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Suspending the Principle of Relativity

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Abstract

A unique hyperbolic geometry paradigm requires suspending the Relativistic principle that absolute velocity is unmeasurable. The idea that of two observers each sees the same constant velocity, therefore there is no absolute velocity, is true only because Relativity uses a particular Lorentz geometry. Our mathematical geometry constructs circle and hyperbola vectors with hyperbolic terms in an original formulation of complex numbers. We use a point on a hyperbola as a frame of reference. A theory of time is given. The physical laws of motion by Galileo, Newton and Einstein are forged using the absolute velocity and the precondition to electromagnetic velocity. The field of real and fictitious force accelerations is established. We utilize Galilean Invariance to measure absolute velocity. An experiment exemplifies the math from the Earth's frame of reference. But Relativity is based on local Lorentz geometry. We discover a possible dark energy and gravitational accelerations and a geometry of gravitational collapse.

Keywords: acceleration, angle, coordinate, time, trigonometry, velocity

1. Introduction

Conventional trigonometry defines coordinates on a circle as:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$
$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

where $r = (x^2 + y^2)^{1/2}$. Leonhard Euler's (1707–1783) formula

$$z = re^{i\theta} = r\cos\theta + ir\sin\theta = x + iy \tag{1}$$

regarding the vector z of length (Marsden, 1999)

$$|z| = (x^2 + y^2)^{1/2}$$

revealed the existence of a profound relationship between complex numbers and trigonometric functions. Technological numerical techniques, like those on a scientific pocket calculator, contrive a trigonometry with hyperbolic coordinates that we can now formulate.

To construct hyperbolic geometry coordinates, we need to devise a whole new complex variables, one not based on circular trigonometry. After rephrasing vectors, we describe how the new complex variables shape up. We apply our hyperbolic trigonometry to electromagnetic velocity. With a precondition velocity, we can derive acceleration. Our geometry requires us to suspend the Special Relativity physics principle that absolute velocity is unmeasurable. The idea that a first observer sees velocity $\tilde{v} = v' + v$ and a second observer, whose velocity relative to the first is a constant v, sees $v' = \tilde{v} - v$, therefore there is no absolute velocity, is contested with our mathematics (Schutz, 2009). Absolute velocity and absolute acceleration are measurable at the specified precondition velocity v of a moving observer. We find a possible dark energy and gravitational accelerations and a geometry of gravitational collapse.

2. Hyperbolic geometry and physics

H.S.M. Coxeter, F.R.S. (1907–2003), explains how the angle of parallelism geodesic $B = \theta = \angle ABC$ makes the asymptotic right triangle ABC (you may draw). Hypotenuse $BA = c$ is of infinite length. The base of the triangle is $BC = a$. $C = \angle BCA$ is a right angle. $A = \angle BAC$ is 0. Parallel lines BA and CA meet in infinity (Coxeter, 1978). Coxeter describes that spherical to hyperbolic triangle transition (Coxeter, 1998) as:

$$\begin{aligned} \tan a &= \tan c \cos B & \cos A &= \cos a \sin B & \tan b &= \sin a \tan B \\ \tanh a &= \tanh \infty \cos B & \cos 0 &= \cosh a \sin B & \tanh b &= \sinh a \tan B \\ \tanh a &= 1 \cos B & 1 &= \cosh a \sin B & 1 &= \sinh a \tan B \\ \tanh a &= \cos B & \operatorname{sech} a &= \sin B & \sinh a &= \cot B. \end{aligned}$$

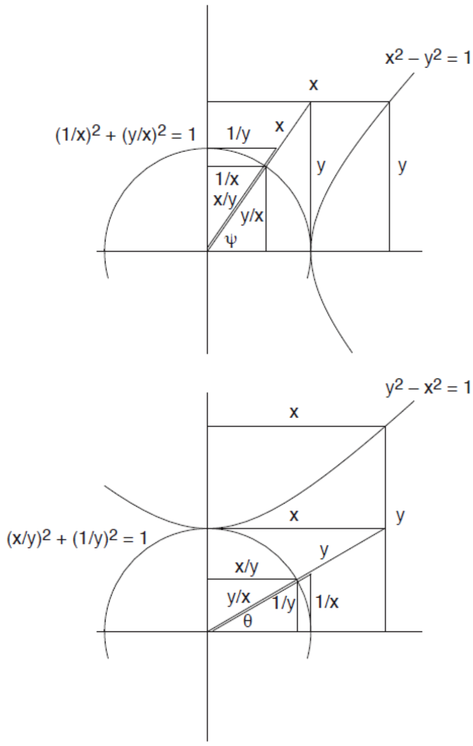

Figure 1. (a) Above, horizontal, and (b) below, vertical hyperbola and circle coordinate angles, as defined in hyperbolic trigonometry. Reproduced with kind permission from Spec in Sci and Tech **21**, 214, 219 (1999) Russell Eskew Copyright 1999 Springer, Springer Science and Business Media

To organize this hyperbolic geometry, before $B = \theta$ let us first consider the angle ψ lying on a spherical circle with a horizontal hyperbola $x^2 - y^2 = 1$ partially used by Christof Gudermann (1798–1852) (Chrystal, 1931). It is called the *gudermannian* and defined (Beyer, 1987) for $0 \leq \psi \leq \frac{\pi}{2}$. Figure 1(a) depicts the four coordinates x, y, ψ and α, which satisfy:

$$\begin{aligned} x &= \sec \psi = \cosh \sinh^{-1} y = \coth \alpha & y &= \tan \psi = \sinh \sinh^{-1} y = \operatorname{csch} \alpha \\ \frac{1}{x} &= \cos \psi = \operatorname{sech} \sinh^{-1} y = \tanh \alpha & \frac{1}{y} &= \cot \psi = \operatorname{csch} \sinh^{-1} y = \sinh \alpha \\ \frac{x}{y} &= \csc \psi = \coth \sinh^{-1} y = \cosh \alpha & \frac{y}{x} &= \sin \psi = \tanh \sinh^{-1} y = \operatorname{sech} \alpha, \end{aligned}$$

when $\sinh^{-1} y = a$ and $\alpha = \ln \frac{1+v}{1-v}$ (see page 87 Theorem 2). We say $\tan\psi = \sinh a = \sinh\sinh^{-1} y = y$ because $\tan 2\frac{\psi}{2} = \frac{2\tan\psi/2}{1-\tan^2\psi/2} = \frac{2\tanh a/2}{1-\tanh^2 a/2} = 2\sinh\frac{a}{2}\cosh\frac{a}{2} = \sinh 2\frac{a}{2}$. But rather than saying $\tan\psi = \tanh a$, we have $\tan\frac{1}{2}\psi = \tanh\frac{1}{2}a$.

Returning to Coxeter's (Coxeter, 1989) angle of parallelism $B = \theta$, Fig. 1(b) displays the classical vertical hyperbola $y^2 - x^2 = 1$ coordinates (Martin, 1972) partially used (Bolyai, 1987) by Nikolai Ivanovic Lobachevskii (1792–1856) (Lobachevskii, 1840) as shown below:

$$\begin{aligned}
x &= \cot\theta = \sinh\sinh^{-1} x = k(\sinh^{-1}\cot\Pi(x)) = k(\sinh^{-1}\sinh\frac{x}{k}) = \operatorname{csch}\alpha \\
\frac{1}{x} &= \tan\theta = \operatorname{csch}\sinh^{-1} x = k(\sinh^{-1}\tan\Pi(x)) = k(\sinh^{-1}\operatorname{csch}\frac{x}{k}) = \sinh\alpha \\
\frac{x}{y} &= \cos\theta = \tanh\sinh^{-1} x = k(\sinh^{-1}\cos\Pi(x)) = k(\sinh^{-1}\tanh\frac{x}{k}) = \operatorname{sech}\alpha \\
y &= \csc\theta = \cosh\sinh^{-1} x = k(\sinh^{-1}\csc\Pi(x)) = k(\sinh^{-1}\cosh\frac{x}{k}) = \coth\alpha \\
\frac{1}{y} &= \sin\theta = \operatorname{sech}\sinh^{-1} x = k(\sinh^{-1}\sin\Pi(x)) = k(\sinh^{-1}\operatorname{sech}\frac{x}{k}) = \tanh\alpha \\
\frac{y}{x} &= \sec\theta = \coth\sinh^{-1} x = k(\sinh^{-1}\sec\Pi(x)) = k(\sinh^{-1}\coth\frac{x}{k}) = \cosh\alpha,
\end{aligned} \tag{2}$$

when $\sinh^{-1} x = \ln\frac{1}{v} = a$ and $\alpha = \ln\frac{1+v}{1-v}$ (see page 87 Theorem 2). Since $\cot\psi = \sinh\alpha = \tan\theta = \cot(\frac{\pi}{2} - \theta)$ we relate angles $\psi = \frac{\pi}{2} - \theta$. We can later demonstrate with $x = \cot\theta = e^{n\pi/2}$. The hyperbolic functions defined by Johann Heinrich Lambert (1728–1777) stem from $\sinh x = (e^x - e^{-x})/2$. By $\sinh^{-1}\sinh x = \ln(\sinh x + \sqrt{(\sinh x)^2 + 1}) = x$, we also have $\sinh\sinh^{-1} x = \underline{\sinh a} = (e^{\sinh^{-1} x} - e^{-\sinh^{-1} x})/2 = x = \cot\theta$. We have $\cosh\sinh^{-1} x = \cosh a = (e^{\sinh^{-1} x} + e^{-\sinh^{-1} x})/2 = y = \sqrt{x^2 + 1} = \csc\theta$ as well. Martin creates the *distance scale* $k = \frac{x}{a}$ for any concentric horocircles of distance x in the Bolyai-Lobachevskii plane (Coxeter, 1989). The angle of parallelism (Eskew, 1999) $\angle ABC$ is of Lobachevskii's $\theta = 2\tan^{-1} e^{-a} = 2\tan^{-1} e^{-\sinh^{-1} x}$ and of Martin's critical function $\Pi(x) = 2\tan^{-1} e^{-x/k} = 2\tan^{-1} e^{-(\sinh a)/k} = 2\tan^{-1} e^{-(x)/(x/a)}$.

A point (x, y) on the vertical hyperbola $y^2 - x^2 = 1$ is understood as an inertial frame of reference, or observer. Unaccelerated inertial observers have a constant velocity with respect to any other observer. Within electromagnetic velocity (Misner, Thorne, & Wheeler, 1970) $\tan\theta = \tanh\alpha = \beta = v/c$, we have (i) a precondition velocity $\tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = v$, (ii) an absolute velocity $u = \tan\theta = (\tanh\alpha)(\cosh\alpha) = \sinh\alpha = \beta/(1-\beta^2)^{1/2} = \frac{1}{x}$, because $\tan 2\frac{\theta}{2} = \frac{2\tan\theta/2}{1-\tan^2\theta/2} = \frac{2\tanh\alpha/2}{1-\tanh^2\alpha/2} = (2v)/(1-v^2) = 2\sinh\frac{\alpha}{2}\cosh\frac{\alpha}{2} = \sinh 2\frac{\alpha}{2}$. We also have (iii) a relative velocity $v' = 1 - v$ and (iv) a relative absolute velocity $u' = \frac{1}{y} = \frac{1}{x+v} = \sin\theta = \tanh\alpha$. Later we challenge these assumptions.

Time can have an analytic quantity $t = e^{\sinh^{-1} x} = x + (x^2 + 1)^{1/2} = x + y$ seconds, which we are saying is about an event $P(t)$. Distance is 1. We are claiming a precondition velocity $v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = 1/(x + y) = 1/t$ distance/time, where $0 \le v \le 1$. Our math utilizes the complex plane rather than the spacetime diagram. We advocate stating $vt = [1/(x + y)][x + y] = 1$ distance and acceleration $a_{\text{frame}} = dv/dt = \frac{d}{dt}t^{-1} = -v^2 = -[1/(x + y)]^2$ distance/time^2 when, say, the Earth's gravitational acceleration is $g = -9.81$ m/s^2.

3. Vectors

Now we will show how hyperbolic coordinates make the vectors of complex numbers. By Eq. (1) the point on a circle is $(x, y) = (r\cos\theta, r\sin\theta)$.

theorem 1 Let (x, y) be a frame of reference point on the vertical hyperbola. Then the hyperbola or circle vectors can be made of the hyperbola's x and y rather than the circle's x and y.

Proof. If a circle vector $z_1 = re^{i\theta} = r(\cos\theta + i\sin\theta) = r(\frac{x}{y} + i\frac{1}{y})$ where (x, y) is on the Eq. (2) vertical hyperbola $y^2 - x^2 = 1$ then we can write:

$$\begin{aligned} z_1 &= re^{i\theta} = r(\cos\theta + i\sin\theta) = r(\frac{x}{y} + i\frac{1}{y}) \\ |z_1| &= ((\cos\theta)^2 + (\sin\theta)^2)^{1/2} \ = \left(\left(\frac{x}{y}\right)^2 + \left(\frac{1}{y}\right)^2\right)^{1/2} = 1 \\ &= ((\csc\theta)^2 - (\cot\theta)^2)^{1/2} \ = (y^2 - x^2)^{1/2} = 1, \end{aligned}$$

rather than Eq. (1). Any point on the complex plane can be reached with $re^{i\theta}$ and translated into the frame of reference (x, y). If a hyperbolic vector $z_2 = \cot\theta + i\csc\theta = x + iy$ where (x, y) is on the Eq. (2) vertical hyperbola $y^2 - x^2 = 1$ then we denote our unconventional *complex number* (Marsden, 1999) thusly:

$$\begin{aligned} z_2 &= (a, b) = a + ib = \cot\theta + i\csc\theta = x + iy \\ |z_2| &= ((a + ib)(a - ib))^{1/2} = (a^2 + b^2)^{1/2} \\ &= ((\csc\theta)^2 + (\cot\theta)^2)^{1/2} = (y^2 + x^2)^{1/2}, \end{aligned}$$

where $r = ((r\cos\theta)^2 + (r\sin\theta)^2)^{1/2}$ and $|z_2|$ are different. With $\sec\theta = \frac{b}{a} = \frac{y}{x}$, rather than $\tan\theta = \frac{y}{x}$, the vector makes an angle $\tan\varphi = \frac{y}{x}$ along the x-axis. Uniting conventional (Palka, 1991) with our unconventional complex numbers, we obtain $x = |z_2|\cos\varphi = \cot\theta$ and $y = (x^2 + 1)^{1/2} = |z_2|\sin\varphi = \csc\theta$. So we have a vector-valued function $f(z) = f(x, y) = f(\cot\theta, \csc\theta) = u(x, y) + iv(x, y)$ for a $Df(x, y)$. Note that $\sin\varphi$ and $\cos\varphi$ have a range of $[-1, 1]$. But $\cot\theta$ has a range of $(-\infty, \infty)$, with similar $\csc\theta$. □

Example: point $(x, y) = (x, \sqrt{x^2 + 1}) = (5, \sqrt{26})$ obtains circle vector $z_1 = re^{i\theta} = r(\cos\theta + i\sin\theta) = r(\frac{5}{\sqrt{26}} + i\frac{1}{\sqrt{26}})$ and hyperbola vector $z_2 = |z_2|e^{i\varphi} = \sqrt{51}(\cos\varphi + i\sin\varphi) = \cot\theta + i\csc\theta = x + iy = 5 + i\sqrt{26}$.

4. Applying velocity in complex numbers

Given precondition velocity v and any two complex numbers $z = a + ib$ and $u = x + iy$ where (x, y) is on the hyperbola $y^2 - x^2 = 1$, if $u^2 = z$ then by taking the square roots of negative numbers (Marsden, 1999) we acquire:

$$\begin{aligned} u^2 &= a + ib \\ &= (x + iy)(x + iy) = (x^2 - y^2) + i2xy \\ &= ((\sinh\ln\frac{1}{v})^2 - (\cosh\ln\frac{1}{v})^2) + i2(\sinh\ln\frac{1}{v})(\cosh\ln\frac{1}{v}) \\ &= ((\cot\theta)^2 - (\csc\theta)^2) + i2(\cot\theta)(\csc\theta) \\ &= -1 + i2(\sinh\ln\frac{1}{v})(\cosh\ln\frac{1}{v}) = -1 + i\sinh 2\ln\frac{1}{v}. \end{aligned}$$

We also have (Ahlfors, 1970) the double-angle in $z = a + ib$ and circular vector $w = re^{i\theta} = r(\cos\theta + i\sin\theta)$ for $w^2 = z = r^2(\cos 2\theta + i\sin 2\theta)$, shown as:

$$\begin{aligned} [1]^2 &= \left[\left(\frac{x}{y}\right)^2 + \left(\frac{1}{y}\right)^2\right]^2 = [(\cos\theta)^2 + (\sin\theta)^2]^2 \\ &= \left[\left(\frac{x}{y}\right)^2 - \left(\frac{1}{y}\right)^2\right]^2 + \left[2\left(\frac{x}{y}\right)\left(\frac{1}{y}\right)\right]^2 = [\cos 2\theta]^2 + [\sin 2\theta]^2 \\ &= [y^2 - x^2]^2 = [(\csc\theta)^2 - (\cot\theta)^2]^2 \\ a^2 + b^2 &= [y^2 + x^2]^2 = [1]^2 + (2xy)^2 = [-1]^2 + (2\cot\theta\csc\theta)^2 \\ (a^2 + b^2)^{1/2} &= y^2 + x^2 = [(-a + (a^2 + b^2)^{1/2})/2] + [(a + (a^2 + b^2)^{1/2})/2], \end{aligned}$$

where $a = -1$ and $b = 2xy = 2\cot\theta\csc\theta = \sinh 2\ln\frac{1}{v}$. The equation $u^2 = z$ solves to $\pm((x^2)^{1/2} + i(y^2)^{1/2}) = ((a + (a^2 + b^2)^{1/2})/2)^{1/2} + i((-a + (a^2 + b^2)^{1/2})/2)^{1/2}$, depending on $\pm b$. We have $z^{1/2} = (x^2)^{1/2} + 0i$ if and only if

$z = (x^2) + 0i > 0$. We have $z^{1/2} = 0 + (y^2)^{1/2}i$ if and only if $z = -((-x)^2 + 1) + 0i < 0$. The two $z^{1/2}$ coincide if and only if $z = 0 + 0i$. Use $x = \cot\theta$, $y = \csc\theta$ rather than $x = r\cos\theta$, $y = r\sin\theta$.

The vertical hyperbola vector has a power of:

$$\begin{aligned} z^n &= (x + iy)^n = (\cot\theta + i\csc\theta)^n \\ &= (\sinh \ln \frac{1}{v} + i \cosh \ln \frac{1}{v})^n. \end{aligned}$$

The logarithmic, exponential, and nth root functions appear as:

$$\begin{aligned} \ln z &= \ln|z| + i\varphi = \ln((\csc\theta)^2 + (\cot\theta)^2)^{1/2} + i\varphi \\ &= \ln(y^2 + x^2)^{1/2} + i\varphi; \end{aligned}$$

$$\begin{aligned} e^z &= e^x e^{iy} = e^{\cot\theta} e^{i\csc\theta} = e^{\cot\theta}(\cos\csc\theta + i\sin\csc\theta) \\ &= e^{\sinh\ln\frac{1}{v}} e^{i\cosh\ln\frac{1}{v}} = e^{\sinh\ln\frac{1}{v}}(\cos\cosh\ln\frac{1}{v} + i\sin\cosh\ln\frac{1}{v}) = |e^z|\frac{e^z}{|e^z|}; \end{aligned}$$

$$\begin{aligned} z^{1/n} &= e^{(\ln z)/n} = e^{(\ln|z|+i\varphi)/n} = e^{(\ln((y^2+x^2)^{1/2})+i\varphi)/n} \\ &= e^{(\ln(y^2+x^2)^{1/2})/n} e^{i\varphi/n} = (y^2 + x^2)^{1/2n} e^{i(\varphi+2\pi k)/n}; \end{aligned}$$

$$\begin{aligned} z &= e^{\ln z} = e^{\ln|z|} e^{i\varphi} = |z|\frac{z}{|z|} \\ &= |z|e^{i\varphi} = (y^2 + x^2)^{1/2}(\cos\varphi + i\sin\varphi) \\ &= \ln e^z = \ln|e^z| + i\arg(e^z) = \ln e^x + iy \\ &= x + iy = \cot\theta + i\csc\theta, \end{aligned}$$

where $\theta = \sin^{-1}\operatorname{sech}\ln\frac{1}{v} = \cos^{-1}\frac{x}{y}$ and $\varphi = \tan^{-1}\frac{y}{x}$, $k = 0, 1, \ldots, n-1$.

5. Accelerated frames of reference

Choosing an arbitrary frame of reference point, say $(x, y) = (5, \sqrt{26})$, can quantify physical laws of motion. Consider Isaac Newton's (1642–1727) second law of motion,

$$F = ma = mg = G\frac{mM}{r^2},$$

about a force F that for gravitation force is exerted on a particle of mass m by one of large mass M, producing an acceleration a at a distance r on a gravitational field g, computed using the gravitational constant G. F and m are the same regardless of which of two observers measures them (Schutz, 2009). It applies to the addition of velocities equation (Gleeson, 2010) of Galileo Galilei (1564–1642)

$$\tilde{v} = v' + v. \tag{3}$$

The quantities are velocity $\tilde{v} = 1$, its "relative velocity" $v' = \tilde{v} - v = 1 - 1/(x + y)$, (i.e., the particle's velocity with respect to a moving reference frame), and the constant "transport velocity" $v = \frac{1}{x+y} = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha$, (i.e., the velocity of the moving reference frame). The "absolute velocity" is $u = \frac{1}{x} = \tan\theta = \sinh\alpha$, (i.e., a moving particle's velocity with respect to a fixed reference frame), where we have:

$$u = 1/[(1/v) - y)] = 1/[(x + y) - y] = 1/x = (2v)/(1 - v^2). \tag{4}$$

This does not say $u = u' + v$, for Eq. (3) and Eq. (4) hold instead (Jackson, 1999). Albert Einstein (1879–1955) and H. A. Lorentz (1853–1928) replaced Eq. (3) with Eq. (5):

$$u = \frac{u' + v}{1 + u'v/c^2}. \tag{5}$$

Like Einstein (Lorentz, 1923) we hold when $v = \frac{1}{x+y} = 1$ distance/time is $c = 299792458$ meters/second, that upon the photon $u' = \frac{1}{y} = \sin\theta = \tanh\alpha = c$, we have (Einstein, 1957) $u = 1/[(1/v) - (1/c)] = 1/[(x+y) - y] = 1/x = c$. If $\tilde{v}$ has the magnitude c, then so does v', regardless of v. So too for u and u'.

Galileo showed that different freely falling bodies experience exactly the same acceleration at a given point in space. At the highest inverse velocity and time, $1/v = t = \infty$, a fictitious inertial force $F_{\text{frame}} = ma_{\text{frame}}$ is added to the real force $F + F_{\text{frame}} = ma'$. Hence $a_{\text{frame}} = \frac{dv}{dt} = \frac{d}{dt}t^{-1} = -1/t^2 = -1/\infty^2 = 0$, relative to an inertial force frame of reference, is at constant velocity, French (1971) and we have $a = a' - a_{\text{frame}} = a' - 0 = a'$. Newton's second law, $F = ma$, may also be written as $F = ma'$, due to the frame force, $F_{\text{frame}} = ma_{\text{frame}}$, that arises simply because the frame of reference is accelerated relative to an inertial frame. Lorentz used the inertial frame of reference $\beta = \tan\theta = \tanh\alpha = v/c$. But we hereby use $v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha$ distance/time as precondition (transport) velocity. Thus we can formulate acceleration (Einstein, 1950).

theorem 2 Let $v = \tan\frac{1}{2}\theta = \tanh\tanh^{-1} v = \tanh\frac{1}{2}\ln\frac{1+v}{1-v} = \tanh\frac{1}{2}\alpha = 1/t$ distance/time be the precondition velocity to $\beta = \tan\theta = \tanh\alpha = v/c$. Then the field of accelerations is shown as:

$$\frac{dv}{d\theta} = \frac{dv}{d\alpha} - \frac{dv}{dt}$$
$$\left(\frac{1}{2} + \frac{v^2}{2}\right) = \left(\frac{1}{2} - \frac{v^2}{2}\right) - \left(-v^2\right)$$
$$a = a' - a_{\text{frame}}$$
$$\text{accel}_{\text{absolute}} = \text{accel}_{\text{relative}} - \text{accel}_{\text{frame}}.$$

Proof. The precondition (Anderson, 2005) to velocity is $v = \tan\frac{1}{2}\theta = \tanh\tanh^{-1} v = \tanh\frac{1}{2}\ln\frac{1+v}{1-v} = \tanh\frac{1}{2}\alpha = 1/(x+y) = 1/t$ distance/time. The Eq. (2) parameter $\alpha = \ln\frac{1+v}{1-v}$ makes for a unique geometry. With the calculus notation u the accelerations are the derivatives resulting in trigonometry and algebra:

$$\begin{aligned}
\frac{dv}{d\theta} &= \frac{dv}{dt}\frac{dt}{d\theta} = \frac{d}{d\theta}\tan\frac{1}{2}\theta = \frac{d}{d\theta}\tan u = (\sec^2 u)\frac{du}{d\theta} = \frac{1}{2}\sec^2\frac{1}{2}\theta = \frac{1}{2} + \frac{v^2}{2}\\
\frac{dv}{d\alpha} &= \frac{dv}{dt}\frac{dt}{d\alpha} = \frac{d}{d\alpha}\tanh\frac{1}{2}\alpha = \frac{d}{d\alpha}\tanh u = (\operatorname{sech}^2 u)\frac{du}{d\alpha} = \frac{1}{2}\operatorname{sech}^2\frac{1}{2}\alpha = \frac{1}{2} - \frac{v^2}{2}\\
\frac{dv}{dt} &= \frac{dv}{d\alpha}\frac{d\alpha}{dt} = \frac{d}{dt}t^{-1} = -\frac{1}{(1/v)^2} = -v^2\\
\frac{d\alpha}{dt} &= \frac{d\alpha}{dv}\frac{dv}{dt} = \frac{d}{dt}\ln\frac{1+(1/t)}{1-(1/t)} = \frac{1}{u}\frac{du}{dt} = \left(\frac{1-(1/t)}{1+(1/t)}\right)\left(\frac{2(1/t^2)}{(1-(1/t))^2}\right) = \frac{2}{t^2-1}\\
\frac{d\theta}{dt} &= \frac{d\theta}{dv}\frac{dv}{dt} = \frac{d}{dt}\sin^{-1}\operatorname{sech}\ln t = \frac{d}{dt}\sin^{-1} u = \frac{u'}{\sqrt{1-u^2}}\\
&= \frac{-((\operatorname{sech}\ln t)(\tanh\ln t))1/t}{\sqrt{1-(\operatorname{sech}\ln t)^2}} = -\frac{2}{t^2+1}.
\end{aligned}$$

The accelerating, noninertial frame of reference becomes:

$$F_\theta = F_\alpha - F_{\text{frame}}$$
$$m\frac{dv}{d\theta} = m\frac{dv}{d\alpha} - m\frac{dv}{dt}.$$

□

This occurrence means that the field $a = a' - a_{\text{frame}}$ is equivalent to an acceleration of the coordinate frame at that event point (Dainton, 2001) in space $P(t) = P(\frac{1}{v}) = 1/(e^{\alpha/2}) = 1/[(1+v)/(1-v)]^{1/2} = P(x+y)$. When a particle decelerates from $a_{\text{relative}} = dv/d\alpha = (1/2 - v^2/2) = 1/2\ \frac{1}{t^2}$ relative to a moving reference frame transporting the particle's fictitious force from an inertial $a_{\text{frame}} = dv/dt = -v^2 = -1/(1/v)^2 = -1/\infty^2 = 0\ \frac{1}{t^2}$, the particle advances a real force acceleration with respect to a fixed observer at $a_{\text{absolute}} = dv/d\theta = (1/2 + v^2/2) = 1/2\ \frac{1}{t^2}$, from a rest frame velocity of $v = 0\ \frac{1}{t}$. As the fictitious force moving reference frame decelerates from 0 to $a_{\text{frame}} = -v^2 = -1/(1/v)^2 = -1/1^2 = -1\ \frac{1}{t^2}$, the a_{relative} of the particle relative to the moving reference frame approaches 0, and the fixed observer sees the particle's real force acceleration at $a_{\text{absolute}} = (1/2 + v^2/2) = 1\ \frac{1}{t^2}$, up to a light velocity of $v = 1\ \frac{1}{t}$. Tables 1 and 2 illustrate our coordinates.

Table 1. Values of a few coordinates at some points (x, y) on $y^2 - x^2 = 1$.

x $\cot\theta$	$t = 1/v$ $x + \sqrt{x^2+1}$	$\theta = \Pi(x)$ $2\tan^{-1} e^{\ln v}$	k x/a
0	1.0	$\pi/2 = 1.5707$	0
e^0	2.414213562	$\pi/4 = 0.7853$	1.134592657
$e^{\pi/2}$	9.723795265	0.204960468	2.11488971
e^{π}	46.30298215	0.043187049	6.033754226
$e^{3\pi/2}$	222.6400485	0.008983049	20.59321402
$e^{2\pi}$	1070.984245	0.001867441	76.75832388
$e^{5\pi/2}$	5151.942712	0.000388203	301.3843059
$e^{3\pi}$	24783.29566	0.000080700	1224.722226
$e^{7\pi/2}$	119219.483	0.000016776	5099.765758
$e^{4\pi}$	573502.6263	0.000003487	21626.07401
$e^{9\pi/2}$	2758821.412	0.000000725	93012.91224
$e^{5\pi}$	13271248	0.000000151	404583.825
$e^{11\pi/2}$	63841038.32	0.000000031	1776134.25
$e^{6\pi}$	307105870.8	0.000000007	7857302.779
$e^{13\pi/2}$	1477325845	0.000000001	34985338.41
$e^{7\pi}$	7106642561	2.81×10^{-10}	156642344.9
$e^{15\pi/2}$	3.41×10^{10}	5.85×10^{-11}	704725077.5
$e^{8\pi}$	1.64×10^{11}	1.21×10^{-11}	3183871713
$e^{17\pi/2}$	7.91×10^{11}	2.52×10^{-12}	1.44×10^{10}
$e^{9\pi}$	3.80×10^{12}	5.25×10^{-13}	6.56×10^{10}
$e^{19\pi/2}$	1.83×10^{13}	1.09×10^{-13}	2.99×10^{11}
$e^{10\pi}$	8.80×10^{13}	2.27×10^{-14}	1.37×10^{12}
$e^{21\pi/2}$	4.23×10^{14}	4.72×10^{-15}	6.28×10^{12}
$e^{11\pi}$	2.03×10^{15}	9.81×10^{-16}	2.89×10^{13}
$e^{23\pi/2}$	9.80×10^{15}	2.04×10^{-16}	1.33×10^{14}
$e^{12\pi}$	4.71×10^{16}	4.24×10^{-17}	6.14×10^{14}
$e^{25\pi/2}$	2.26×10^{17}	8.81×10^{-18}	2.83×10^{15}
$e^{13\pi}$	1.09×10^{18}	1.83×10^{-18}	1.31×10^{16}
$e^{27\pi/2}$	5.24×10^{18}	3.80×10^{-19}	6.08×10^{16}
$e^{14\pi}$	2.52×10^{19}	7.92×10^{-20}	2.82×10^{17}
$e^{29\pi/2}$	1.21×10^{20}	1.64×10^{-20}	1.31×10^{18}
$e^{15\pi}$	5.84×10^{20}	3.42×10^{-21}	6.11×10^{18}
$e^{31\pi/2}$	2.81×10^{21}	7.11×10^{-22}	2.84×10^{19}
$e^{16\pi}$	1.35×10^{22}	1.47×10^{-22}	1.32×10^{20}
$e^{33\pi/2}$	6.50×10^{22}	3.07×10^{-23}	6.19×10^{20}
$e^{17\pi}$	3.12×10^{23}	6.39×10^{-24}	2.89×10^{21}
$e^{35\pi/2}$	1.50×10^{24}	1.32×10^{-24}	1.35×10^{22}
$e^{25\pi}$	2.57×10^{34}	7.77×10^{-35}	1.62×10^{32}
∞	2∞	0	∞

$$
\begin{aligned}
x &= \cot\theta = e^{n\pi/2} \\
\text{time} &= t = 1/v = x + (x^2+1)^{1/2} = x + y \\
a &= \sinh^{-1} x = \frac{x}{k} = \ln\frac{1}{v} \\
k &= \frac{x}{a} \\
\angle ABC &= \theta = 2\tan^{-1} e^{-a} = \cos^{-1}\frac{x}{y} \\
&= \Pi(x) = 2\tan^{-1} e^{-x/k} = 2\tan^{-1} e^{-(x)/(x/a)}
\end{aligned}
$$

Table 2. Velocity and addition of accelerations.

velocity v 1/time	absolute accel $a_{\text{absolute}} = dv/d\theta$	relative accel $a_{\text{relative}} = dv/d\alpha$	-frame accl $-a_{\text{frm}} = v^2$
1.0	1.0	0	1.0
0.414213562	0.5 + 0.08578643	0.5 − 0.08578643	0.1715728
0.102840503	0.5 + 0.00528808	0.5 − 0.00528808	0.0105761
0.021596881	0.5 + 0.00023321	0.5 − 0.00023321	0.0004664
0.004491555	0.5 + 0.00001008	0.5 − 0.00001008	0.0000201
0.001867443	0.5 + 0.00000174	0.5 − 0.00000174	0.0000034
0.000194102	0.5 + 0.00000001	0.5 − 0.00000001	3×10^{-8}
0.000040350	$0.5 + (8 \times 10^{-10})$	$0.5 - (8 \times 10^{-10})$	2×10^{-9}
0.000008388	$0.5 + (3 \times 10^{-11})$	$0.5 - (3 \times 10^{-11})$	7×10^{-11}
0.000001744	$0.5 + (1 \times 10^{-12})$	$0.5 - (1 \times 10^{-12})$	3×10^{-12}
0.000000362	$0.5 + (6 \times 10^{-14})$	$0.5 - (6 \times 10^{-14})$	1×10^{-13}
0.000000151	$0.5 + (1 \times 10^{-14})$	$0.5 - (1 \times 10^{-14})$	2×10^{-14}
0.000000016	$0.5 + (1 \times 10^{-16})$	$0.5 - (1 \times 10^{-16})$	2×10^{-16}
0.000000003	$0.5 + (5 \times 10^{-18})$	$0.5 - (5 \times 10^{-18})$	1×10^{-17}
6.76×10^{-10}	$0.5 + (2 \times 10^{-19})$	$0.5 - (2 \times 10^{-19})$	4×10^{-19}
1.40×10^{-10}	$0.5 + (9 \times 10^{-21})$	$0.5 - (9 \times 10^{-21})$	1×10^{-20}
2.92×10^{-11}	$0.5 + (4 \times 10^{-22})$	$0.5 - (4 \times 10^{-22})$	8×10^{-22}
6.08×10^{-12}	$0.5 + (1 \times 10^{-23})$	$0.5 - (1 \times 10^{-23})$	3×10^{-23}
1.26×10^{-12}	$0.5 + (7 \times 10^{-25})$	$0.5 - (7 \times 10^{-25})$	1×10^{-24}
2.62×10^{-13}	$0.5 + (3 \times 10^{-26})$	$0.5 - (3 \times 10^{-26})$	6×10^{-26}
5.46×10^{-14}	$0.5 + (1 \times 10^{-27})$	$0.5 - (1 \times 10^{-27})$	2×10^{-27}
1.13×10^{-14}	$0.5 + (6 \times 10^{-29})$	$0.5 - (6 \times 10^{-29})$	1×10^{-28}
2.36×10^{-15}	$0.5 + (2 \times 10^{-30})$	$0.5 - (2 \times 10^{-30})$	5×10^{-30}
4.90×10^{-16}	$0.5 + (1 \times 10^{-31})$	$0.5 - (1 \times 10^{-31})$	2×10^{-31}
1.02×10^{-16}	$0.5 + (5 \times 10^{-33})$	$0.5 - (5 \times 10^{-33})$	1×10^{-32}
2.12×10^{-17}	$0.5 + (2 \times 10^{-34})$	$0.5 - (2 \times 10^{-34})$	4×10^{-34}
4.40×10^{-18}	$0.5 + (9 \times 10^{-36})$	$0.5 - (9 \times 10^{-36})$	1×10^{-35}
9.16×10^{-19}	$0.5 + (4 \times 10^{-37})$	$0.5 - (4 \times 10^{-37})$	8×10^{-37}
1.90×10^{-19}	$0.5 + (1 \times 10^{-38})$	$0.5 - (1 \times 10^{-38})$	3×10^{-38}
3.96×10^{-20}	$0.5 + (7 \times 10^{-40})$	$0.5 - (7 \times 10^{-40})$	1×10^{-39}
8.23×10^{-21}	$0.5 + (3 \times 10^{-41})$	$0.5 - (3 \times 10^{-41})$	6×10^{-41}
1.71×10^{-21}	$0.5 + (1 \times 10^{-42})$	$0.5 - (1 \times 10^{-42})$	2×10^{-42}
3.55×10^{-22}	$0.5 + (6 \times 10^{-44})$	$0.5 - (6 \times 10^{-44})$	1×10^{-43}
7.39×10^{-23}	$0.5 + (2 \times 10^{-45})$	$0.5 - (2 \times 10^{-45})$	5×10^{-45}
1.53×10^{-23}	$0.5 + (1 \times 10^{-46})$	$0.5 - (1 \times 10^{-46})$	2×10^{-46}
3.19×10^{-24}	$0.5 + (5 \times 10^{-48})$	$0.5 - (5 \times 10^{-48})$	1×10^{-47}
6.64×10^{-25}	$0.5 + (2 \times 10^{-49})$	$0.5 - (2 \times 10^{-49})$	4×10^{-49}
3.88×10^{-35}	$0.5 + (7 \times 10^{-70})$	$0.5 - (7 \times 10^{-70})$	1×10^{-69}
0	0.5	0.5	0

$$\text{velocity} = v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = \left(\frac{1}{x+y}\right)\frac{1}{\text{time}}$$

$$\text{kinetic}\quad \text{absoluteaccel} = a_{\text{absolute}} = \frac{dv}{d\theta} = \left(\frac{1}{2} + \frac{v^2}{2}\right)\frac{1}{\text{time}^2}$$

$$\text{potential}\quad \text{relativeaccel} = a_{\text{relative}} = \frac{dv}{d\alpha} = \left(\frac{1}{2} - \frac{v^2}{2}\right)\frac{1}{\text{time}^2}$$

$$\text{frameaccel} = a_{\text{frame}} = \frac{dv}{dt} = -v^2\frac{1}{\text{time}^2}.$$

6. Absolute velocity and acceleration

theorem 3 By Galilean Invariance there is no experiment that can be performed that can measure the velocity of a moving observer. We can detect the presence of accelerations and measure the relative velocity between two bodies but we cannot measure the absolute velocity (Gleeson, 2010). However we can measure the velocity $v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = 1/(x+y) = 1/t$ of a moving observer. We can detect the presence of accelerations like $\frac{d}{d\theta}\tan\frac{1}{2}\theta$, $\frac{d}{d\alpha}\tanh\frac{1}{2}\alpha$ and $\frac{d}{dt}t^{-1}$ and measure the relative velocity $v' = 1 - 1/(x+y)$ between two bodies and we can measure the absolute velocity $u = \tan\theta = (\tanh\alpha)(\cosh\alpha) = 1/x = (2v)/(1-v^2)$ with $u' = 1/y = \sin\theta = \tanh\alpha$.

Proof. Velocity can only be measured in relation to some specified point of rest, therefore, it is said, in physics absolute velocity does not exist. Just as it happens that absolute velocity $u = 1/x = 0$ when $x = \infty$, in mathematics absolute velocity does not exist only when $v = 0$. With Galileo's $\tilde{v} = v' + v = 1$ relative velocity $v' = \tilde{v} - v = 1 - 1/(x+y)$ exists. Eq. (4) absolute velocity $u = \tan\theta = (2v)/(1-v^2)$ is measured with the velocity $v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = 1/t$ distance/time of a moving observer. By Galilean Invariance we cannot measure a velocity, so $v = 0$ is at rest state and absolute velocity $1/x = 1/\infty = 0$ doesn't exist. Using mathematics, though, when $0 < v$ velocity is measurable and absolute velocity does exist. In Einstein's Special Relativity the velocity of light $v = 1$ is invariant; as we said about Eq. (5), absolute velocity is absolute motion $u = 1/x = 1/0 = \infty = c$ and $u' = 1/y = \sin\theta = c$. We have also stated that absolute acceleration is $\left[\frac{dv}{d\theta} = \frac{d}{d\theta}\tan\frac{1}{2}\theta = \frac{1}{2}\sec^2\frac{1}{2}\theta = \frac{1}{2} + \frac{v^2}{2}\right] = \left[\frac{dv}{d\alpha} = \frac{d}{d\alpha}\tanh\frac{1}{2}\alpha = \frac{1}{2}\mathrm{sech}^2\frac{1}{2}\alpha = \frac{1}{2} - \frac{v^2}{2}\right] - \left[\frac{dv}{dt} = \frac{d}{dt}t^{-1} = -v^2\right]$. □

To quote Einstein's (1911) principle (Misner, Thorne, & Wheeler, 1970) of the local equivalence between a "gravitational field" and an acceleration:

> "We arrive at a very satisfactory interpretation of this law of experience, if we assume that the systems K and K' are physically exactly equivalent, that is, if we assume that we may just as well regard the system K as being in a space free from gravitational fields, if we then regard K as uniformly accelerated. This assumption of exact physical equivalence makes it impossible for us to speak of the absolute acceleration of the system of reference, just as the usual theory of relativity forbids us to talk of the absolute velocity of a system; and it makes the equal falling of all bodies in a gravitational field seem a matter of course."

The accepted theory of gravity is Einstein's theory of General Relativity. The Einstein field equation is $G = 8\pi T$, where T is the Riemann stress-energy tensor (Misner, Thorne, & Wheeler, 1970). We will conclude that Relativity is based on local Lorentz geometry. This essay, however, can express absolute velocity and absolute acceleration because of our unique hyperbolic geometry Eq. (2). We conjecture an accelerated coordinate frame of a precondition velocity $v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = 1/t$ for a field of accelerations. The vectors $z_2 = x + iy$ of complex variables are basic science because our event points in space $P(t) = P(\frac{1}{v}) = 1/(e^{\alpha/2}) = 1/[(1+v)/(1-v)]^{1/2} = P(x+y)$ stem from hyperbolic properties $\frac{1}{v} = t = x + y$.

7. An experiment

When 1 distance/time is 299792458 meters/second = c, the speed of light, we advocate stating a fictitious force acceleration $a_{\text{frame}} = dv/dt = \frac{d}{dt}t^{-1} = -1/(1/v)^2 = -v^2 = -[1/(x+y)]^2$ distance/time^2 when, say, the Earth's gravitational acceleration is $g = (-v^2)(299792458) = -299792458/30559883.58 = -9.81$ m/s^2 at $x = 2764.04963722$. (Use $(x+y)^2 = 30559883.58$ and $y^2 = x^2 + 1$.) Let $(x, y) = (x, \sqrt{x^2+1}) = (2764.04963722, 2764.04981811)$ be the frame of reference of the Earth. The Earth's particle wave is moving at a precondition velocity $v = 1/(x+y) = 1/t = 1/(2764.04963722 + 2764.04981811) = 1/5528.099455$ distance/time = 54230.65566 meters/second. Earth's orbital velocity is 29.8 km/s. A particle on Earth moves forward at a relative velocity $v' = 1 - v = 1 - 1/5528.099455 = 0.999819106$ distance/time = 299738227.3 meters/second. We say the slice of time is $t = 1/v = x + y = 2764.04963722 + 2764.04981811 = 5528.099455$ seconds. As the Earth passes a fixed observer, the observer sees the particle's absolute velocity at $u = 1/x = (2v)/(1-v^2) = (2\frac{1}{t})/(1-(\frac{1}{t})^2) = 1/2764.04963722$ distance/time = 108461.3149 meters/second. The relative absolute velocity is $u' = 1/y = 1/2764.04981811$ distance/time = 108461.3078 meters/second. The Earth's particle real force absolute acceleration is $a_{\text{absolute}} = \frac{1}{2} + \frac{v^2}{2}$ distance/time^2 = $(\frac{1}{2} + \frac{v^2}{2})(299792458) = 149896233.9$ m/s^2. The relative acceleration is $a_{\text{relative}} = \frac{1}{2} - \frac{v^2}{2}$ distance/time^2 = $(\frac{1}{2} - \frac{v^2}{2})(299792458) = 149896224.1$ m/s^2.

8. Conclusion

This work contends that there is an underlying theory of v within velocity $\beta = \tan\theta = \tanh\alpha = v/c$ for an absolute velocity $u = \tan\theta = \sinh\alpha$. A slice of time can have an analytic quantity $t = e^{\sinh^{-1} x} = x+(x^2+1)^{1/2} = x+y$ seconds, with $\Delta t = y - x$. We are claiming a precondition velocity $v = \tan\frac{1}{2}\theta = \tanh\frac{1}{2}\alpha = 1/(x + y) = 1/t$ distance/time, where $0 \le v \le 1$. We advocate stating $vt = [1/(x + y)][x + y] = 1$ distance and a particle's acceleration as $a_{\text{absolute}} = dv/d\theta = 1/2 + v^2/2$ distance/time^2. All of the coordinates in Figure 1(b) move together. The angle of parallelism θ graphs both the circle and the $y^2 - x^2 = 1$ vertical hyperbola in hyperbolic terms. The complex plane point (x, y) makes $\sec\theta = \frac{y}{x}$, rather than $\tan\theta = \frac{y}{x}$, while vector angle $\tan\varphi = \frac{y}{x}$ holds. Our circle vector $z_1 = re^{i\theta} = r(\cos\theta + i\sin\theta) = r\left(\frac{x}{y} + i\frac{1}{y}\right)$ and hyperbola vector $z_2 = |z_2|e^{i\varphi} = (y^2 + x^2)^{1/2}(\cos\varphi + i\sin\varphi) = x + iy = \cot\theta + i\csc\theta$ are made with the hyperbola's x and y, rather than the circle's. Any point on the complex plane can be reached with $re^{i\theta}$ and translated into the frame of reference (x, y).

In the local Lorentz frame of reference every particle moves in a straight line with uniform velocity β. In hyperbolic geometry the "straight line" is due to the angle of parallelism geodesic θ or $\Pi(x)$. The Lorentz frame of reference ("inertial frame of reference") is rectified as $\beta = v/c = \tan\theta = \tanh\alpha$ by absolute velocity $u = \frac{1}{x} = \tan\theta = (\tanh\alpha)(\cosh\alpha) = \sinh\alpha = \beta/(1-\beta^2)^{1/2} = (2v)/(1-v^2)$, relative velocity $v' = 1-v = 1-1/(x+y)$, relative absolute velocity $u' = \frac{1}{y} = \frac{1}{x+v} = \sin\theta = \tanh\alpha$, and vector velocity $\frac{dt}{d\tau} = 1/(1-\beta^2)^{1/2} = \frac{1}{\tau} = \frac{y}{x} = \sec\theta = \cosh\alpha = \gamma$. Event points in space (Misner, Thorne, & Wheeler, 1970) happen $P(t) = P(\frac{1}{v}) = 1/(e^{\alpha/2}) = 1/[(1+v)/(1-v)]^{1/2} = P(x+y)$. We have τ is *proper time* and s is *proper distance* with *time dilation* Δt and *length contraction* s' as:

$$\tau = \Delta t\frac{1}{\gamma} = (y - x)\frac{x}{y} \qquad \Delta t = \tau\gamma = \frac{s}{v} = y - x$$
$$s = v\Delta t = \frac{1}{x + y}(y - x) \qquad s' = s\frac{1}{\gamma} = v\tau = (\frac{y - x}{x + y})(\frac{x}{y}).$$

Hyperbolic geometry has $(\cos\theta)^2 + (\sin\theta)^2 = (x/y)^2 + (1/y)^2 = 1$ with $x = \cot\theta$ and $y = \csc\theta$. Local Lorentz geometry holds the interval $-\tau^2 = s^2 = (AB)^2 = -(\cos\theta)^2 = (AC)^2 + (BC)^2 = (CZ)^2 + (BC)^2 = -1^2 + (\sin\theta)^2 = -(AQ)(A\grave{P}) = (\sin\theta + 1)(\sin\theta - 1) = -\tau\tau$ between vector events $B - A$ (or AB). A light ray calculated from events $\grave{P}B$ to events BQ lies with B off and with $ACZ = A\grave{P}QZ$ on the particle's *world line* $P(\tau)$. Vectors $z_2 = x + iy$ are alternately made with $t = \frac{1}{v} = x + y$ than spacetime events $P(\tau) = P(\frac{\Delta t}{\gamma})$. We may have Lorentzian $B - A = P(\tau) - P(0) = P(\cos\theta) - P(0) = P(1) - P(0)$ at one second but hyperbolic $P(t) = P(x + y) = P(0 + 1)$ events happen on the complex plane. The combination of hyperbolic geometry and local Lorentz geometry might be called *pregeometry*, gravitational collapse (Misner, Thorne, & Wheeler, 1970). We assert that absolute velocity and absolute acceleration exist in mathematics and are measurable. A particle's real force acceleration is $a_{\text{absolute}} = (1/2 + v^2/2)(299792458)\ \ \text{m/s}^2$ with a gravitational fictitious force acceleration $g = (-v^2)(299792458)\ \ \text{m/s}^2$. Can this expanding real force be dark energy?

Joshua Frieman, director of the Dark Energy Survey, Frieman (2015) describes the particle, with my brackets, as follows:

> "Dark energy takes the form of a so far undetected ['quintessence'] particle that could be a distant cousin of the recently discovered Higgs boson . . . a particle acting like a ball rolling down a hill at each point in space. The rolling ball carries both kinetic energy (because of its motion) [like our absolute acceleration] and potential energy (because of the height of the hill it is rolling down) [like our relative acceleration]; the higher an object is, the greater its potential energy is. As it rolls down, its potential energy declines, and its kinetic energy rises . . . . If the quintessence particle is extremely light, . . . then it would be rolling very slowly today, with relatively little kinetic versus potential energy. In that case, its effect on cosmic expansion would be similar but not identical to that of vacuum energy and would lead to acceleration [of the universe]."

If dark energy (Riess & Livio, 2016) is the energy of the vacuum (General Relativity's cosmological constant) (Misner, Thorne, & Wheeler, 1970), then the acceleration of the moving reference frame $a_{\text{frame}} = -v^2 = -1$ will be constant at $v = 1$. Eq. (2) is a projective geometry, which does not include Riemannian geometry, nor topology (Coxeter, 1989). General Relativity is a theory determined by relative acceleration $a_{\text{relative}} = \frac{dv}{d\alpha} = 1/2 - v^2/2$. The Einstein field equation is $G = 8\pi T$, where T is the Riemann stress-energy tensor. With Riess and Livio (2016) we alternatively hypothesize dark energy may be an energy field that pervades the universe, imbuing every point in space $P(t) = P(\frac{1}{v}) = 1/(e^{\alpha/2}) = 1/[(1+v)/(1-v)]^{1/2} = P(x+y)$ with a property $a_{\text{absolute}} = \frac{dv}{d\theta} = 1/2 + v^2/2$ that counteracts the pull of gravity $g = -v^2$.

References

Ahlfors, L. V. (1970). *Complex analysis* (3rd ed., pp. 3-13). New York: McGraw-Hill.

Anderson, J. W. (2005). *Hyperbolic geometry* (Chapter 3, p. 124). London: Springer-Verlag.

Beyer, W. H. (1987). *CRC handbook of mathematical sciences*. Boca Raton, FL: CRC Press.

Bolyai, J. (1987). *Appendix: the theory of space*. no. 138 Amsterdam: North-Holland Mathematical Studies.

Chrystal, G. (1931). *Algebra* (vol. 2, pp. 312-313). London: Black.

Coxeter, H. S. M. (1978). Parallel lines. *Canad. Math. Bull, 21*, 385-397.

Coxeter, H. S. M. (1989). *Introduction to geometry* (2nd ed., pp. 92-94, 230, 267, 291, 301, 315, 372, 376-377). Toronto: Univ. Toronto Press.

Coxeter, H. S. M. (1998). *Non-euclidean geometry* (6th ed., pp. 238, 277, 295, 306, 312). Washington, D.C.: Math. Assoc. of America.

Dainton, B. (2001). *Time and space*. Montreal: McGill-Queen's Univ. Press.

Einstein, A. (1950). On the generalized theory of gravitation. *Sci. Am., 182*(4), 13-17. http://dx.doi.org/10.1038/scientificamerican0450-13

Einstein, A. (1957). *Relativity: the special and the general theory* (15th ed., pp. 26-30). London: Methuen.

Eskew, R. C. (1999). Time, gravity and the exterior angle of parallelism. *Speculations in Science and Technology, 21*, 213-225. http://dx.doi.org/10.1023/A:1005401228301

French, A. P. (1971). *Newtonian mechanics*. Cambridge, MA: Massachusetts Institute of Technology, Chapter 12.

Frieman, J. (2015). Seeing in the dark. *Sci. Am., 313*(5), 40-47.

Gleeson, A. M. (2010). *Introduction to modern physics*. Austin: Univ. TX, 185, 205-218. Retrieved from www.utexas.edu/faculty

Jackson, J. D. (1999). *Classical electrodynamics* (3rd ed., pp. 524-532). New York: Wiley.

Lobachevskii, N. I. (1840). *Geometrical researches on the theory of parallels*. Berlin: Karzan; (Russian, English translation available).

Lorentz, H. A. (1923). *The principle of relativity* (pp. 41, 46, 50). New York: Dover.

Marsden, J. E. (1999). *Basic complex analysis* (3rd ed., pp. 3, 7, 12, 16). New York: W. H. Freeman.

Martin, G. E. (1972). *The foundations of geometry and the non-euclidean plane* (pp. 300-302, 318-319, 414-434). New York: Intext Educational.

Palka, B. P. (1991). *An introduction to complex function theory* (pp. 5-15). New York: Springer-Verlag. http://dx.doi.org/10.1007/978-1-4612-0975-1

Riess, A. G., & Livio, M. (2016). The puzzle of dark energy. *Sci. Am., 314*(3), 38-43. http://dx.doi.org/10.1038/scientificamerican0316-38

Schutz, B. F. (2009). *A first course in general relativity* (pp. 1-2). Cambridge Univ.. http://dx.doi.org/10.1017/cbo9780511984181

Taylor, E. F., & Wheeler, J. A. (1966). *Spacetime physics* (pp. 1-58). San Francisco: Freeman.

Misner, C. W., Thorne, K. S. & Wheeler, J. A. (1970). *Gravitation* (pp. 17, 20, 50, 67, 416, 707, 1203). San Francisco: Freeman.

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