

## Nuclear Properties of $vg_{9/2}^+$ Isomers in Odd $^{59}\text{Cr}$ to $^{69}\text{Se}$ Nuclei for $N=35$

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### Abstract

We have reported the systematic magnetic quadrupole reduced transition probabilities  $B(M2)$  between  $9/2^+$  and  $5/2^-$  in odd  $^{59}\text{Cr}$  to  $^{69}\text{Se}$  nuclei for neutron  $N=35$ . At present work the calculated  $B(M2)$  values are 0.079 W.U., 0.067 W.U., 0.230 W.U., 0.412 W.U., 0.060 W.U., and 0.044 W.U. for  $^{59}\text{Cr}$ ,  $^{61}\text{Fe}$ ,  $^{63}\text{Ni}$ ,  $^{65}\text{Zn}$ ,  $^{67}\text{Ge}$ , and  $^{69}\text{Se}$  nuclei respectively. The theoretical calculations of the reduced transition probabilities were compared to the previous experimental values. The width of isomeric levels, mean-life and Weisskopf hindrance factors in the odd  $^{59}\text{Cr}$  to  $^{69}\text{Se}$  nuclei were also calculated. The isomeric levels  $9/2^+$  were systematically studied as a function of even atomic number from 24 to 34. It is shown that the isomeric levels are increases from  $Z=24$  to  $Z=28$  and then decreases monotonically up to  $Z=34$ .

**Keywords:** reduced transition probabilities, isomeric levels, mean-life, hindrance factors, width of isomeric levels

### 1. Introduction

Understanding the nuclear properties and its structure has been a crucial study. A theoretical study of nuclear structure can be predicted through several different nuclear models but some of them can explain some limited features of the nucleus. Nuclear shell model has explained about the properties of nuclear isomers in the vicinity of closed shells excited near to the magic numbers. The low-lying excited states in nuclei are widely spaced and can be explained by the shell model by considering several potential. A long-lived excited state of a nucleus which has been called isomers is near to shell closure or magic numbers (Baglin, 2002; Bhat, 1999; Bhat & Tuli, 2000).

Broda et al. found sub-shell closure in  $^{68}\text{Ni}$  for  $N=40$  experimentally (Broda et al., 1995). This nucleus containing double magic nucleons ( $Z=28$ ,  $N=40$ ) and  $2^+$  level in  $^{68}\text{Ni}$  is very high compare to neighbor nucleus. Usually isomers belong to shell closure. Even-odd nuclei  $^{59}\text{Cr}$ ,  $^{61}\text{Fe}$ ,  $^{63}\text{Ni}$ ,  $^{65}\text{Zn}$ ,  $^{67}\text{Ge}$ , and  $^{69}\text{Se}$  belong near to  $^{68}\text{Ni}$ . The  $M2$  transitions of those nuclei were established due to  $vg_{(9/2)^+}$  configuration (Baglin, 2002; Bhat, 1999; Bhat and Tuli, 2000; Broda et al., 1995; Brown, 2010; Erjun, 2001; Junde et al., 2005; Hossain et al., 2011).

The isomeric properties of odd-even Arsenic (As) nuclei of  $\pi g_{(9/2)^+}$  configuration raised the possibility to calculate the isomerism of even-odd nuclei  $vg_{(9/2)^+}$  configuration. Single particle nuclear transition is governed when the nuclear excitation is due to only one nucleon. A study of multipolarity transition in neutron rich nuclei has been made by using shell model calculation (Hossain et al., 1998). It showed that proton motion and neutron motion are collective for quadrupole transition (Umeya et al., 2006). Quadrupole magnetic ( $M2$ ) transition between the state  $9/2^+$  to  $5/2^-$  of odd As isotopes with  $A=67-79$  were observed (Hossain et al., 1998). Moreover, the systematic mean lives, reduced transition probabilities, width of isomeric levels, and Weisskopf hindrance factors in odd  $^{59}\text{Cr}$  to  $^{69}\text{Se}$  nuclei were not investigated yet. In the present work we investigate the properties of isomers of those nuclei systematically by theoretical calculation.

## 2. Theoretical Methods of Calculations

### 2.1 Reduce Transition Probabilities $B(M2)$

In order to obtain the total transition probabilities that determine the lifetime of the initial state we have to sum over all the possible substate  $\mu$  and  $m_f$ . Now the total transition probabilities are

$$P(L; I_i \rightarrow I_f) = \sum_{\mu, m_f} P(L, \mu; I_i m_i \rightarrow I_f m_f) \\ = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{1}{\hbar} \left( \frac{\omega}{c} \right)^{2L+1} B(L, I_i \rightarrow I_f) \dots \quad (1)$$

$\hbar = h/2\pi$ ,  $h$  is planck constant,  $c$  is speed of light,  $\omega$  is angular frequency.

where the quantity

$$B(L, I_i \rightarrow I_f) = \sum_{\mu, m_f} |\langle I_f m_f | M(L, \mu) | I_i m_i \rangle|^2 \dots \quad (2)$$

is called the “reduced transition probability”. While the transition probability depends on the transition energy, being related to  $E^{2L+1}$ , the reduced transition probability does not depend on energy, but is the square of transition matrix element. Theoretical evaluation of reduced transition probability involves nuclear quantizes and depends on specific models of the nucleus.

$$B(ML) = 4.15 \times 10^{-20} \frac{L[(2L+1)!!]^2}{8\pi(L+1)} \left( \frac{197}{E(MeV)} \right)^{2L+1} [P(ML) \text{sec}^{-1}] \mu_N^2 (fm)^{2L-2} \dots \quad (3)$$

By taking the assumption that the nucleus is spherical and the gamma emission is the result of a transition of a single particle from one state to another, Weisskopf estimated for the reduced transition probabilities:

$$B^w(ML) = \frac{10}{\pi} \left( \frac{3}{L+3} \right)^2 (1.2)^{2L-2} A^{(2L-2)/3} \mu_N^2 (fm)^{2L-2} \dots \quad (4)$$

The reduced transition probabilities  $B(M2)$  are determine the structure and the wave functions of nuclei, comparison between experimental and theoretical of  $B(M2)$  is a good investigation. The equation below is used to find the  $B(M2)$  of the isomers,

$$B(M2; I_i \rightarrow I_f) = 7.381 \times 10^{-8} E_\gamma^{-5} P_\gamma(M2; I_i \rightarrow I_f) \text{ in } \mu_N^2 fm^{2L-2} \dots \quad (5)$$

where,  $I_i$  and  $I_f$  are the spin of the initial and final states respectively,

$E_\gamma$  = gamma energy

$$P_\gamma = \text{transition probability } (\lambda) = \left( \frac{0.693}{t_{1/2}} \right)$$

The Weisskopf estimate of  $B^w(M2)$  is given by

$$B^w(M2) = 1.65 A^{2/3} \mu_N^2 fm^2$$

So, the magnetic reduce transition probability for M2 transition is

$$B(M2) = \frac{B(M2) \mu_N^2 fm^2}{B^w(M2)} W.u. \dots \quad (6)$$

### 2.2 Mean-Life Time, $\tau$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \equiv 0.693 \tau \dots \quad (7)$$

where,  $T_{1/2}$  = half-life

$$\text{So, } \tau_\gamma = \frac{T_{1/2}}{0.693}$$

### 2.3 Width of Isomeric Levels, $\Gamma_\gamma$

Width of isomeric level can be calculated by using equation below which indicates the thickness of gamma ray produced from the transition.

$$\frac{1}{\tau_\gamma} = \frac{\Gamma_\gamma}{\hbar} \quad \dots \quad (8)$$

where;  $\tau_\gamma$  = Mean life

$\Gamma_\gamma$  = Width of isomeric level

$\hbar = \frac{h}{2\pi}$ ; where  $h$  is the Plank constant

### 2.4 Weisskopf Hindrance Factor, $F_w$

Hindrance factor is useful to give a test with which different  $\gamma$ -ray transition probabilities can be compared and others that might perturb the lifetime of the nuclear state. Weisskopf Hindrance Factor can be determined by;

$$F_w = \frac{B(M)w}{B(M)_{\text{theoretical}}} \quad \dots \quad (9)$$

where,  $B(M)w = 1.65 A^{2/3}$

## 3. Results and Discussion

Table 1 shows the calculations of reduced transition probabilities  $B(M2)$ , M2 gamma transition between  $9/2^+$  to  $5/2^-$ , isomeric levels, mean life, width of isomeric levels and Weisskopf hindrance factor of odd  $^{59}\text{Cr}$  to  $^{69}\text{Se}$  for  $N=35$  which were presented at the conference (Ghani et al., 2012).

### 3.1 Isomeric Levels

The  $^{59}\text{Cr}$ ,  $^{61}\text{Fe}$ ,  $^{63}\text{Ni}$ ,  $^{65}\text{Zn}$ ,  $^{67}\text{Ge}$  and  $^{69}\text{Se}$  nuclei consist of neutron number  $N=35$  and the atomic numbers 24, 26, 28, 32, and 34 respectively. The odd number of neutron governs the ground state spin  $1/2^-$ ,  $3/2^-$ ,  $1/2^-$ ,  $5/2^-$ ,  $1/2^-$  respectively according to shell model. The isomeric M2 transitions between the  $9/2^+$  and  $5/2^-$  states were observed earlier in the odd neutron  $N=35$  with even atomic number  $Z=24$  to 34 (Baglin, 2002; Bhat, 1999; Bhat and Tuli, 2000; Brown, 2010; Erjun, 2001; Junde et al., 2005; Hossain et al., 2011). Figure 1 shows the isomeric levels at  $9/2^+$  is plotted as a function of the odd mass number 59, 61, 63, 65, 67, and 69 for  $Z=24, 26, 28, 30, 32, 34$  respectively. It is shown that the energy of isomeric levels of  $9/2^+$  increases from  $A=59$  to  $A=63$  and decreases towards mass number  $A=69$ . The maximum isomeric level is observed for magic number  $Z=28$  in  $^{63}\text{Ni}$ . The isomeric levels occur when there is a large difference in angular momentum ( $L$ ) between the successive two states or when the energy difference ( $E$ ) between the two states is relatively small. The isomeric level decreases after  $Z=28$  due to fill-up shell closure.

### 3.2 Systematic Reduced Transition Probabilities

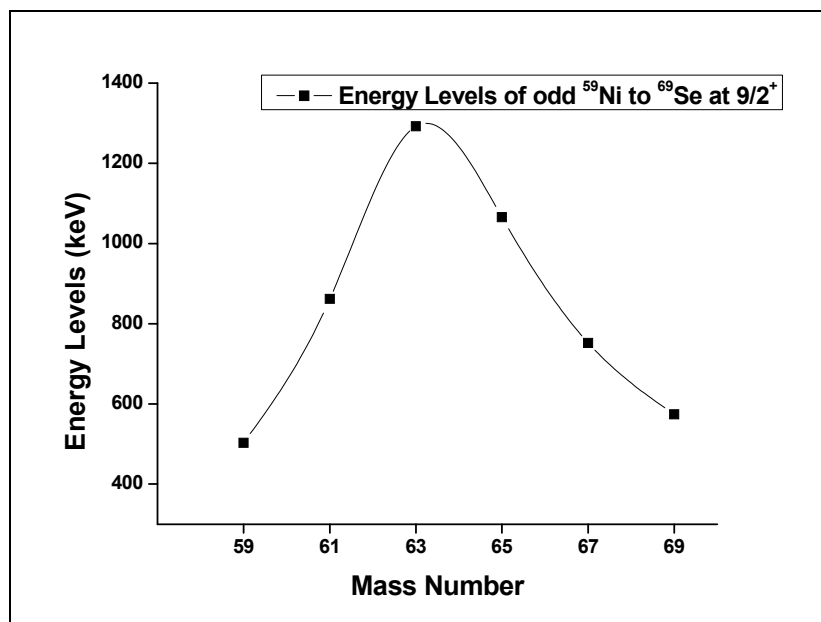
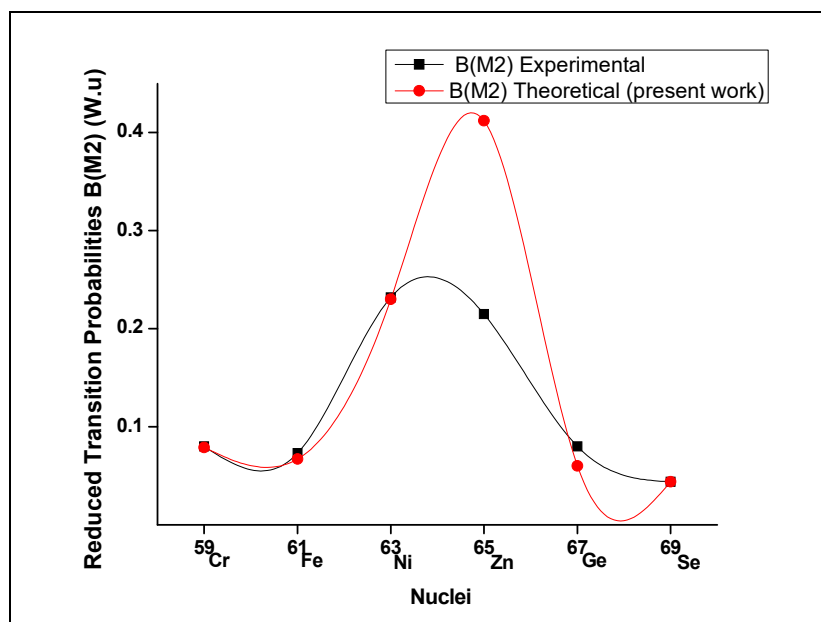
Experimental data have shown the reduced transition probabilities of odd  $^{59}\text{Cr}$  to  $^{69}\text{Se}$  nuclei with neutron  $N=35$  (Baglin, 2002; Bhat, 1999; Bhat and Tuli, 2000; Brown, 2010; Erjun, 2001; Junde et al., 2005; Hossain et al., 2011). All transitions between  $9/2^+$  to  $5/2^-$  have been assigned to M2-type based on the selection rule. At present work the  $B(M2)$  values are 0.079 W.U., 0.067 W.U., 0.230 W.U., 0.412 W.U., 0.060 W.U., and 0.044 W.U. for  $^{59}\text{Cr}$ ,  $^{61}\text{Fe}$ ,  $^{63}\text{Ni}$ ,  $^{65}\text{Zn}$ ,  $^{67}\text{Ge}$ , and  $^{69}\text{Se}$  nuclei respectively. Figure 2 shows the comparison between the experimental and theoretical data. The calculated data showed that  $B(M2)$  values of the isomers are good agreement with the previous experimental results except for  $^{65}\text{Zn}$ . Because branching ratio of gamma ray in  $^{65}\text{Zn}$  is very small (10%), which showed strong deviation from experimental values. The maximum magnetic quadrupole reduced transition probability is at  $^{65}\text{Zn}$ . The  $B(M2)$  values as well as the isomeric level  $9/2^+$  increases towards magic number  $Z=28$ , and then M2 strength and isomeric level decreases as neutron number increases except for  $^{65}\text{Zn}$ . The  $B(M2)$  values in  $^{65}\text{Zn}$  seem to relate to the configuration of ground state of  $5/2^-$  states as well as to the deformation of the nuclei.

### 3.3 Width of Isomeric Levels ( $\Gamma_\gamma$ )

The widths of isomeric levels were calculated using the Equation (8). The width indicates the thickness of gamma rays produced from the transition. The value of width of isomeric levels are 47.64 eV, 18.30 eV,  $1.3 \times 10^3$  eV,  $7.9 \times 10^3$  eV, 31.33 eV and 4.76 eV for  $^{59}\text{Cr}$ ,  $^{61}\text{Fe}$ ,  $^{63}\text{Ni}$ ,  $^{65}\text{Zn}$ ,  $^{67}\text{Ge}$ , and  $^{69}\text{Se}$  nuclei respectively. It indicates that width of isomeric levels have no correlation with increase of neutrons of  $^{59}\text{Cr}$ ,  $^{61}\text{Fe}$ ,  $^{63}\text{Ni}$ ,  $^{65}\text{Zn}$ ,  $^{67}\text{Ge}$ , and  $^{69}\text{Se}$  nuclei.

Table 1. Isomeric properties of odd  $^{59}\text{Cr}$  to  $^{69}\text{Se}$  nuclei for  $N=35$ 

Nucl.	Isomeric levels $g_{9/2^+}$ (keV)	Energy levels $f_{5/2^-}$ (keV)	$E_\gamma$ ( $9/2^+-5/2^-$ ) (keV)	B.R (%)	$T_{1/2}$ ( $_{\text{exp}}$ )	Mean-life time, $\tau$	B(M2)		$\Gamma_\gamma$ (eV)	$F_w$
							W.U. ( $_{\text{Exp}}$ )	W.U. ( $_{\text{present}}$ )		
$^{59}\text{Cr}$	503	310	193	100	96(20) $\mu\text{s}$	139(29) $\mu\text{s}$	0.080 (17)	0.079	47.64	12.62
$^{61}\text{Fe}$	862	207	655	100	0.25 $\mu\text{s}$	0.36 $\mu\text{s}$	0.073 (3)	0.067	18.30	15.13
$^{63}\text{Ni}$	1292	87	1205	10	3.33(21)ns	4.8(3) ns	0.232(15)	0.230	$1.37 \times 10^3$	4.34
$^{65}\text{Zn}$	1066	0.0	1066	100	575ps	830 ps	0.215 (24)	0.412	$7.9 \times 10^3$	2.43
$^{67}\text{Ge}$	752	18	734	100	0.146 $\mu\text{s}$	0.21 $\mu\text{s}$	0.079 (1)	0.060	31.33	16.57
$^{69}\text{Se}$	574	39	535	100	0.96 $\mu\text{s}$	1.39 $\mu\text{s}$	0.044 (1)	0.044	4.76	22.83

Figure 1. Isomeric levels versus mass number of odd  $^{59}\text{Cr}$  to  $^{69}\text{Se}$  for  $N=35$ Figure 2. B(M2) values versus mass number of odd  $^{59}\text{Cr}$  to  $^{69}\text{Se}$  isomers

### 3.4 Weisskopf Hindrance Factor, $F_w$

The Weisskopf hindrance Factor,  $F_w$  was calculated according to Equation (9). The values of hindrance factors are 12.62, 15.13, 4.34, 2.41, 16.57, and 22.83 of  $^{59}\text{Cr}$ ,  $^{61}\text{Fe}$ ,  $^{63}\text{Ni}$ ,  $^{65}\text{Zn}$ ,  $^{67}\text{Ge}$ , and  $^{69}\text{Se}$  nuclei, respectively. The minimum value and maximum value of hindrance factor is 2.41 and 22.83 for  $^{63}\text{Ni}$  and  $^{69}\text{Se}$ , respectively.

### 4. Conclusion

The systematic reduced transition probabilities  $B(M2)$ , width of isomeric level, mean-life and Weisskopf factor hindrance were calculated in odd  $^{59}\text{Cr}$  to  $^{69}\text{Se}$  nuclei for  $N=35$ . The theoretical calculation of reduced transition probabilities  $B(M2)$  is good agreement with the previous experimental values (Baglin, 2002; Bhat, 1999; Bhat and Tuli, 2000; Brown, 2010; Erjun, 2001; Junde et al., 2005; Hossain et al., 2011). The isomeric level and  $B(M2)$  values are increases with proton towards magic number  $Z=28$ , and then  $M2$  strength and isomeric level decreases as proton number increases except for  $^{65}\text{Zn}$ .

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