

Is the Lorentz Factor a Probability Function in Superfluid Spacetime?

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Abstract

A number of studies indicate that spacetime may have properties resembling that of a superfluid, suggesting that percolation theory may provide a useful approach to studying the relationship between velocity and time. By hypothesizing that the effect described by the Lorentz factor may represent an increase in the viscosity of spacetime, it was possible to model time dilation in terms of the movement of a fluid through porous media. Using a random resistor network to equate superfluid percolation with conductance, it is shown that the Lorentz factor corresponds to a probability function involving the phase transition of the superfluid to a normal fluid with insulating properties.

Keywords: Lorentz factor, superfluid, random resistor network, percolation theory, phase transition

1. Introduction

The possibility that spacetime may have the properties of a superfluid suggests that percolation theory may provide a meaningful approach to finding a physical correlate for the Lorentz factor, as applied to the dimension of time (Liberati & Maccione, 2014; Volovik, 2013; Huang, Low, & Roh-Suan, 2012). The foundations of percolation theory are derived from the random action of fluids as they migrate through a lattice composed of regularly spaced channels (Bunde & Kantelhart, 2005; Kapitulnik, Aharony, Deutscher, & Stauffer, 1983). The concentration of fluid-filled conduits determines the probability that they will become linked together in a network that permits the fluid to percolate from one end of the lattice to the other. This process of forming a confluent stream from isolated channels produces a model for chemical and physical interactions that may be repeated at different levels of scale. At some critical point in the evolution of the percolation network, a phase transition can occur, which corresponds to an entirely new pattern of material behavior (Sahimi, 2009).

The increase in the viscosity of a fluid, as it undergoes a phase transition, is analogous to a loss of conductivity in a random resistor network, a subset of percolation models (de Gennes, 1976; Farago & Kantor, 2000). If we hypothesize that the effect of the Lorentz factor at velocities approaching the speed of light corresponds to an increase in the viscosity of spacetime, it may be possible to model time dilation in terms of the properties of such a network. Taking this analogy one step further, it is conceivable that the Lorentz factor may correspond to a probability function involving the phase transition of a superfluid as it moves toward a nonconducting state.

2. The Lorentz Factor as a Probability Function

The notion that spacetime may have hydrodynamic properties suggests a potential drag effect on the speed of light (c). The presence of even a minimal degree of viscosity would produce changes in this parameter that are inconsistent with current measurements (Liberati & Maccione, 2014). Consequently, spacetime should have properties associated with a superfluid, which lacks viscosity.

Nevertheless, for inertial frames traveling at velocities close to c , where the speed of light and the movement of time precipitously decline toward zero, a phase transition might occur, in which the superfluid component is replaced by normal fluid, resulting in an increase in viscosity (Huang, 1995).

To model this process, we employed a 3-dimensional random resistor network composed of uniform interconnecting bonds that are indiscriminately cut to simulate the rise in fluid viscosity (de Gennes, 1976; Farago & Kantor, 2000). The use of this model is justified by the similarity between such networks and fluid percolation systems. The movement of fluid through a porous medium has been shown to be mathematically identical to current flow in a random resistor network (Golden, 1997). Thus, the porous medium in our model of spacetime can be replaced by conducting bonds, and an external source of electromotive force across these bonds can be substituted for flow pressure, such that the current density (j) may be expressed as:

$$j = \rho_s v_s + \rho_n v_n \quad (1)$$

where $\rho_s v_s$ and $\rho_n v_n$ represent the density (ρ) and velocity (v) of the superfluid and normal fluid components, respectively (Huang, 1995).

If the channels in the lattice are sufficiently small, $v_n = 0$, and the normal fluid component is removed from the equation. Thus,

$$j = \rho_s \nabla \Phi \quad (2)$$

corresponding to Ohm's law, where Φ represents the electrostatic potential.

As conducting bonds are cut indiscriminately in the random resistor network, the total cross-sectional area of these bonds is reduced, resulting in a decrease in current density that approaches a critical point where conductance becomes zero. Conductance is therefore a function of the relative cross-sectional area of conducting bonds in the network. Furthermore, the density of the normal fluid increases as the superfluid density decreases, where

$$\rho_s + \rho_n = \text{constant} \quad (3)$$

According to Poiseuille's law (Sutera & Skalak, 1993), the cross-sectional area may be incorporated into the equation for superfluid flow through the network, as follows:

$$VA(k\mu) = P_1 - P_2 \quad (4)$$

where V is the velocity of the fluid, A is the cross-sectional area of the intact bonds, μ is the fluid viscosity, k is a constant incorporating the uniform radius and length of the bonds, and $P_1 - P_2$ is the pressure differential across the network (analogous to electrostatic potential).

From equation (4), the resistance to superfluid flow through the network is proportional to the product of both the relative cross-sectional area of disconnected bonds and the relative density of normal fluid, and may be expressed as:

$$\frac{VA_x \left(\frac{\rho_{nx}}{\rho_{nt}} \right)}{VA_t} = \frac{P_{1x} - P_{2x}}{P_{1t} - P_{2t}} \quad (5)$$

where A_x is the cross-sectional area of the disconnected bonds at a given point, ρ_{nx} is the normal fluid density at the same point, A_t is the maximum cross-sectional area of the disconnected bonds, ρ_{nt} is the maximum normal fluid density, $P_{1x} - P_{2x}$ is the pressure differential at the given point, and $P_{1t} - P_{2t}$ is the maximum pressure differential. The contribution to viscosity in equation (4) is a function of the cross-sectional area represented by normal fluid, and is therefore directly proportional to normal fluid density.

Since V is fixed by the physical properties of the uniform bonds,

$$\frac{A_x \left(\frac{\rho_{nx}}{\rho_{nt}} \right)}{A_t} = \frac{P_{1x} - P_{2x}}{P_{1t} - P_{2t}} \quad (6)$$

For random resistor networks, the relative cross-sectional area of non-conducting bonds is a measure of the probability (p) that a random bond is disconnected (Figure 1). The same probability applies to the relative density of normal fluid, which is dependent on the cross-sectional area of disconnected bonds. Thus,

$$p = \frac{A_x}{A_t} = \frac{\rho_{nx}}{\rho_{nt}} \quad (7)$$

And, from equation (6):

$$p^2 = \frac{P_{1x} - P_{2x}}{P_{1t} - P_{2t}} \quad (8)$$

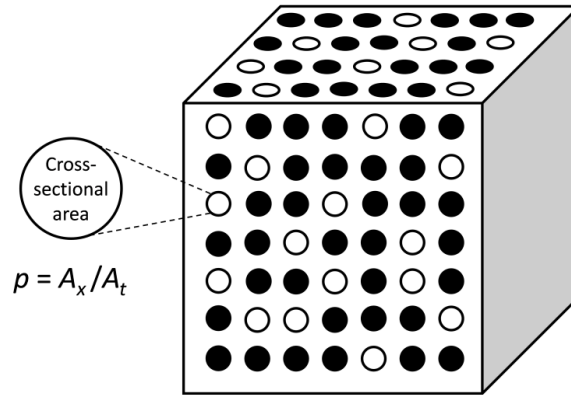


Figure 1. Diagram of the 3-dimensional random resistor network model, showing uniform interconnecting bonds that are either intact (white) or disconnected (black). At a given point (x), the cross-sectional area of the nonconducting bonds (A_x), as a proportion of the total cross-sectional area (A_t), is a measure of the probability (p) that a random bond is disconnected

According to Bernoulli's principle, an inertial frame moving at different velocities through fluid spacetime of constant density (from equation 3) would exhibit changes in pressure based on the square of its velocity. A frame moving from rest to the speed of light (equation 9, denominator) would show the maximum pressure change, whereas another frame with a lower velocity differential would have a smaller effect on pressure (equation 9, numerator).

$$p^2 = \frac{c^2 - v^2}{c^2} \quad (9)$$

Or,

$$p = \sqrt{1 - \frac{v^2}{c^2}} \quad (10)$$

As the velocity differential between the two inertial frames (represented in the numerator and denominator) increases, the p -value for the opposite frame becomes larger, consistent with a relative increase in viscosity.

Compared to time dilation, the effect of velocity is reversed in equation (10) (i.e., a higher velocity is associated with decreased viscosity). However, this aberration may be removed by incorporating the probability metric, $1-p$, which is used in percolation networks when the critical probability is approached from the opposite direction.

With regard to equation (10), there is ample theoretical precedent for probabilistic modeling of phase transitions. For a variety of media, the critical exponent involved in such transitions is $1/2$, and the process may be expressed as follows:

$$\left| \frac{p_c - p}{p_c} \right|^{1/2} \quad p_c - p \ll 1 \quad (11)$$

where p_c is the critical probability at the transition point (Papon, Leblond, and Meijer, 2006; Annett, 2004).

This equation also applies to superfluid conductivity (G) in a random resistor network:

$$G \propto \left| 1 - \frac{p}{p_c} \right|^{1/2} \quad (12)$$

where substitution of v^2 for p and c^2 for p_c yields the Lorentz factor,

$$G \propto \left| 1 - \frac{v^2}{c^2} \right|^{1/2} \quad (13)$$

3. Conclusion

The current paper suggests that superfluid spacetime may have features that are analogous to those associated with random resistor networks. With regard to this possibility, several intriguing questions immediately arise. Are there infinitesimal percolation channels, perhaps at the level of the Planck length, that control the viscosity of spacetime? Would such a network provide a structural mechanism for the action of dark matter or dark energy? Does the maximum superfluid velocity change over time, and what effect would this have on the speed of light? The development of appropriate theoretical constructs to address these questions will require a clearer understanding of the potential relationship between fluid dynamics and spacetime.

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