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Nonperturbative QED: Muon Structure and Decay

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Abstract

A nonperturbative QED is presented which is free of the divergences of perturbative QED. The theory comprises two equations of motion (EOM's) for a relativistic electron which are mutually coupled by gauge-invariant self-electromagnetic interaction terms calculated from Maxwell's equations. The first is Dirac's equation itself, which Dirac required to be compatible with the material equation of continuity and which accounts for the electron's rest-mass energy and spin-1/2 nature. The second is an EOM with Dirac form for a mass-0, spin-1/2 particle which I require to be compatible with the electromagnetic equation of continuity and which accounts for the electron's charge and electromagnetic self energy such that the combined equations account for the electron's rest-mass energy, spin, charge, and self energy and may therefore be regarded as a complete relativistic-electron theory. Due to the transverse nature of Maxwell's equations the second EOM actually comprises two EOM's: an EOM with magnetic-field interaction (MFEOM) accounting for radiative contributions to atomic structure and an EOM with electric-field interaction (EFEOM) investigated in this paper. The use of all three EOM's together may be considered to describe radiation-dressed states of matter as opposed to radiation-bare states of matter as described by Dirac's or Schroedinger's equation alone. I argue that the EFEOM is physically appropriate for a neutrino for large separations of the two particles and is possibly associated with the electroweak force. On the other hand I show that for small interparticle separations the EFEOM shows binding on a GeV energy and Fermi-unit length scale such that one bare electron and two neutrinos with net spin-1 are possibly related to the muon or "heavy" electron. A model is constructed for muon structure and decay.

Keywords: radiation-dressed states of matter, electron, neutrino, muon

1. Introduction

Nonperturbative QED is normally associated with the coupled Dirac-Maxwell equations. This approach, in which the electron's self energy is assumed to arise from the classical Maxwell field whose source is the electron's current calculated from Schroedinger's or Dirac's equation, was first investigated by Edwin Jaynes and co-workers (neoclassical radiation theory) over fifty years ago and then by Assim Barut and co-workers over forty years ago (self-field quantum electrodynamics). These theories have been severely criticized by workers who use the quantized radiation field and the renormalization methodology proposed by Bethe in the late nineteen forties to calculate the Lamb shift and other radiative corrections to atomic structure.

Light and charged matter are inextricably interwoven in classical electrodynamics. Matter contributions were originally understood using classical mechanics and then were later updated using quantum mechanics to arrive at an overall semiclassical theory (classical theory of radiation combined with a quantum theory of matter). This situation changed dramatically in 1927 when Dirac quantized the free radiation field to calculate the Einstein A and B coefficients for emission and absorption of radiation as discrete photons by matter (Dirac, 1927). Dirac's theory failed however when it was applied to the Lamb shift some twenty years later, requiring a renormalization methodology in order to obtain agreement of theory with Lamb's experimental data (Louisell, 1973). It is possible to interpret Lamb's experiments not as the result of the interaction of free photons and matter, which is the standard interpretation following the success of the renormalization methodology, but rather as the observation that radiation is a permanent part of the structure of matter, which of course is not compatible with the prior development of the quantum theory of matter as a radiation-free theory. It is possible to criticize QED, notwithstanding its success in practical calculations following renormalization, on the basis not only that theory must be augmented by physical argument and *ad hoc* mathematical procedures to obtain agreement with experiment but that its pioneers failed to confront Lamb's experiments in the first place as revolutionary for the structure of matter and decided to use the matter-free photon theory and the photon-free matter theory which were on hand (Dirac, 1927; Louisell, 1973).

2. Equation of Motion for the Electron's Material Nature (MEOM) Accounting for Its Rest-Mass Energy and Spin (Dirac's Equation)

First I show that insight can be gleaned into Dirac's equation by inferring it from a postulated Lorentz invariant found from the scalar product of the 4-gradient and a 4-potential postulated to comprise the electron's intrinsic properties such as its rest-mass energy and spin. The Lorentz invariant equation has the form of the equation bearing Lorentz's name,

$$\left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)\cdot(\Phi, \vec{A}) = \frac{1}{c}\frac{\partial\Phi}{\partial t} + \vec{\nabla}\cdot\vec{A} = 0, \tag{1}$$

which further elucidates the relationship of Dirac's equation for a relativistic electron and the spinorial form of Maxwell's equation, a subject which has been studied continuously (Darwin, 1928; Laporte & Uhlenbeck, 1931; Armour Jr., 2004; Ritchie, 2006) since Dirac's equation first appeared in 1928 (Dirac, 1928). The scalar and vector potentials can be written in the form of carrier-wave expansions for an assumed dominant frequency component, thusly,

$$\Phi = \chi(\vec{r},t)e^{-i\omega t} + \psi(\vec{r},t)e^{i\omega t} \tag{2a}$$

$$\vec{A} = \vec{X}_{-}(\vec{r},t)e^{-i\omega t} + \vec{X}_{+}(\vec{r},t)e^{i\omega t} \tag{2b}$$

On substituting Equation (2) into Equation (1) and separately setting the coefficients of the exponential factors equal to zero, one obtains,

$$\left(i\hbar\frac{\partial}{\partial t} - mc^2\right)\psi(\vec{r},t) + i\hbar c\vec{\sigma}\cdot\vec{\nabla}\chi(\vec{r},t) = 0 \tag{3a}$$

$$\left(i\hbar\frac{\partial}{\partial t} + mc^2\right)\chi(\vec{r},t) + i\hbar c\vec{\sigma}\cdot\vec{\nabla}\psi(\vec{r},t) = 0, \tag{3b}$$

which are identically Dirac's pair of first-order equations for a free electron on setting $\omega = \frac{mc^2}{\hbar}$, $\vec{X}_{+}(\vec{r},t) = \vec{\sigma}\chi(\vec{r},t)$, $\vec{X}_{-}(\vec{r},t) = \vec{\sigma}\psi(\vec{r},t)$, where *m* is the electron's mass, $\boldsymbol{\sigma}$ is Pauli's vector, and the wave functions are the Dirac 2-component spinors. Hence one has a Lorentz-invariant relativistic equation of motion for a material particle (MEOM) if the carrier-wave frequency belonging to the posited 4-potential is equal to the rest-mass energy of the material particle divided by $\hbar$. The present optical-physics derivation of Dirac's equation gives us uniquely a corollary of Einstein's mass-energy relation by stating an equivalency between a material particle's posited *radiant* carrier-wave energy and the electron's rest-mass energy, $\hbar\omega = mc^2$, which is manifest in Zitterbewegung and as confirmed experimentally in positron-electron annihilation with the emission of two gamma photons. Also notice that the electron's spin operator as given by Pauli's vector, $\boldsymbol{\sigma}$, is a quantized polarization vector for the electron's posited 4-potential as given by Equation (1). The posited 4-potential obviously refers to an *intrinsic* property of the electron as opposed to its extrinsic properties, which will be given below for its interaction with the familiar electromagnetic 4-potential, $(\Phi_{ex}, \boldsymbol{A}_{ex})$ [usually written $(\Phi, \boldsymbol{A})$] due to electromagnetic forces *external* to the electron. I have to make a distinction here between the usual electromagnetic 4-potential due to forces external to the electron and my posited intrinsic 4-potential from which I am able to infer Dirac's equation. Dirac's own derivation, which flows from the tradition of matter as opposed to optical physics, follows from his demands that a correct relativistic electron equation of motion should satisfy the relationship between energy and momentum of special relativity ($E = \gamma mc^2$, for Lorentz factor $\gamma = \sqrt{1+\frac{p^2}{m^2c^2}}$, subject to the quantization rules $E \to i\hbar\frac{\partial}{\partial t}$ and $\boldsymbol{p} \to -i\hbar\nabla$) and further should be compatible with the material equation of continuity given by the Lorentz invariant found from the scalar product of the 4-gradient and the material 4-current,

$$\left(\frac{1}{c}\frac{\partial}{\partial t},\nabla\right)\cdot(c\rho,\boldsymbol{j})=\frac{\partial\rho}{\partial t}+\nabla\cdot\boldsymbol{j}=0\,. \tag{4}$$

It is well known that the latter demand is satisfied by Dirac's equation, giving a current,

$$\boldsymbol{j}(\boldsymbol{r},t)=c\left[\psi^{+}(\boldsymbol{r},t)\boldsymbol{\sigma}\chi(\boldsymbol{r},t)+\chi^{+}(\boldsymbol{r},t)\boldsymbol{\sigma}\psi(\boldsymbol{r},t)\right], \tag{5}$$

where the superscripts denote Hermitian conjugates.

3. Equation of Motion for the Electron'S Radiant Nature (REOM) Accounting for Its Charge and Self Electromagnetic Field

Dirac's relativistic electron theory presented in Section 2 is obviously incomplete since it does not account for the electron's charge. Its self electromagnetic field is accounted for in standard QED by using the quantized radiation field (Dirac, 1927), but as I have already noted this theory fails in absence of renormalization (Louisell, 1973). My approach is to infer a second relativistic EOM for the electron which depends explicitly on the charge from a postulated Lorentz invariant found from the scalar product of a renormalized 4-gradient and a second 4-potential postulated to account for the electron's intrinsic properties which are omitted in Dirac's equation, namely its charge and self electromagnetic interaction henceforth referenced as the electron's radiant nature. I write this EOM as follows,

$$\left(\frac{1}{c}\frac{\partial}{\partial t},\vec{\nabla}\pm\frac{e}{mc^{2}}\vec{E},\vec{H}\right)\cdot(\Phi_{\nu},\vec{A}_{\nu})=\frac{1}{c}\frac{\partial}{\partial t}\Phi_{e}+\left(\hbar\vec{\nabla}\pm\frac{e}{mc^{2}}\vec{E},\vec{H}\right)\cdot\vec{A}_{\nu}=0\,, \tag{6}$$

where the upper (lower) sign stands for neutrino (antineutrino) – for as we shall see later the EOM appears to be physically acceptable for a neutrino for large electron-neutrino separations - and the notation $\vec{E},\vec{H}$ means electric *or* magnetic field respectively which is understood to be external to the neutrino due to the presence of the electron. This is the electron's self electromagnetic-field interaction. (The subscripts denoting interactions external to the neutrino are henceforth dropped.) The scalar product of the renormalized 4-gradient, $(\frac{1}{c}\frac{\partial}{\partial t},\vec{\nabla}\pm\frac{e}{mc^{2}}\vec{E},\vec{H})$, and the electromagnetic 4-current, $(c(u+\int_0^t dt\ \vec{j}\cdot\vec{E}),\vec{S})$, gives the electromagnetic equation of continuity,

$$\frac{\partial u}{\partial t}+\vec{\nabla}\cdot\vec{S}+\vec{j}\cdot\vec{E}=0\,, \tag{7}$$

since the scalar product of the $\vec{E}$ or $\vec{H}$ with $\vec{S}$ vanishes, where $u=\frac{1}{8\pi}(E^{2}+H^{2})$ is the electromagnetic energy density and $\vec{S}=\frac{c}{4\pi}\vec{E}\times\vec{H}$ is the electromagnetic 3-current. Hence the EOM for the electron's radiant nature (its REOM as opposed to Dirac's equation for its material nature or MEOM) is compatible with the electromagnetic equation of continuity as given by Equation (7) as opposed to Dirac's equation itself, which is compatible with the material equation of continuity as given by Equation (4). When Dirac's equation and the REOM inferred from Equation (6) are used to calculate the Lamb shift (Ritchie, 2007) and the electron's anomalous magnetic moment (Ritchie, 2008), it is found that the new results are free of divergences. It appears therefore that a complete description of a relativistic electron, in which its radiant nature responsible for the Lamb shift and its anomalous magnetic moment are properly understood without needing to remove infinite contributions using renormalization schemes, should also comprise equations of motion (REOM's) which are compatible with the electromagnetic equation of continuity given by Equation (7).

As with the electron the posited 4-potential given in Equation (6) can be written in the form of carrier-wave expansions,

$$\Phi_{\nu}=\Phi_{\nu-}\mathrm{e}^{-i\omega_{\nu}t}+\Phi_{\nu+}\mathrm{e}^{i\omega_{\nu}t} \tag{8a}$$

$$\boldsymbol{A}_{\nu}=\boldsymbol{A}_{\nu-}\mathrm{e}^{-i\omega_{\nu}t}+\boldsymbol{A}_{\nu+}\mathrm{e}^{i\omega_{\nu}t}\,, \tag{8b}$$

from which on substituting Equation (8) into Equation (6) and separately setting the coefficients of the exponen-tial factors equal to zero, we obtain,

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+i\frac{\omega_{\nu}}{c}\right)\Phi_{\nu+}+\left(\vec{\nabla}\pm\frac{e}{mc^{2}}\vec{E},\vec{H}\right)\cdot\vec{A}_{\nu+}=0 \tag{9a}$$

$$(\frac{1}{c}\frac{\partial}{\partial t} - i\frac{\omega_\nu}{c})\Phi_{\nu-} + (\vec{\nabla} \pm \frac{e}{mc^2}\vec{E},\vec{H})\cdot\vec{A}_{\nu-} = 0\,. \tag{9b}$$

On setting $\Phi_{\nu+} = \xi_{E,H}$, $\boldsymbol{A}_{\nu+} = \boldsymbol{\sigma}\zeta_{E,H}$, $\Phi_{\nu-} = \zeta_{E,H}$, $\boldsymbol{A}_{\nu-} = \boldsymbol{\sigma}\xi_{E,H}$ we obtain the Dirac form for the REOM,

$$\frac{\partial\xi_{E,H}}{c\partial t} + i\frac{\omega_\nu}{c}\xi_{E,H} + \vec{\sigma}\cdot(\vec{\nabla} \pm \frac{e}{mc^2}\vec{E},\vec{H})\zeta_{E,H} = 0 \tag{10a}$$

$$\frac{\partial\zeta_{E,H}}{c\partial t} - i\frac{\omega_\nu}{c}\zeta_{E,H} + \vec{\sigma}\cdot(\vec{\nabla} \pm \frac{e}{mc^2}\vec{E},\vec{H})\xi_{E,H} = 0\,. \tag{10b}$$

Writing $\xi_{E,H} = e^{-i\omega t}\psi_{E,H}$ and $\zeta_{E,H} = e^{-i\omega t}\chi_{E,H}$ in Equation (10) we derive stationary equations for $\psi_{E,H}$ and $\chi_{E,H}$; then we eliminate the equation for $\chi_{E,H}$ in favor of a second-order equation for $\psi_{E,H}$,obtaining equations for the electric and magnetic neutrino wave functions which have the Helmholtz form,

$$\{\nabla^2 + \frac{\omega^2 - \omega_\nu^2}{c^2} \pm \frac{e}{mc^2}[\vec{\nabla}\cdot\vec{E} + 2\vec{E}\cdot\vec{\nabla} + i\sigma\cdot(\vec{\nabla}\times\vec{E}) \pm \frac{e}{mc^2}E^2]\}\psi_E = 0 \tag{11a}$$

$$\{\nabla^2 + \frac{\omega^2 - \omega_\nu^2}{c^2} \pm \frac{e}{mc^2}[\vec{\nabla}\cdot\vec{H} + 2\vec{H}\cdot\vec{\nabla} + i\sigma\cdot(\vec{\nabla}\times\vec{H}) \pm \frac{e}{mc^2}H^2]\}\psi_H = 0\,, \tag{11b}$$

where I have used the identity $(\vec{\sigma}\cdot\vec{A})\,(\vec{\sigma}\cdot\vec{B}) = \vec{A}\cdot\vec{B} + i\vec{\sigma}\cdot(\vec{A}\times\vec{B})$. Equation (11b), for $\hbar\omega_\nu = 0$, was used in previously published physical applications to calculate a *divergence-free* Lamb shift (Ritchie, 2007) and electron's anomalous magnetic moment (Ritchie, 2008).

The success of the use Equation (11b) to calculate divergence-free radiative properties of matter (Ritchie, 2007; Ritchie, 2008) suggests that the concept of radiation as a permanent part of the structure of matter is a valid one. Recall that this is identically the concept of mass renormalization used in standard QED to remove infinite contributions to the electron's energy arising from the logic, which appears to be unphysical, that *first-quantized* states of matter exist which are totally free of radiation. As I have shown here it is possible to present a theory in which the electron does not exist in a bare or radiation-free state and whose material and radiant properties are described by a pair of relativistic, Lorentz-invariant first-quantized MEOM and REOM respectively.

Notice that in Equation (9)-(11) external electromagnetic fields and not external electromagnetic potentials occur such that there is no question of a gauge dependence of self-electromagnetic interactions in the REOM. As shown in the electric-field and magnetic-field equations of motion with second-order Helmholtz form [Equation (11)], the same-parity, $\vec{\nabla} \to \vec{\nabla} \pm \frac{e}{mc^2}\vec{E}$, and mixed-parity, $\vec{\nabla} \to \vec{\nabla} \pm \frac{e}{mc^2}\vec{H}$, addition vectors contribute, among other terms, all four of Maxwell's equations as interaction terms, the same parity addition vector contributing $\vec{\nabla}\cdot\vec{E} = 4\pi e\rho$ and $\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}$ and the mixed parity, addition vector contributing $\vec{\nabla}\cdot\vec{H} = 0$ and $\vec{\nabla}\times\vec{H} = \frac{4\pi e}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$.

4. Dirac's Equation (MEOM) With External Electromagnetic Interactions

Finally it remains to follow through with the optical derivation of Dirac's equation for the material particle (MEOM) in the presence of external electromagnetic potentials, which is given by

$$(i\hbar\frac{\partial}{\partial t} - e\Phi_{ex} - mc^2)\psi(\vec{r},t) + \vec{\sigma}\cdot(i\hbar c\vec{\nabla} + e\vec{A}_{ex})\chi(\vec{r},t) = 0 \tag{12a}$$

$$(i\hbar\frac{\partial}{\partial t} - e\Phi_{ex} + mc^2)\chi(\vec{r},t) + \vec{\sigma}\cdot(i\hbar c\vec{\nabla} + e\vec{A}_{ex})\psi(\vec{r},t) = 0 \tag{12b}$$

Notice that Equation (12) follow from Equation (1) on renormalizing the 4-gradient as follows, $\frac{1}{c}\frac{\partial}{\partial t} \to \frac{1}{c}\frac{\partial}{\partial t} - \frac{e}{i\hbar c}\Phi$ and $\vec{\nabla} \to \vec{\nabla} + \frac{e}{i\hbar c}\vec{A}$. Dirac's own derivation of course follows from the Lorentz invariant found from the scalar product of the electron's 4-momentum and the Dirac gamma matrices operating on his 4-component wave function comprising his 2-component spinors given in Equation (3) or (12).

Finally I should end this section by presenting an orthogonality theorem for Equation (10), which is different from that of Dirac for Equation (12) owning to the renormalization of the 4-gradient using $(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla} \pm \frac{e}{mc^2}\vec{E}, \vec{H})$ instead of the renormalization just given using the electromagnetic 4-potential. Readers will recall that Dirac's orthogonality theorem is proved by addition of Equation (12) (with the replacement $i\hbar\frac{\partial}{\partial t} \rightarrow E$) left-multiplied by ψ^+ and χ^+ respectively and integrated over all space followed by subtraction of the resulting equations giving

$$\begin{aligned}&(E - E')(<\psi'\mid\psi> + <\chi'\mid\chi>) + i\hbar c(<\psi'\mid\vec{\sigma}\cdot\vec{\nabla}\mid\chi> + <\chi'\mid\vec{\sigma}\cdot\vec{\nabla}\mid\psi> + \\ &<\psi\mid\vec{\sigma}\cdot\vec{\nabla}\mid\chi'> + <\chi\mid\vec{\sigma}\cdot\vec{\nabla}\mid\psi'>) = 0\end{aligned} \tag{13}$$

where the primed and unprimed quantities denote different states. Parts integration with vanishing wave functions at the boundaries shows that the term multiplying $i\hbar c$ vanishes by cancellation such that Dirac's orthogonality theorem states that

$$(E - E')(<\psi'\mid\psi> + <\chi'\mid\chi>) = 0 \tag{14}$$

which is satisfied by the vanishing of the term in the second parentheses if $E \neq E'$ (orthogonality of the wave functions) or by the normalization of the wave function to unity if $E = E'$.

The theorem given by Equation (14) is proved for Equation (10) using the procedure just outlined (with the replacement $\frac{1}{c}\frac{\partial}{\partial t} \rightarrow -i\frac{\omega}{c}$) giving

$$\begin{aligned}&(\frac{\omega}{c} - \frac{\omega'}{c})(<\psi'_{E,H}\mid\psi_{E,H}> + <\chi'_{E,H}\mid\chi_{E,H}>) + i(<\psi'_{E,H}\mid\vec{\sigma}\cdot\vec{\nabla} \pm \frac{e}{mc^2}\vec{E},\vec{H}\mid\chi_{E,H}> + <\chi'_{E,H}\mid\vec{\sigma}\cdot\vec{\nabla} \pm \frac{e}{mc^2}\vec{E},\vec{H}\mid\psi_{E,H}> + \\ &<\psi_{E,H}\mid\vec{\sigma}\cdot\vec{\nabla} \pm \frac{e}{mc^2}\vec{E},\vec{H}\mid\chi'_{E,H}> + <\chi_{E,H}\mid\vec{\sigma}\cdot\vec{\nabla} \pm \frac{e}{mc^2}\vec{E},\vec{H}\mid\psi'_{E,H}>) = 0\end{aligned} \tag{15}$$

Just as in Equation (13) the gradient terms cancel by parts integration with vanishing wave functions at the boundaries. The electric-field terms similarly vanish when $\omega = \omega'$. For $\omega' \neq \omega$ the orthogonality theorem is

$$\begin{aligned}&(\frac{\omega}{c} - \frac{\omega'}{c})(<\psi'_{E,H}\mid\psi_{E,H}> + <\chi'_{E,H}\mid\chi_{E,H}>) \pm \frac{ie}{mc^2}(<\psi'_{E,H}\mid\vec{\sigma}\cdot\vec{E},\vec{H}\mid\chi_{E,H}> + <\chi'_{E,H}\mid\vec{\sigma}\cdot\vec{E},\vec{H}\mid\psi_{E,H}> + \\ &<\psi_{E,H}\mid\vec{\sigma}\cdot\vec{E},\vec{H}\mid\chi'_{E,H}> + <\chi_{E,H}\mid\vec{\sigma}\cdot\vec{E},\vec{H}\mid\psi'_{E,H}>) = 0\end{aligned} \tag{16}$$

5. MEOM-REOM Bound States and Neutrino Emission

I consider that the coupled-equations solution of the REOM, as given by Equation (11) for $\hbar\omega_r = 0$ appropriate for a mass-0 particle, and the MEOM, as given by Equation (12), describe radiation-dressed as opposed to radiation-bare states of matter, as given by the MEOM alone. First I comment on the scaling of the interaction terms in Equation (11a). $\vec{E} = -\vec{\nabla}\Phi$ is the self electric field, where $\Phi = \frac{e}{r}$ is the self Coulomb potential of a REOM particle in the presence of a single bare electron in vacuo. The nearest classical analog to Equation (11a) is Maxwell's equation for an electromagnetic wave passing through charged matter. In Equation (12) the scalar potential is zero as appropriate for a single electron in vacuo, and the self interaction is calculated from Maxwell's equation for the vector potential with source calculated from the current inferred from the time-dependent REOM given by Equation (10),

$$\vec{j}_{E,H}(\vec{r},t) = c[\xi^+(\vec{r},t)\vec{\sigma}\zeta(\vec{r},t) + \zeta^+(\vec{r},t)\vec{\sigma}\xi(\vec{r},t)]. \tag{17}$$

Notice that the term $\frac{e^2\hbar^2}{m^2c^2}E^2 = \frac{e^2\hbar^2}{m^2c^2}(\vec{\nabla}\Phi)^2$ in Equation (11a), which is in units of squared energy, is always attractive such that it can support bound states of the REOM. The interaction scales according to the quantum expectation value of the attractive interaction term, $\frac{e^2\hbar^2}{m^2c^2}(\vec{\nabla}\Phi)^2$, as $\frac{e^4\hbar^2w^4}{m^2c^2}$, where w is a variational parameter chosen for exponentially decaying (i. e. 1S) trial radial wave functions for both Equation (12) and Equation (11a) for zero orbital angular momentum. The attraction scaling as $\frac{e^4\hbar^2 w^4}{m^2c^2}$ can overcome with increasing w the kinetic energy scaling as $\hbar^2c^2w^2$. The square root of $\frac{e^4\hbar^2 w^4}{m^2c^2}$ is an energy in the GeV regime for a variational parameter w which is equal to the reciprocal of the Compton wavelength $w = \frac{m_p c}{\hbar} \cong 4.755cm^{-1}$, as the reader may easily verify.

Results are presented in Figures 1-2 for an approximate variational solution of the coupled EOM's for MEOM (Dirac's equation) and REOM particles. Figure 1 shows electron energy versus variational parameter, w, from the variational solution of Equation (12). Notice in Figure 1 that it is possible to find a minimum energy for a single electron which lies within the bound regime (0<E<0.511 MeV) of the MEOM. The energy is not determined uniquely however since the magnetic interaction in the MEOM depends on the normalization of the REOM solution, which depends on a shifted variational parameter $w' = 1.6w$. The energy curve in Figure 1 is calculated using a strength parameter of 2116.894 for the magnetic interaction term in the MEOM.

Notice also that the minimum occurs at a w value which is approximately close to the inverse Compton wavelength of the proton such that the bound solution is on a nuclear rather than an atomic length scale. Figure 2 shows a plot of $\hbar^2c^2$ times the terms operating on ψ_E containing the electric field in Equation (11a), omitting the cross product since only electrostatic terms are retained in the calculation. This curve occurs at the w value in Figure 1 where $w' = 1.6w$ is the value of the trial wave function used in Equation (11a). The $\frac{2e\hbar^2}{m}\vec{E}\cdot\vec{\nabla}\psi_E$ term is evaluated approximately by replacing it with $-\frac{2e\hbar^2 w}{m}\vec{E}\cdot\hat{r}\psi_E$ in which it is assumed that $w' >> |\frac{d}{dr}h_{-1}(r)|$ using $g_{-1}(r) = e^{-w'r}h_{-1}(r)$ for the trial radial wave function for Equation (11a). Note the magnitude of the ordinate for the squared energy of Figure 2, whose square root is on a GeV energy scale in agreement with the scaling analysis given in the previous paragraph. Also note that the shape of the squared energy in Figure 2 with attractive well less than one fermi from the electron position at the origin agrees with observation of the short-range nature of nuclear forces.

The reciprocal of the additive rate of tunneling through the barrier is roughly equal to the lifetime of the muon (2×10^{-6} s) for an energy behind the barrier of 38 MeV per neutrino such that twice 38 MeV is roughly 76% of the muon's rest-mass energy of 100 MeV. This rate is calculated using the WKB approximation discussed in Bethe and Salpeter (2008), for which the energy of 38 MeV is chosen to give a lifetime roughly equal to the lifetime per neutrino. In an exact numerical calculation of the two coupled equations the two energies and the lifetime against breakup would be determined and thus free of any adjustments of the REOM solution to give the muon's lifetime.

Future work should focus on the exact solution and investigate whether the MEOM-REOM solution can accurately describe the muon, which is a "heavy electron" which decays into an electron and two neutrinos.

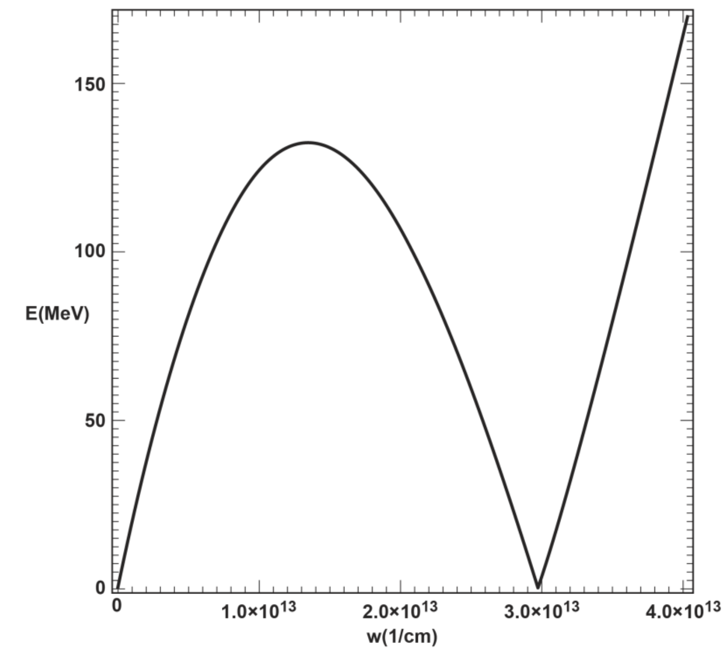

Figure 1. Electron energy versus variational parameter, w, from the variational solution of Equation (12). Notice that it is possible to find a minimum energy which lies within the bound regime (0<E<0.511 MeV) of Dirac's equation (MEOM). The energy is not determined uniquely however since the interaction in the MEOM depends on the normalization of the REOM solution, which depends on a shifted variational parameter $w' = 1.6w$. The energy curve is calculated using a strength parameter of 2116.894 for the magnetic interaction term in the MEOM

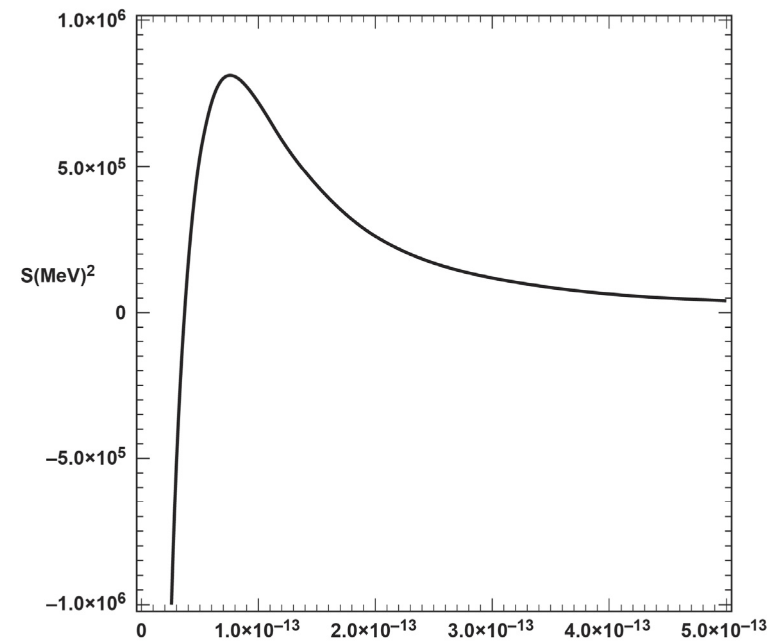

Figure 2. The squared potential energy function in the neutrino REOM given by Equation (11a) versus r corresponding to the w value at the minimum of the MEOM energy in Figure 1. The inverse rate of tunneling through the barrier per neutrino is equal to $1.01\times10^{-6}s$ for an energy behind the barrier 38 MeV such that the total lifetime against muon decay into an electron and two neutrinos is $2.02\times10^{-6}s$ for a total energy of 76 MeV or 76% of the observed muon rest-mass energy. This rate is calculated using the WKB approximation discussed in Bethe and Salpeter (2008). In an exact numerical calculation of the two coupled equations the two energies and the lifetime against breakup would be determined and thus free of any adjustments of the REOM solution for an energy behind the barrier which is tuned to match the lifetime of a neutrino trapped behind the barrier

6. Conclusions

I have formulated a nonperturbative, divergence-free QED which describes the permanent contribution of radiation to the structure of matter and as well shows nuclear-scale binding. The foundation of the new QED is based on my belief that relativistic electron theory is incomplete if it is compatible only with the material equation of continuity and accounts only for the electron's rest-mass energy and spin. A complete relativistic-electron theory should also be compatible with the electromagnetic equation of continuity such that *in toto* it accounts for the electron's rest-mass energy, spin, charge, and self electromagnetic interaction. The theory presented here accounts for these properties by including an EOM which appears to be physically appropriate for a neutrino at large electron-neutrino separations. The observed weakness of a free neutrino's interaction with matter would be manifest here in the smallness of the rate of tunnelling of free neutrinos from right to left back through the squared potential energy barrier shown in Figure 2 and can easily be estimated from the uncertainty principle $\Delta E \Delta t \cong \hbar$, where Δt is the right-to left back tunnelling rate theough the potential barrier and ΔE is an estimate of the strength of the interaction between free neutrinos and matter. On the other hand neutrinos can be strongly bound in the potential-well region of the barrier with the electron and are possibly related to the structure of the muon. Future work should address this point.

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