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Possible Forms of the Solution for Spherically Symmetric Static Problem in General Relativity

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Abstract

The paper is concerned with analysis of various forms of solution for the spherically symmetric static problem of the General Theory of Relativity (GTR). The problem under consideration is reduced to three equations for the components of the Einstein tensor which include three components of the metric tensor. However, only two of these three equations are mutually independent which means that the solution is not unique. Three possible forms of the solution are derived and analyzed in paper. One of these solutions is the traditional singular Schwarzchild solution, whereas two other solutions are not singular.

Keywords: general relativity, spherically symmetric problem

1. Introduction

Consider a spherical solid with radius R surrounded by an infinite empty space. The basic line element in spherical coordinates r,θ,ϕ has the following general form:

$$ds^2 = g^2dr^2 + \rho^2(d\theta^2 + \sin^2\theta d\phi^2) - h^2c^2dt^2 \tag{1}$$

where $g(r)$, $\rho(r)$ and $h(r)$ are the coefficients of the metric tensor of the Riemannian space induced by gravitation, t is time and c is the velocity of light. These coefficients must satisfy the following equations that specify the components of the Einstein tensor (Synge, 1960):

$$E_1^1 = \frac{1}{\rho^2} - \frac{1}{g^2}\frac{\rho'}{\rho}\left(\frac{\rho'}{\rho} + \frac{2h'}{h}\right) \tag{2}$$

$$E_2^2 = \frac{1}{g^2}\left[\frac{g'h'}{gh} + \frac{\rho'}{\rho}\left(\frac{g'}{g} - \frac{h'}{h}\right) - \frac{\rho''}{\rho} - \frac{h''}{h}\right] \tag{3}$$

$$E_4^4 = \frac{1}{\rho^2} - \frac{1}{g^2}\left[\left(\frac{\rho'}{\rho}\right)^2 + \frac{2\rho''}{\rho} + \frac{2\rho' g'}{\rho g}\right] \tag{4}$$

where, $(...)' = d(...)/dr$.

Consider first the external ($R \le r < \infty$) empty space for which $E_j^i = 0$ in Equations (2)-(4). Solving Equations (2) and (4), we can express the metric coefficients g and h in terms of the function $\rho(r)$ as (Vasiliev & Fedorov, 2014)

$$g_e = \rho_e'\sqrt{\frac{\rho_e}{\rho_e + C_1^e}}\,, \quad h_e = C_2^e\sqrt{1 + \frac{C_1^e}{\rho_e}} \tag{5}$$

Here, index *"e"* corresponds to the external space and C_1^e, C_2^e are the integration constants. It looks natural to try do determine the function $\rho_e(r)$ from Equation (3). However, direct substitution of Equations (5) in

Equation (3) shows that this equation is satisfied identically for any function $\rho_e(r)$. Thus, the function $\rho_e(r)$ cannot be found from Equation (3) and no other equation than can be used for this purpose exists in GTR.

The same situation takes place for the internal ($0 \le r \le R$) problem. For the sphere consisting of a perfect fluid with constant density μ, $E_1^1 = E_2^2 = -\chi p$ and $E_4^4 = \chi\mu c^2$, where $p(r)$ is the pressure in the fluid and

$$\chi = 8\pi\gamma / c^4 \tag{6}$$

is the GTR gravitational constant. Solution of Equations (2) and (4) is

$$g_i = \frac{\rho_i \rho_i^{/}}{\sqrt{\rho_i - (\chi\mu c^2/3)\rho_i^3 + C_1^i}} \tag{7}$$

$$\frac{h_i^{/}}{h_i} = \frac{\rho_i}{2\rho_i^{/}}\left[g_i^2\left(\frac{1}{\rho_i^2} + \chi p\right) - \left(\frac{\rho_i^{/}}{\rho_i}\right)^2\right] \tag{8}$$

where index *"i"* corresponds to the internal space and C_1^i is the integration constant. The pressure can be found from the following conservation equation (Synge, 1960):

$$p^{/} + \frac{h_i^{/}}{h}(p + \mu c^2) = 0 \tag{9}$$

Substituting Equations (7) and (8) in Equation (3) and taking into account Equation (9), we can readily prove that Equation (3) is satisfied identically for any function $\rho_i(r)$. Thus, the function $\rho(r)$ cannot be determined from GTR equations for external and internal spaces. Note that this is the intrinsic property of GTR equations. The covariant derivative of the Einstein tensor (2)-(4) is zero which results in the following equation for the tensor components (Synge, 1960):

$$(E_1^1)^{/} - \frac{2\rho^{/}}{\rho}(E_2^2 - E_1^1) + \frac{h^{/}}{h}(E_1^1 - E_4^4) = 0 \tag{10}$$

Because the left-hand parts of Equations (2)-(4) are linked by Equation (10), only two of three Equations (2)-(4) are mutually independent which means that one of the metric coefficients in Equation (1) can be preassigned, whereas two other cotfficents can be found from Equations (5), (7) and (8). Depending on this choice, we can construct at least three possible forms of solution which are presented below.

2. The First Form of the Solution

To derive the first form of the solution, take $g = 1$, i.e., apply the so-called polar coordinates (Synge, 1960). Assume that the function $\rho(r)$ satisfies the following asymptotic and boundary conditions:

$$\rho_e(r \to \infty) = r, \quad \rho_i(r = 0) = 0, \quad \rho_i(r = R) = \rho_e(r = R) \tag{11}$$

and take into account that for $r \to \infty$ (and for $\rho \to \infty$) the time metric coefficient $h(r)$ in Equation (5) for the external space must reduce to the solution of the classical gravitation theory according to which (Landau & Lifshitz, 1988)

$$h_\infty(r) = \sqrt{1 - r_g / r} \tag{12}$$

Here,

$$r_g = 2mG / c^2 \tag{13}$$

is the gravitational radius which depends on the sphere mass m and the classical gravitational constant G. Matching the second Equation (5) and (12), we can conclude that $C_1^e = -r_g$, $C_2^e = 1$ and the final form of Equations (5) is

$$g_e = \rho_e^{/}\sqrt{\frac{\rho_e}{\rho_e - r_g}}, \quad h_e = \sqrt{1 - \frac{r_g}{\rho_e}} \tag{14}$$

Taking $g_e = 1$ in the first of these equations and integrating, we arrive at

$$\rho_e\sqrt{1-\frac{r_g}{\rho_e}}+\frac{r_g}{2}\ln\left[\rho_e\left(1-\frac{r_g}{2\rho_e}\right)+\sqrt{1-\frac{r_g}{\rho_e}}\right]=r+C_3^e \tag{15}$$

where C_3^e is the integration constant which is determined further. Asymptotic analysis of Equation (15) allows us to conclude that $\rho_e(r\to\infty)=r$, i.e. that the first condition in Equations (11) is satisfied.

Consider the internal space. Taking into account the second condition in Equations (11) and applying the regularity condition for the solution at the sphere center $\rho_i=0$, we must put $C_1^i=0$ in Equation (7). Introducing the Euclidean mass of the sphere m_0 and the corresponding gravitational radius analogous to Equation (13), we get

$$m_0=(4/3)\pi\mu R^3,\quad r_g^0=2m_0G/c^2 \tag{16}$$

Note that Equation (13) includes, in contrast to Equations (16), the Riemannian mass of the sphere

$$m=4\pi\mu\int_0^R\rho_i^2 g_i dr \tag{17}$$

Now, using Equations (6) and (16), we can reduce Equation (7) to the following form:

$$g_i=\frac{\rho_i^{/}}{\sqrt{1-r_g^0\rho_i^2/R^3}} \tag{18}$$

Taking $g_i=1$ and integrating under the second condition in Equations (11), we have

$$\rho_i=\frac{1}{\sqrt{r_g^0}}\sin\frac{r}{\sqrt{r_g^0}} \tag{19}$$

Substituting $\rho_i^{/}$ from Equation (18) in Equation (8), we arrive at the following equation for the time metric coefficient:

$$\frac{h_i^{/}}{h_i}=\frac{\chi\rho_i}{2\sqrt{1-r_g^0\rho_i^2/R^3}}\left(p+\frac{\mu c^2}{3}\right) \tag{20}$$

To finish the derivation, we must determine the integration constant in Equation (15) using the third condition in Equations (11). The final result is

$$r=R+\rho_e\sqrt{1-\frac{r_g}{\rho_e}}-\rho_R\sqrt{1-\frac{r_g}{\rho_R}}+\frac{r_g}{2}\ln\frac{\rho_e\left(1-\frac{r_g}{2\rho_e}+\sqrt{1-\frac{r_g}{\rho_e}}\right)}{\rho_R\left(1-\frac{r_g}{2\rho_R}+\sqrt{1-\frac{r_g}{\rho_R}}\right)} \tag{21}$$

where $\rho_R=\rho_e(r=R)=\rho_i(r=R)$. Thus, the first form of the solution derived under the condition $g=1$ is specified by Equations (14), (19), (20) and (21).

Substituting the metric coefficient (18) in Equation (17) for the sphere mass and taking $g_i=1$, we can find the sphere mass in Equation (17) and the corresponding value of the gravitational radius in Equation (13) as

$$m=\frac{3m_0R}{2r_g^0}f_1(r_g^0),\quad r_g=\frac{3}{2}Rf_1(r_g^0) \tag{22}$$

in which

$$f_1(r_g^0)=1-\frac{\sin(2\sqrt{r_g^0/R})}{2\sqrt{r_g^0/R}}$$

Decomposing the right-hand sides of Equations (22) into power series, we arrive at

$$m = m_0\left[1-\frac{r_g^0}{5R}+\frac{2}{105}\left(\frac{r_g^0}{R}\right)^2-...\right],\qquad r_g = r_g^0\left[1-\frac{r_g^0}{5R}+\frac{2}{105}\left(\frac{r_g^0}{R}\right)^2-...\right]$$

where m_0 and r_g^0 are specified by Equations (16). As can be seen, the sphere mass corresponding to the solution under consideration is less than the Euclidean mass m_0.

As follows from Equation (21), the real solution exists if the minimum valure of ρ_e which is ρ_R is higher than r_g. Taking $r = R$ and $\rho_i = \rho_R$ in Equation (19) and using Equation (22) for r_g, we can present the corresponding condition as

$$\frac{1}{\sqrt{r_g^0/R}}\sin\sqrt{r_g^0/R} \geq r_g = \frac{3}{2}Rf_1(r_g^0)$$

The solution of this inequality, i.e., $r_g^0 = 1,0293R$ specifies the minimum possible value of the gravitational radius $r_g^m = 0,8731R$. Thus, the minimum sphere radius is $R_g^{(1)} = 1,195r_g$. For the sphere with radius $R < R_g^{(1)}$, the solution becomes imaginary.

Introduce normalized variables

$$\overline{r} = \frac{r}{R},\ \overline{r}_g = \frac{r_g}{R},\ \overline{r}_g^0 = \frac{r_g^0}{R},\ \overline{\rho} = \frac{\rho}{R},\ \overline{\rho}_R = \frac{\rho_R}{R} \tag{23}$$

and the function

$$\delta(\overline{r}) = 1-\frac{\overline{\rho}}{\overline{r}} \tag{24}$$

showing the deviation of the obtained metric coefficient $\overline{\rho}$ from the Euclidean metric coefficient $\overline{r}$. For the sphere with the limiting radius $R = R_g^{(1)}$, the function $\delta(\overline{r})$ is plotted in Figure 1 (line 1). As can be seen, $\overline{\rho}$ coincides with $\overline{r}$ at the sphere center $\overline{r} = 0$ and asymptotically approaches $\overline{r}$ for $\overline{r} \to \infty$. The radial metric coefficient $g = 1$ is presented in Figure 2 (line 1). Note that the obtained metric coefficients do not demonstrate singular behavior.

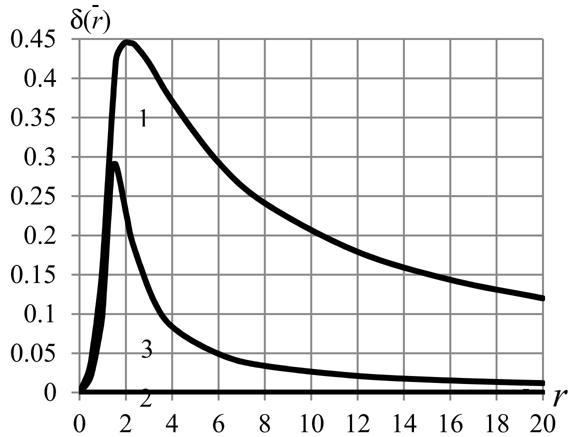

Figure 1. Dependences of function $\delta(\overline{r})$ on the radial coordinate for the first (1), the second (2), and the third (3) forms of the solution

The pressure inside the sphere can be found from Equation (9). Using Equation (8) and changing the independent variable r to ρ_i, we arrive at

$$\frac{dp}{d\rho_i}+\frac{\mu c^2 r_g^0\rho_i}{2(R^3-r_g^0\rho_i^2)}\left(1+\frac{3p}{\mu c^2}\right)\left(1+\frac{p}{\mu c^2}\right)=0 \tag{25}$$

According to the boundary condition, the pressure must be zero on the sphere surface $r = R$ or $\rho_i = \rho_R$. The solution of Equation (25) satisfying this condition is

$$\overline{p} = \frac{p}{\mu c^2} = \frac{\sqrt{1 - r_g^0 \overline{\rho}_i^2} - \sqrt{1 - r_g^0 \overline{\rho}_R^2}}{3\sqrt{1 - r_g^0 \overline{\rho}_R^2} - \sqrt{1 - r_g^0 \overline{\rho}_i^2}} \tag{26}$$

Finally, using Equation (19) which specifies ρ_i, we get

$$\overline{p} = \frac{\cos(\overline{r}\sqrt{\overline{r}_g^0}) - \cos\sqrt{\overline{r}_g^0}}{3\cos\sqrt{\overline{r}_g^0} - \cos(\overline{r}\sqrt{\overline{r}_g^0})}$$

Dependence $\overline{p}(r)$ for the ultimate case $R = R_g^{(1)}$ and $\overline{r}_g^0 = 1,0293$ is plotted in Figure 3 (line 1).

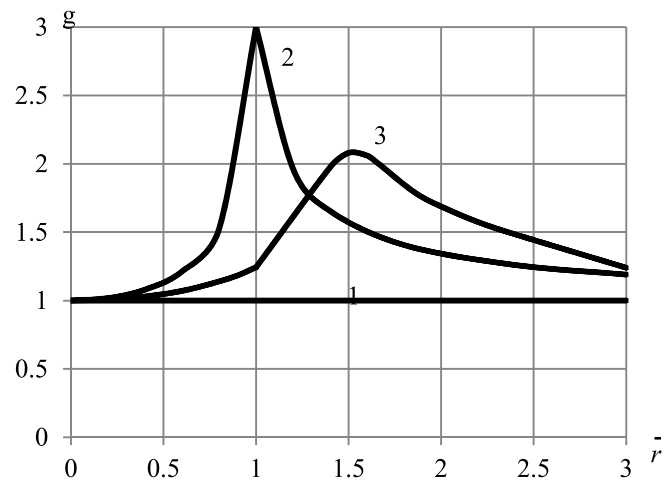

Figure 2. Dependences of the metric coefficient on the radial coordinate for the first (1), the second (2), and the third (3) forms of the solution

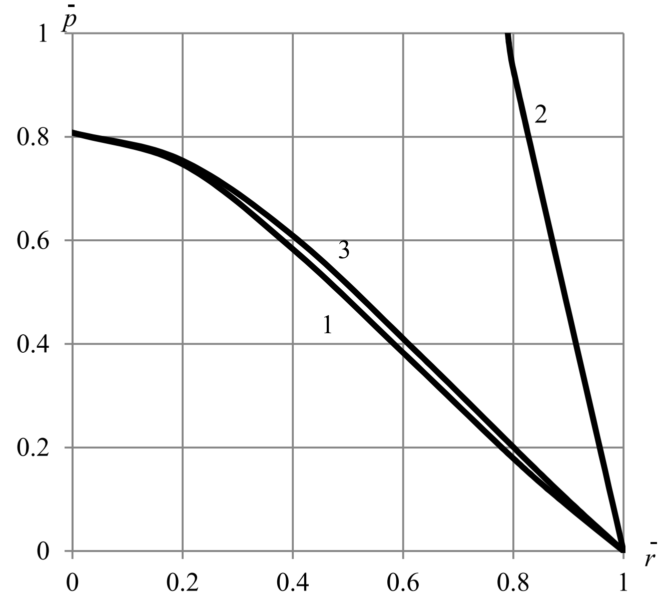

Figure 3. Distribution of the normalized pressure in the fluid over the sphere radius for the first (1), the second (2), and the third (3) forms of the solution

3. The Second Form of the Solution

To present the second possible form of the solution, apply the so-called curvature coordinates (Synge, 1960) and take $\rho = r$. The solution under consideration was obtained in 1916 by K. Schwarzschild. For the external space, the metric coefficients g_e and h_e in Equation (1) which asymptotically reduce to the corresponding results of the classical gravitation theory are (Synge, 1960)

$$g_e = \frac{1}{\sqrt{1 - r_g / r}} \quad , \quad h_e = \sqrt{1 - r_g / r} \tag{27}$$

where r_g is specified by Equation (13). For the internal space, Equation (18) in which $\rho_i = r$ yields

$$g_i = \frac{1}{\sqrt{1 - r_g^0 r^2 / R^3}} \tag{28}$$

Now, we need to satisfy the boundary condition on the sphere surface according to which

$$g_i(r = R) = g_e(r = R) \tag{29}$$

Substituting Equations (27) and (28) in this condition, we get

$$r_g = r_g^0 \tag{30}$$

in which r_g and r_g^0 are specified by Equations (13) and (16), respectively. Thus, we can conclude that Equation (30) is equivalent to the condition $m = m_0$. However, this is not the case for the solution under study, because Equations (17) and (28) give the following expression for the sphere mass:

$$m = \frac{3m_0 R}{2r_g^0}\left(\sqrt{\frac{R}{r_g^0}}\sin^{-1}\sqrt{\frac{r_g^0}{R}} - \sqrt{1 - \frac{r_g^0}{R}}\right) \tag{31}$$

Decomposing the right-hand part of these equations into power series, we arrive at

$$m = m_0\left[1 + \frac{3r_g^0}{10R} + \frac{9}{56}\left(\frac{r_g^0}{R}\right)^2 + \ldots\right]$$

As can be seen, in general, m is higher then m_o and $m = m_o$ only if $r_g^o = 0$. So, we can conclude that the classical Schwarzschild solution does not satisfy either the boundary condition in Equation (29) or Equation (31) for the sphere mass. Because the boundary condition, Equation (29), must not be violated (Synge, 1960), proceed the analysis taking that $r_g^0 = r_g$ or $m = m_0$. As follows from Equations (27) and (28), the metric coefficients can become singular if the radius of the sphere surface R reaches r_g which is referred to as the radius of the Black Hole event horizon. However, as follows from the solution specifying the pressure inside the fluid sphere, R cannot reach r_g. Using Equation (8) in which $\rho_i = r$ to transform Equation (9), we arrve at

$$p' + \frac{\mu c^2 r_g r}{2(R^3 - r_g r^2)}\left(1 + \frac{3p}{\mu c^2}\right)\left(1 + \frac{p}{\mu c^2}\right) = 0$$

The solution of this equation that satisfies the boundary condition on the sphere surface, i.e. $p(r = R) = 0$, has the following form in notations (23) (Synge, 1960):

$$\overline{p}(\overline{r}) = \frac{p}{\mu c^2} = \frac{\sqrt{1 - \overline{r}_g \overline{r}^2} - \sqrt{1 - \overline{r}_g}}{3\sqrt{1 - \overline{r}_g} - \sqrt{1 - \overline{r}_g \overline{r}^2}}$$

As known (Weinberg, 1972), this solution is singular. Taking $\overline{r} = 0$, determine the pressure at the sphere center

$$\overline{p}(0) = \frac{1 - \sqrt{1 - \overline{r}_g}}{3\sqrt{1 - \overline{r}_g} - 1}$$

The denominator of this expression becomes zero at $\bar{r}_g = 8/9$ which corresponds to the minimum possible sphere radius $R_g^{(2)} = 1{,}125 r_g$.

The results of calculation for the sphere with the critical radius $R = R_g^{(2)}$ are presented in Figures 1-3 (lines 2). The function $\delta(\bar{r})$ in Equation (24) is zero for the case under study, and line 2 coincides, naturally, with the r-axis in Figure 1. Dependence of g on $\bar{r}$ is shown in Figure 2. As follows from Figure 3, the pressure becomes infinitely high at the center of the sphere with the critical radius.

4. The Third Form of the Solution

Finally, consider the solution obtained by Vasiliev and Fedorov (2014). This solution corresponds to the general line element in Equation (1). The metric coefficients are specified by Equations (5) and (7), (8). Using the first two conditions in Equations (11) we can can arrive at Equations (14) for the external space and Equation (18) for the internal space, i.e.,

$$g_e = \rho_e^{/} \sqrt{\frac{\rho_e}{\rho_e - r_g}} \quad , \quad g_i = \frac{\rho_i^{/}}{\sqrt{1 - r_g^0 \rho_i^2 / R^3}} \tag{32}$$

in which $\rho_e(r)$ and $\rho_i(r)$ are some unknown functions. Recall that in Section 3 Equation (30) following from the boundary condition in Equation (29) could not be satisfied because $m \neq m_0$. Now, we have the possibility to satisfy Equation (30) by the proper selection of functions $\rho_e(r)$ and $\rho_i(r)$. Assume that the following conditions are valid for the internal and the external spaces:

$$g_i = \frac{r^2}{\rho_i^2} \quad , \quad g_e = \frac{r^2}{\rho_e^2} \tag{33}$$

Two main consequences follow from these equations. First, the boundary condition for g in Equation (29) is automatically satisfied if the function $\rho(r)$ is continuous on the sphere surface $r = R$. Second, substituting the first of Equations (33) in Equation (17) for the sphere mass, we can conclude that $m = m_0$ and hence $r_g = r_g^0$. This result has the following physical interpretation: gravitation, changing the Euclidean space inside the sphere to the Riemannian space, does not affect the sphere mass. Thus, the metric coefficients corresponding to the solution under study are specified by Equations (32) in which $r_g = r_g^0$. Sudstituting Equations (32) in Equations (33), we arrive at the following two equations for the functions ρ_e and ρ_i:

$$\rho_e^{/} \rho_e^2 = r^2 \sqrt{1 - r_g^0 / \rho_e} \ , \quad \rho_i^{/} \rho_i^2 = r^2 \sqrt{1 - r_g^0 \rho_i^2 / R^3} \tag{34}$$

For the internal space, the solution of the second equation in Equations (34) satisfying the condition $\rho_i(r=0) = 0$, can be presented as

$$F_i(\rho_i) = \frac{2 r_g^0 r^3}{3 R^4} \tag{35}$$

where

$$F_i(\rho_i) = \sqrt{\frac{R}{r_g^0}} \sin^{-1}\left(\frac{\rho_i}{R}\sqrt{\frac{r_g^0}{R}}\right) - \frac{\rho_i}{R}\sqrt{1 - \frac{r_g^0 \rho_i^2}{R^3}}$$

Taking $r = R$ and $\rho_i = \rho_R$ in Equation (35), we arrive at the following expression for ρ_R corresponding to the sphere surface:

$$F_e(\rho_R) = \frac{2 r_g^0}{3R} \tag{36}$$

For the external space, the solution of the first equation in Equations (34) satisfying the third condition in Equations (11), i.e. $\rho_e(r = R) = \rho_R$, has the form

$$F_e(\rho_e) - F_e(\rho_R) = \frac{1}{3}\left(\frac{r^3}{R^3} - 1\right) \tag{37}$$

where

$$F_e(\rho) = \frac{1}{R^3}\left(\frac{\rho^2}{3} + \frac{5r_g^0\rho}{12} + \frac{5(r_g^0)^2}{8}\right)\sqrt{\rho\left(\rho - r_g^0\right)} + \frac{5(r_g^0)^3}{8R^3}\ln\left(\sqrt{\frac{\rho - r_g^0}{R}} + \sqrt{\frac{\rho}{R}}\right) \tag{38}$$

Asymptotic analysis of Equation (37) shows that $\rho_e \to r$ for $r \to \infty$ which means that the obtained solution reduces to the Schwarzschild solution (Section 3) at a distance from the sphere. As follows from Equation (38), the solution is real if the minimum possible value $\rho_e = \rho_R$ is higher than r_g^0. Thus, in the limiting case, we have $\rho_R = r_g^0$. Then, Equation (36) yields the minimum possible sphere radius $R_g^{(3)} = 1{,}115 r_g^0$. For the sphere with radius $R < R_g^{(3)}$, the solution becomes imaginary. As can be seen, no singularity appears for $R = R_g^{(3)}$. In the limiting case, i.e. for $R = R_g^{(3)}$, the dependence of function $\delta(\overline{r})$ in Equation (24) on $\overline{r}$ is plotted in Figure 1 (curve 3). The corresponding dependence of the radial metric coefficient g is shown in Figure 2 (curve 3).

The normalized pressure in the fluid sphere satisfies Equation (25), and the solution is specified by Equation (26). The pressure distribution over the sphere radius is presented in Figure 3 (curve 3). As can be seen, the pressure is not singular and is close to the pressure corresponding to the first solution discussed in Section 2.

5. Conclusion

Because the solution of GTR equations for the spherically symmetric static problem is not unique, three possible forms of the solution are derived and compared.

For the first solution, the radial coefficient of the fundamental metric form in the Riemannian space is taken the same that for the corresponding Euclidean space ($g = 1$). The minimum possible sphere radius for this solution is $R_g^{(1)} = 1{,}195 r_g$ in which r_g is the gravitational radius. The solution is not singular and becomes imaginary if $R < R_g^{(1)}$. .

The second form of the solution is the classical Schwarzschild solution obtained under under the assumption according to which the circumferential metric coefficient of the Riemannian line element form corresponds to the Euclidean space ($\rho = r$). The solution becomes singular for the sphere with radius $R = r_g$. The solution exists if the sphere mass corresponds to the Euclidean space which is actually not the case because the space inside the sphere is Riemannian. The minimum possible sphere radius for this solution is $R_g^{(2)} = 1{,}125 r_g$. For $R = R_g^{(2)}$, the solution for the internal space gives infinitely high pressure at the center of the fluid sphere.

The third solution is based on the condition under which the sphere mass corresponds to the Euclidean space, whereas the actual space inside the sphere is Riemannian. The minimum possible sphere radius for this solution is $R_g^{(3)} = 1{,}115 r_g$. The solution is not singular. If $R < R_g^{(2)}$, the solution becomes imaginary.

As can be shown (the corresponding cumbersome analysis is not presented in the paper) all three considered solutions provide the results which agree with experiments supporting the General Theory of Relativity.

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