numbers, the statement above exists. The statement that orders the times in an interval mirrors the statement of the chain of implications in the same interval: $t_1 \supset t_2 \supset t_3 \supset t_4 \ldots \supset t_f$,. This statement says that the state of the universe at time t_1 implies the state of the universe at time t_2, etc.

In the case of continuous time, every statement $t_1 < t_2 \ldots$ etc. is incomplete and the chain of implications $t_1 \supset t_2 \ldots$ etc. is broken at some point because between t_1 and t_2, however they are chosen, there is a continuum of time points left out (and similarly for every such pair). This is not a consequence of the larger cardinality of the points on the continuous interval, but instead it follows from the order. If the points are restricted to the rationals, between t_1 and t_2 there is necessarily a countable infinity of rationals left out (an interval's worth of rationals). In the same way that there is a gap in the chain of implications between t_1 and t_2, the chain is broken at every later point as well. I claim that because the chain is broken, causes fail to imply their effects in a continuous time world.

3. Space Quantization

In special relativity, whether two events are simultaneous depends on the state of constant motion of the observer. To one observer, the events may be separated only by a distance (they are simultaneous), while to another observer, they are separated by a distance and a time (they are not simultaneous). If the distance between two events that are simultaneous to observer A may vary continuously, then the time between the two events as seen by observer B, to whom they are not simultaneous, may also vary continuously. This is because the time component of the spacetime interval between the events is related to the space components by the distance formula in four dimensions. Continuous space components imply a continuous time component, however, we have argued that time is discrete based on the order needed for cause and effect to function. So it follows by Modus Tollens (if P then Q, not Q, therefore not P) that space is quantized as well.

4. Conclusion

We have argued for the quantization of space and time based on a connection between mathematical proof by induction and causality in nature.