

# Dynamic scattering by cluster of small particles: local perturbation approach

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## Abstract

The wave scattering by moving particles (dynamic scattering) is a well known physical problem routinely occurring in practice. For the particles which are much smaller than the incident wavelength, the static scattering problem can be solved by using the local perturbation method. In this paper we apply the local perturbation approach to the problem of the dynamic scattering by the cluster of small particles. We calculate the fields scattered by the cluster of moving particles. As an example, the scattered light field and its resonance frequency are calculated for moving sphere in scalar approximation and in vector case.

**Keywords:** dynamic wave scattering, moving small particles, local perturbation

## 1. Introduction

Wave propagation and scattering in inhomogeneous media is a classical physical problem constantly reoccurring in many practical areas such as adaptive optics, free space communication, biology, and medicine. In many practical cases the inhomogeneous medium is actually homogeneous host medium (infinite or bounded) filled with the finite size inhomogeneities like dust particles, water droplets, air bubbles, snow flakes, and living cells.

The wave scattering by stationary inhomogeneities (static scattering) was studied extensively, and there are many papers devoted to this problem (Kerker, 1969; Born & Wolf, 1999). In reality, however, some scatterers do move: snow falls, blood cells flow, and cosmic dust rovers the space.

The wave scattering by moving bodies (dynamic scattering) is a long standing problem with many practical applications (Bladel, 2007; Brown, 1993). For example, the scattering properties of the moving particles are routinely used for velocity and object size measurements (Lee et al., 2012; Yokoi et al., 2001). The statistical properties of the dynamic scattering are discussed by Bladel (2007), and in works Ishimaru (1978) and Rytov et al. (1989), while the used scattering function is essentially of the static particle. The general theory of the scattering by single tree-dimensional object in translation motion was presented by Zutter (1980), and only recently the exact theory of the scattering by moving sphere was presented (see the work Handapangoda et al. (2011) and references therein). The dynamic scattering by the cluster of particles was not studied yet. When the characteristic size of the inhomogeneity is much smaller than the incident wavelength, the local perturbation method (LPM) can be used. The LPM was applied initially by Fermi for calculation of atomic spectra (Fermi, 1934, 1936). Later, the method was applied in crystal theory by Kosevich (2005) and in solid state physics by Maleev (1965) and by Bass et al. (2007). Most recently, the local perturbation method was applied for wave scattering by cluster of static particles (Bass & Fix, 1997; Bass et al., 2000).

There are, to authors knowledge, no studies where the LPM was used for study of the wave propagation in the media filled with moving local perturbations. The LPM allows, in principle, to take into account multiple scattering by moving particles, the shape of each moving scatterer, and the resonance properties of the dynamic scattering.

In this paper we use the LPM to study the dynamic wave scattering by the cluster of the particles which characteristic sizes are small compared to the incident wavelength. The general formalism is presented for this problem. As an example, we apply our method for calculation of the light field scattered by moving sphere in scalar approximation and in vector case.

In the following discussion we will make no distinction between particle and perturbation.

## 2. General Formalism: the Scattering by the Local Perturbations Moving With Arbitrary Speeds

The wave propagation in the medium filled with the  $N$  small particles can be described by the following equation

$$\widehat{H}_0 \left\{ \frac{\partial}{i\partial \mathbf{r}}, \frac{-\partial}{i\partial t} \right\} \mathbf{E}(\mathbf{r}, t) + \sum_{n=1}^N \widehat{H}_1 \left\{ \frac{\partial}{i\partial \mathbf{r}}, \frac{-\partial}{i\partial t} \right\} U_n(\mathbf{r} - \mathbf{r}_n(t)) \times \quad (1)$$

$$\widehat{H}_2 \left\{ \frac{\partial}{i\partial \mathbf{r}}, \frac{-\partial}{i\partial t} \right\} \mathbf{E}(\mathbf{r}, t) = \mathbf{j}(\mathbf{r}, t)$$

where the operators  $\widehat{H}_0$ ,  $\widehat{H}_1$ , and  $\widehat{H}_2$  are the tensors of the second order,  $\mathbf{E}$  and  $\mathbf{j}$  are the field and source vectors respectively depending on the space and time coordinates  $\mathbf{r}$  and  $t$ . The function  $U_n$  describes the properties of the  $n$ -th local perturbation and its dimensions,  $\mathbf{r}_n(t)$  is the position of the  $n$ -th perturbation and this position varies in time.

We note that the operator  $\widehat{H}_0$  in the Equation (1) describes the field propagation in the homogeneous medium, while the operators  $\widehat{H}_1$  and  $\widehat{H}_2$  are related to the perturbation.

We emphasize that the Equation (1) is quite general one and it can be reduced to partial differential equation, to integral equation, or to difference equations (Bass et al., 2008). As a consequence, the solution of the Equation (1) can describe the broad class of the fields related to different physical phenomena.

In this section we solve the Equation (1) by using the local perturbation method. For completeness, we note that the local perturbation method (LPM) is valid for the particles (perturbations) which characteristic size  $L_n$  is much smaller compared to the incident wavelength  $\lambda$  and that in this case the following relation holds (Bass & Fix, 1997)

$$U_n(\mathbf{r} - \mathbf{r}_n(t))\mathbf{E}(\mathbf{r}, t) \approx U_n(\mathbf{r} - \mathbf{r}_n(t))\mathbf{E}(\mathbf{r}_n(t), t), \quad (L_n/\lambda \ll 1) \quad (2)$$

By multiplying the Equation (1) by the operator  $\widehat{H}_0^{-1}$  inverse to the operator  $\widehat{H}_0$  and by using the LPM relation (2) we can present the field  $\mathbf{E}$  in the following form

$$\mathbf{E}(\mathbf{r}, t) = \widehat{H}_0^{-1} \mathbf{j}(\mathbf{r}, t) - \sum_{n=1}^N \widehat{H}_0^{-1} \widehat{H}_1 U_n(\mathbf{r} - \mathbf{r}_n(t)) \mathbf{F}_n(t). \quad (3)$$

Here the field  $\mathbf{F}_n$  is defined as

$$\mathbf{F}_n(t) \equiv \widehat{H}_2 \mathbf{E}(\mathbf{r}_n(t), t) \quad (4)$$

and  $\widehat{H}_0^{-1} \widehat{H}_0 = \widehat{I}$ , where  $\widehat{I}$  is the unity operator.

The field  $\mathbf{E}$  in the Equation (3) can be presented as the sum of the incident  $\mathbf{E}_{in}$  and the scattered  $\mathbf{E}_{sc}$  fields calculated via the Green's tensors, i. e. as

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_{in}(\mathbf{r}, t) + \mathbf{E}_{sc}(\mathbf{r}, t), \quad (5)$$

where

$$\mathbf{E}_{in}(\mathbf{r}, t) \equiv \int \widehat{G}_0(\mathbf{r} - \mathbf{r}', t - t') \mathbf{j}(\mathbf{r}', t') d\mathbf{r}' dt', \quad (6)$$

$$\mathbf{E}_{sc}(\mathbf{r}, t) = \sum_{n=1}^N \mathbf{E}_{sc,n}(\mathbf{r}, t), \quad (7)$$

$$\mathbf{E}_{sc,n}(\mathbf{r}, t) \equiv - \int \widehat{G}_1(\mathbf{r} - \mathbf{r}', t - t') U_n(\mathbf{r}' - \mathbf{r}_n(t')) \mathbf{F}_n(t') d\mathbf{r}' dt'. \quad (8)$$

Here  $\mathbf{E}_{sc,n}$  is the field scattered by the  $n$ -th particle,  $\widehat{G}_0$  is the Green's tensor of the homogeneous medium, and  $\widehat{G}_1$  is the Green's tensor related to the inhomogeneity

$$\widehat{G}_0(\mathbf{r} - \mathbf{r}', t - t') \equiv \frac{\pi^{-4}}{16} \int \widehat{H}_0^{-1}(\mathbf{q}, \omega) e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}')-i\omega(t-t')} d\mathbf{q}d\omega, \quad (9)$$

$$\widehat{G}_1(\mathbf{r} - \mathbf{r}', t - t') \equiv \frac{\pi^{-4}}{16} \int \widehat{H}_0^{-1}(\mathbf{q}, \omega) \widehat{H}_1(\mathbf{q}, \omega) e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}')-i\omega(t-t')} d\mathbf{q}d\omega. \quad (10)$$

Here and below we use infinite limits for integration and we do not write them explicitly. The expressions (5)-(9) allow to calculate the total field  $\mathbf{E}$  in the medium when the fields  $\mathbf{F}_n$  are known. To find the fields  $\mathbf{F}_n$  we multiply the Equation (3) by the operator  $\widehat{H}_2$  and get the following equation for the fields  $\mathbf{F}_n$

$$\mathbf{F}_m(t) = \mathbf{J}_m(\mathbf{r}_m(t), t) - \sum_{n=1}^N \int \widehat{G}_{21}(\mathbf{r}_m(t) - \mathbf{r}', t - t') U_n(\mathbf{r}' - \mathbf{r}_n(t')) \mathbf{F}_n(t') d\mathbf{r}' dt', \quad (11)$$

where the vector  $\mathbf{J}_m$  and the Green's tensor  $\widehat{G}_{21}$  are defined as

$$\mathbf{J}_m(\mathbf{r}_m(t), t) \equiv \int \widehat{G}_2(\mathbf{r}_m(t) - \mathbf{r}', t - t') \mathbf{j}(\mathbf{r}', t') d\mathbf{r}' dt', \quad (12)$$

$$\widehat{G}_{21}(\mathbf{r}_m - \mathbf{r}', t - t') \equiv \frac{\pi^{-4}}{16} \int \widehat{H}_2(\mathbf{q}, \omega) \widehat{H}_0^{-1}(\mathbf{q}, \omega) \widehat{H}_1(\mathbf{q}, \omega) \times e^{i\mathbf{q}(\mathbf{r}_m(t)-\mathbf{r}')-i\omega(t-t')} d\mathbf{q}d\omega, \quad (13)$$

$$\widehat{G}_2(\mathbf{r}_m - \mathbf{r}', t - t') \equiv \frac{\pi^{-4}}{16} \int \widehat{H}_2(\mathbf{q}, \omega) \widehat{H}_0^{-1}(\mathbf{q}, \omega) \times e^{i\mathbf{q}(\mathbf{r}_m(t)-\mathbf{r}')-i\omega(t-t')} d\mathbf{q}d\omega. \quad (14)$$

We note that the expression (11) is actually the system of equations with respect to the unknown vectors  $\mathbf{F}_n$  and it can be presented in the compact form

$$\sum_{n=1}^N \int \widehat{W}_{mn}(t, t') \mathbf{F}_n(t') dt' = \mathbf{J}_m(\mathbf{r}_m(t), t), \quad (15)$$

where the operators  $\widehat{W}_{mn}$  are

$$\widehat{W}_{mm}(t, t') \equiv \widehat{I} \delta(t - t') + \int \widehat{G}_{21}(\mathbf{r}_m(t) - \mathbf{r}_m(t') - \mathbf{r}', t - t') U_m(\mathbf{r}') d\mathbf{r}', \quad (16)$$

$$\widehat{W}_{mn}^{(m \neq n)}(t, t') \equiv V_n \widehat{G}_{21}(\mathbf{r}_m(t) - \mathbf{r}_n(t'), t - t'). \quad (17)$$

Here  $\widehat{I}$  is the unity operator and  $V_n$  is the volume of the  $n$ -th particle calculated as

$$V_n = \int U_n(\mathbf{r}') d\mathbf{r}'. \quad (18)$$

We note that the fields (5)-(7) and the fields  $\mathbf{F}_n(t)$  (solutions of the system (15)) give complete solution of the dynamic multiple scattering problem in the local perturbation approximation.

We note also that, the solution of the system (15), in general case, can not be expressed in analytical form and it should be solved numerically. However, in particular case when the perturbations move with the constant speed, the system (15) can be resolved analytically. This solution will be discussed in the following subsection.

### 2.1 The Scattering by the Local Perturbations Moving With Constant Velocities

Consider the situation when the perturbations move with constant velocities. In this case their coordinates  $\mathbf{r}_n(t)$  are

$$\mathbf{r}_n(t) = \mathbf{r}_{0n} + \mathbf{v}_n t, \quad |\mathbf{v}_n| = \text{const}, \quad (\mathbf{v}_n \neq \mathbf{v}_m) \quad (19)$$

where  $\mathbf{r}_{0n}$  is the initial position of the  $n$ -th perturbation at time  $t = 0$  and  $\mathbf{v}_n$  is the velocity of the  $n$ -th perturbation. Substituting relation for coordinates (19) into general expressions (16) for operators  $\widehat{W}_{mn}$ , we can recast the system of equations (15) into the following one

$$\widehat{W}_{mm}(\omega)\widetilde{\mathbf{F}}_m(\omega) + \sum_{n \neq m}^N \int \widehat{f}_{mn}(\mathbf{q}, \omega)\widetilde{\mathbf{F}}_n(\omega + \mathbf{q}(\mathbf{v}_m - \mathbf{v}_n))d\mathbf{q} = \widetilde{\mathbf{J}}_m(\omega), \quad (20)$$

or in vector components

$$W_{mm,ij}(\omega)\widetilde{F}_{m,j}(\omega) + \sum_{n \neq m}^N \int f_{mn,ij}(\mathbf{q}, \omega)\widetilde{F}_{n,j}(\omega + \mathbf{q}(\mathbf{v}_m - \mathbf{v}_n))d\mathbf{q} = \widetilde{J}_{m,i}(\omega). \quad (21)$$

Here  $\widetilde{\mathbf{F}}_n$  is the Fourier transform of the field  $\mathbf{F}_n$  and the operator  $\widehat{W}_{mn}$  is

$$\widehat{W}_{mm}(\omega) = \widehat{I} + \int \widehat{H}_2(\mathbf{q}, \omega + \mathbf{q}\mathbf{v}_m)\widehat{H}_0^{-1}(\mathbf{q}, \omega + \mathbf{q}\mathbf{v}_m) \times \widehat{H}_1(\mathbf{q}, \omega + \mathbf{q}\mathbf{v}_m)\widetilde{U}_m(\mathbf{q})d\mathbf{q}, \quad (22)$$

and

$$\widehat{f}_{mn}(\mathbf{q}, \omega) = \frac{V_n}{8\pi^3}\widehat{H}_2(\mathbf{q}, \omega + \mathbf{q}\mathbf{v}_m)\widehat{H}_0^{-1}(\mathbf{q}, \omega + \mathbf{q}\mathbf{v}_m) \times \widehat{H}_1(\mathbf{q}, \omega + \mathbf{q}\mathbf{v}_m)e^{i\mathbf{q}\mathbf{r}_{mn}}, \quad (23)$$

$$\mathbf{r}_{mn} \equiv \mathbf{r}_{0m} - \mathbf{r}_{0n}. \quad (24)$$

The Fourier transforms of the source function  $\mathbf{J}_m(\mathbf{r}_m(t), t)$  and the function describing the shape of the particle  $U_m(\mathbf{r})$  respectively are

$$\widetilde{\mathbf{J}}_m(\omega) = \frac{1}{2\pi} \int \mathbf{J}_m(\mathbf{r}_m(t), t)e^{i\omega t} dt, \quad (25)$$

$$\widetilde{U}_m(\mathbf{q}) = \frac{1}{8\pi^3} \int U_m(\mathbf{r})e^{-i\mathbf{q}\mathbf{r}} d\mathbf{r}. \quad (26)$$

We note that the expressions (20) and (21) are the system of equations with respect to the unknown fields  $\widetilde{\mathbf{F}}_n$ , and even these systems can not be solved analytically without further simplification.

#### 2.1.1 Local Perturbations Moving as One Body (All Particles Have the Same Velocity)

To simplify the systems (20) and (21) further, we assume that the speeds of the particles are such that the following condition holds

$$\frac{|\mathbf{v}_m - \mathbf{v}_n|}{c} \ll 1. \quad (27)$$

This condition is automatically satisfied for the particles with small speeds, and it is also correct for the particles with large but similar speeds. By using the condition (27), we can approximate the Fourier transform  $\widetilde{\mathbf{F}}_n$  as

$$\widetilde{\mathbf{F}}_n(\omega + \mathbf{q}(\mathbf{v}_m - \mathbf{v}_n)) \approx \widetilde{\mathbf{F}}_n(\omega) + \left. \frac{\partial \widetilde{\mathbf{F}}_n(\omega + \mathbf{q}(\mathbf{v}_m - \mathbf{v}_n))}{\partial \omega} \right|_{\mathbf{v}_m = \mathbf{v}_n} \mathbf{q}(\mathbf{v}_m - \mathbf{v}_n), \quad (28)$$

where the second term is much smaller than the first one and it can be neglected. Neglecting by the second term in Equation (28) we effectively apply condition that all the particles have the same velocity.

Taking into account the relation (28), we present the system (21) in the following form

$$\sum_{n=1}^N W_{mn,i}(\omega) \widetilde{F}_{n,j}(\omega) = \widetilde{J}_{m,i}(\omega), \quad (29)$$

and its solution for the field components  $\widetilde{F}_{n,i}$  is

$$\widetilde{F}_{n,i}(\omega) = \sum_{m=1}^N \frac{\widehat{A}_{nm,ij} \widetilde{J}_{m,j}(\omega)}{\det \widehat{W}(\omega)}. \quad (30)$$

Here the tensor  $\widehat{W}$  has components  $W_{mn,ij}$  (see the formula (29)) and  $\widehat{A}_{nm,ij}$  is the matrix of cofactors. Finally, taking into account the expression (30) for the fields  $\widetilde{\mathbf{F}}_n$ , the scattered field (7) can be presented in the form

$$\mathbf{E}_{sc}(\mathbf{r}, t) = \sum_{n=1}^N \mathbf{E}_{sc,n}(\mathbf{r}, t), \quad (31)$$

where the field  $\mathbf{E}_{sc,n}$  scattered by the  $n$ -th particle is

$$\begin{aligned} \mathbf{E}_{sc,n}(\mathbf{r}, t) = & -\frac{V_n}{8\pi^3} \int \widehat{H}_0^{-1}(\mathbf{q}, \omega + \mathbf{q}\mathbf{v}_n) \widehat{H}_1(\mathbf{q}, \omega + \mathbf{q}\mathbf{v}_n) \times \\ & e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}_{0n}-\mathbf{v}_n t)-i\omega t} d\mathbf{q} \sum_{m=1}^N \frac{\widehat{A}_{nm} \widetilde{\mathbf{J}}_m(\omega)}{\det \widehat{W}(\omega)} d\omega. \end{aligned} \quad (32)$$

Furthermore, we note that the field (31) can be integrated over  $\omega$  space by using the residue theorem and in this case the scattered field is

$$\mathbf{E}_{sc}(\mathbf{r}, t) = \int \frac{\mathbf{Q}(\omega, t) e^{-i\omega t}}{\det \widehat{W}(\omega)} d\omega = 2\pi i \sum_q \frac{\mathbf{Q}(\omega_q, t) e^{-i\omega_q t}}{\left. \frac{d \det \widehat{W}(\omega)}{d\omega} \right|_{\omega=\omega_q}}, \quad (33)$$

where the vector  $\mathbf{Q}$  is defined as

$$\begin{aligned} \mathbf{Q}(\omega, t) \equiv & -\sum_{n,m=1}^N \frac{V_n}{8\pi^3} \widehat{A}_{nm} \widetilde{\mathbf{J}}_m(\omega) \int \widehat{H}_0^{-1}(\mathbf{q}, \omega + \mathbf{q}\mathbf{v}_n) \times \\ & \widehat{H}_1(\mathbf{q}, \omega + \mathbf{q}\mathbf{v}_n) e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}_{0n}-\mathbf{v}_n t)} d\mathbf{q}. \end{aligned} \quad (34)$$

Here  $\omega_q$  is the  $q$ -th root of the equation  $\det \widehat{W}(\omega) = 0$ . Furthermore, we note that the resonance frequencies of the dynamic scattering are defined by the equation

$$\det \widehat{W}(\omega) = 0. \quad (35)$$

We note that the formula (33) is the essence of this section. The formula gives analytical expression for the field scattered by the particles moving with the same speed in the local perturbation approximation.

### 3. Example 1: Scattering by Moving Sphere in Scalar Approximation

In this section we consider the scattering by moving sphere in scalar approximation. We assume that the particle moves in the infinite homogeneous medium with the constant velocity  $\mathbf{v}$  in  $x$  direction, and that the radius and the volume of the sphere is  $L$  and  $V$  respectively. The position of the sphere is described by the radius vector

$\mathbf{r}_1(t) = \mathbf{r}_{01} + \mathbf{v}t$ , and  $\mathbf{r}_{01}$  is the position of the particle at time  $t = 0$ . In this case, the equation for the scalar field  $E(\mathbf{r}, t)$  is

$$\left(\Delta - \frac{\varepsilon_h}{c^2} \frac{\partial^2}{\partial t^2}\right) E(\mathbf{r}, t) - \frac{(\varepsilon_{sc} - \varepsilon_h)}{c^2} \frac{\partial^2}{\partial t^2} U(\mathbf{r} - \mathbf{r}_1(t)) E_1(\mathbf{r}_1, t) = j(\mathbf{r}, t), \quad (36)$$

where  $\varepsilon_h$  and  $\varepsilon_{sc}$  are the permittivities of the host medium and the particle respectively,  $U$  is the function describing the shape of the sphere. Comparing Equation (36) with the general Equation (1) we can see that the operators  $\widehat{H}_0$ ,  $\widehat{H}_1$ , and  $\widehat{H}_2$  are

$$\widehat{H}_0 \left\{ \frac{\partial}{i\partial \mathbf{r}}, \frac{-\partial}{i\partial t} \right\} = \Delta - \frac{\varepsilon_h}{c^2} \frac{\partial^2}{\partial t^2}, \quad \widehat{H}_2 \left\{ \frac{\partial}{i\partial \mathbf{r}}, \frac{-\partial}{i\partial t} \right\} = 1, \quad (37)$$

$$\widehat{H}_1 \left\{ \frac{\partial}{i\partial \mathbf{r}}, \frac{-\partial}{i\partial t} \right\} = -\frac{(\varepsilon_{sc} - \varepsilon_h)}{c^2} \frac{\partial^2}{\partial t^2} \quad (38)$$

and as the result

$$\widehat{H}_0 \{ \mathbf{q}, \omega \} = -q^2 + k^2, \quad k \equiv \sqrt{\varepsilon_h} \frac{\omega}{c}, \quad (39)$$

$$\widehat{H}_1 \{ \mathbf{q}, \omega \} = \frac{(\varepsilon_{sc} - \varepsilon_h)\omega^2}{c^2}, \quad \widehat{H}_2 \{ \mathbf{q}, \omega \} = 1. \quad (40)$$

By using the obtained results (32) and the expressions (39)-(40) we get for the scattered field  $E_{sc}$  the following expression

$$E_{sc}(\mathbf{r}, t) = \frac{(\varepsilon_{sc} - \varepsilon_h)V}{8\pi^3 c^2} \int \frac{(\omega + \mathbf{q}\mathbf{v})^2 e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}_{01}) - i(\omega+\mathbf{q}\mathbf{v})t}}{q^2 - \left(k + \sqrt{\varepsilon_h} \frac{\mathbf{q}\mathbf{v}}{c}\right)^2} d\mathbf{q} \widetilde{E}_1(\omega) d\omega, \quad (41)$$

where  $\widetilde{E}_1(\omega)$  is the field inside the particle and it is

$$\widetilde{E}_1(\omega) = \widetilde{E}_{in,1}(\omega) + \frac{(\varepsilon_{sc} - \varepsilon_h)}{c^2} \widetilde{E}_1(\omega) \int \frac{\widetilde{U}(\mathbf{q})(\omega + \mathbf{q}\mathbf{v})^2}{q^2 - \left(k + \sqrt{\varepsilon_h} \frac{\mathbf{q}\mathbf{v}}{c}\right)^2} d\mathbf{q}. \quad (42)$$

Here  $\widetilde{E}_{in,1}(\omega)$  is the Fourier transform of the field incident on the particle at the point  $\mathbf{r}_1$ . We note that the integral in Equation (41) can be calculated with the help of the stationary phase method for the large distances when  $kR \gg 1$  ( $R \equiv |\mathbf{r} - \mathbf{r}_1(t)|$ ). Integrating both formulae (41) and (42) over  $\mathbf{q}$  we get for the scattered field and the field inside particle  $\widetilde{E}_1(\omega)$  the following expressions respectively

$$E_{sc}(\mathbf{r}, t) = \frac{(\varepsilon_{sc} - \varepsilon_h)V}{4\pi c^2 \rho(t)} \mu(\mathbf{r}, t) \int \omega^2 \widetilde{E}_1(\omega) e^{i\omega(\varphi(t)-t)} d\omega, \quad (kR \gg 1) \quad (43)$$

$$\widetilde{E}_1(\omega) = \frac{\widetilde{E}_{in,1}(\omega)}{W(\omega)}, \quad R \equiv |\mathbf{r} - \mathbf{r}_1(t)|, \quad (44)$$

where the coefficients  $\mu$  and  $\varphi$  are

$$\mu(\mathbf{r}, t) \equiv \frac{\left(1 + \beta \frac{R_x(t)}{\rho(t)}\right)^2}{(1 - \beta^2)}, \quad \rho(t) \equiv \sqrt{R^2 - \beta^2 R_{\perp}^2}, \quad \beta \equiv \sqrt{\varepsilon_h} \frac{v}{c}, \quad (45)$$

$$\varphi(t) \equiv \frac{\sqrt{\varepsilon_h} \rho(t)}{c(1 - \beta^2)} \left(1 + \beta \frac{R_x(t)}{\rho(t)}\right), \quad R_x(t) \equiv x - x_1(t), \quad (46)$$

$$R_{\perp} \equiv |\mathbf{r}_{\perp} - \mathbf{r}_{1\perp}(t)|, \quad v = |\mathbf{v}|, \quad (47)$$

and the denominator  $W$  is

$$W(\omega) = 1 - (\varepsilon_{sc} - \varepsilon_h) \left\{ \begin{array}{l} \frac{1}{\varepsilon_h} \left( \ln\left(\frac{1+\beta}{1-\beta}\right) / 2\beta - 1 \right) \\ + \frac{\omega^2 L^2}{2c^2} \frac{1}{(1-\beta^2)^2} \\ + i \frac{\omega^3 L^3}{3c^3} \frac{\sqrt{\varepsilon_h(1+\beta^2/3)}}{(1-\beta^2)^3} \end{array} \right\}. \quad (48)$$

We note that for the static particle when  $\beta = 0$ , the formulae (43) and (48) reproduce well known result presented, for example, in Bass et al. (2000). We note also that the formula (43) can be obtained from the vector case (see for example Zutter (1980)) when the depolarization term is neglected, and in addition, the double Doppler shift (see for example Bladel (1984)) can be seen in the phase of the scattered field (43).

### 3.1 The Resonance

The formula (48) shows that the dynamic scattering in the scalar case has resonance when

$$\text{Re } W(\omega_r) = 0. \quad (49)$$

From the resonance condition (49) we can calculate the resonance frequency of the field scattered by the moving sphere in scalar approximation

$$\omega_r = \frac{\sqrt{2}c(1-\beta^2)}{L\sqrt{\varepsilon_{sc}-\varepsilon_h}} \left[ 1 - \frac{(\varepsilon_{sc}-\varepsilon_h)}{\varepsilon_h} \left( \ln\left(\frac{1+\beta}{1-\beta}\right) / 2\beta - 1 \right) \right]^{1/2}. \quad (50)$$

The expression (50) clearly shows that the resonance frequency  $\omega_r$  decreases with the speed of the particle and that the resonance frequency can be even zero. Moreover, the higher the optical contrast of the particle, the faster decrease of the frequency.

The expression (50) can be simplified for the small speeds when  $\beta \ll 1$  (while  $\varepsilon_{sc} - \varepsilon_h \gg 1$ ), and in this case the resonance frequency  $\omega_r$  of the field scattered by the moving particle is

$$\omega_r = \frac{\sqrt{2}c}{L\sqrt{\varepsilon_{sc}-\varepsilon_h}} \left( 1 - (\varepsilon_{sc}-\varepsilon_h) \frac{\beta^2}{3\varepsilon_h} \right)^{1/2}, \quad (\beta \ll 1, \varepsilon_{sc} - \varepsilon_h \gg 1) \quad (51)$$

and the resonance width  $\xi$  is

$$\xi \equiv \left. \frac{\text{Im } W}{\frac{\partial \text{Re } W}{\partial \omega}} \right|_{\omega=\omega_r} = \frac{2c\sqrt{\varepsilon_h}}{9L(\varepsilon_{sc}-\varepsilon_h)} \frac{(3+\beta^2)}{(1-\beta^2)^3} \left( 1 - (\varepsilon_{sc}-\varepsilon_h) \frac{\beta^2}{3\varepsilon_h} \right). \quad (52)$$

We note that the resonance frequency and the resonance width are the functions of the particle's speed  $v$ . At zero speed when  $\beta = 0$ , the formula (51) reproduces the result obtained previously for the resonance scattering by static particle (Bass et al., 2000). Here assumed that the refractive indexes of the particle and the host medium are real values.

The formula (51) shows that the resonance frequency decreases with the speed of the particle (we consider the most commonly encountered case when  $\varepsilon_{sc} > \varepsilon_h$ ), and for particles with relatively high speeds the resonance frequency may be even zero. Physically this means that light propagating inside particle with the speed about  $c/\sqrt{\varepsilon_{sc}}$  does not interact with boundaries of the particle moving with the speed  $v$ .

The resonance width (52) is more complicated function of the particle's speed: it can increase or decrease its value at some conditions. For the small speeds when  $\beta \ll 1$ , we have

$$\xi \approx \frac{2c\sqrt{\varepsilon_h}}{3L(\varepsilon_{sc}-\varepsilon_h)} \left( 1 + \frac{\beta^2}{3} \left( 11 - \frac{\varepsilon_{sc}}{\varepsilon_h} \right) \right), \quad (\beta \ll 1, \varepsilon_{sc} - \varepsilon_h \gg 1) \quad (53)$$

meaning that the width increases with the increase of the particle's speed when  $\varepsilon_{sc} < 11\varepsilon_h$ . On the contrary, when  $\varepsilon_{sc} > 11\varepsilon_h$ , the resonance width decreases with the increase of the speed of the particle when  $\beta \ll 1$ .

### 3.2 The Scattered Intensities

The scattered field (43) can be calculated even further when the incident field  $\widetilde{E}_{in,1}$  is somehow specified. Consider two most common cases below.

#### 3.2.1 Case 1: Monochromatic Incident Light

Suppose that the incident field is a monochromatic light with the angular frequency  $\Omega$ . In this case the incident field can be presented in the following form

$$\widetilde{E}_{in,1}(\omega) = E_1 \delta(\omega - \Omega), \quad (54)$$

where  $E_1$  is the amplitude of the field and  $\delta$  is the delta function. In accordance with (43) and (54) the expression the scattered field and its intensity  $I_{sc} \equiv |E_{sc}(\mathbf{r}, t)|^2$  is

$$E_{sc}(\mathbf{r}, t) = \frac{(\varepsilon_{sc} - \varepsilon_h)V}{4\pi c^2} \frac{\Omega^2}{\rho(t)} \frac{\mu(\mathbf{r}, t)E_1}{W(\Omega)} e^{i\Omega(\varphi(t)-t)}, \quad (55)$$

and

$$I_{sc}(\mathbf{r}, t) = \frac{|\varepsilon_{sc} - \varepsilon_h|^2 V^2}{16\pi^2 c^4} \frac{\Omega^4}{\rho^2(t)} \frac{\mu^2(\mathbf{r}, t) |E_1|^2}{|W(\Omega)|^2}. \quad (56)$$

The formula (56) shows that the intensity of the scattered field vary in space and time via the coefficient  $\mu(\mathbf{r}, t)$ . The intensity increases when the particle heads in the direction of observer and it goes down when the particle flies away from the observer. The scattered intensity is maximal then the frequency of the incident light  $\Omega$  coincides with the resonance frequency  $\omega_r$  of the field scattered by the moving particle (Equation (51)), because in this case the denominator  $W$  is minimal.

#### 3.2.2 Case 2: Broad Band Light

Suppose now that the incident field is relatively broad function in frequency domain and that the resonance frequency  $\omega_r$  of the particle is inside this frequency band. In this case the integral in (43) can be calculated with the help of the residue theorem and we get the following expressions for the scattered field and its intensity

$$E_{sc}(\mathbf{r}, t) = i \frac{(\varepsilon_{sc} - \varepsilon_h)V}{2c^2} \frac{\omega_0^2 \mu(\mathbf{r}, t)}{\rho(t)} \frac{\widetilde{E}_{in,1}(\omega_0)}{\left. \frac{\partial W}{\partial \omega} \right|_{\omega=\omega_0}} e^{i\omega_0(\varphi(t)-t)}, \quad (57)$$

and

$$I_{sc}(\mathbf{r}, t) = \frac{|\varepsilon_{sc} - \varepsilon_h|^2 V^2}{4c^4} \frac{|\omega_0^2|^2 \mu^2(\mathbf{r}, t)}{\rho(t)^2} \left| \frac{\widetilde{E}_{in,1}(\omega_0)}{\left. \frac{\partial W}{\partial \omega} \right|_{\omega=\omega_0}} \right|^2 e^{-2\text{Im}[\omega_0(\varphi(t)-t)]}. \quad (58)$$

where  $\omega_0$  is the solution of the equation  $W(\omega) = 0$  (see Equation (48)) and  $\omega_0$  is, in principle, complex number. The formula (58) for the intensity of the scattered field is correct when the condition

$$\text{Im}[\omega_0(\varphi(t) - t)] \geq 0 \quad (59)$$

is satisfied. The formula (58) shows that the intensity of the scattered field decreases exponentially for the times which are not equal to  $t = \varphi(t)$ .

When the resonance is narrow,  $\text{Im} \omega_0 = -\xi$  meaning that the intensity (58) decrease is related to the resonance width: the broader the resonance the faster the scattered intensity drops. In the limit, when the resonance width tends to zero, the scattered intensity does not decay exponentially in time.

## 4. Example 2: Scattering by Moving Sphere in Vector Case

In this section we consider the light scattering by moving sphere in vector case. As well as in the scalar case, we assume that the particle moves in the infinite homogeneous medium with the constant velocity  $\mathbf{v}$  in  $x$  direction, and that the radius and the volume of the sphere is  $L$  and  $V$  respectively. The position of the sphere is described by the



radius vector  $\mathbf{r}_1(t) = \mathbf{r}_{01} + \mathbf{v}t$ , where  $\mathbf{r}_{01}$  is the position of the particle at time  $t = 0$ . In this case, the equation for the vector field  $\mathbf{E}(\mathbf{r}, t)$  is

$$\left( \Delta - \nabla \otimes \nabla - \frac{\varepsilon_h}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{E}(\mathbf{r}, t) - \frac{(\varepsilon_{sc} - \varepsilon_h)}{c^2} \frac{\partial^2}{\partial t^2} U(\mathbf{r} - \mathbf{r}_1(t)) \mathbf{E}_1(\mathbf{r}_1, t) = \mathbf{j}(\mathbf{r}, t). \quad (60)$$

Here  $\Delta$  and  $\nabla$  are the Laplacian and nabla operators,  $\otimes$  defines tensor product,  $\varepsilon_h$  and  $\varepsilon_{sc}$  are the permittivities of the host medium and the particle respectively,  $U$  is the function describing the shape of the sphere. Comparing Equation (60) with the general Equation (1) we can see that the operators  $\widehat{H}_0$ ,  $\widehat{H}_1$ , and  $\widehat{H}_2$  are

$$\widehat{H}_0 \left\{ \frac{\partial}{i\partial \mathbf{r}}, \frac{-\partial}{i\partial t} \right\} = \Delta - \nabla \otimes \nabla - \frac{\varepsilon_h}{c^2} \frac{\partial^2}{\partial t^2}, \quad \widehat{H}_2 \left\{ \frac{\partial}{i\partial \mathbf{r}}, \frac{-\partial}{i\partial t} \right\} = 1, \quad (61)$$

$$\widehat{H}_1 \left\{ \frac{\partial}{i\partial \mathbf{r}}, \frac{-\partial}{i\partial t} \right\} = -\frac{(\varepsilon_{sc} - \varepsilon_h)}{c^2} \frac{\partial^2}{\partial t^2} \quad (62)$$

and as the result the operators  $\widehat{H}_0$ ,  $\widehat{H}_1$ , and  $\widehat{H}_2$  are

$$\widehat{H}_0 \{ \mathbf{q}, \omega \} = -q^2 + \mathbf{q} \otimes \mathbf{q} + k^2, \quad k \equiv \sqrt{\varepsilon_h} \frac{\omega}{c}, \quad (63)$$

$$\widehat{H}_1 \{ \mathbf{q}, \omega \} = \frac{(\varepsilon_{sc} - \varepsilon_h) \omega^2}{c^2}, \quad \widehat{H}_2 \{ \mathbf{q}, \omega \} = 1. \quad (64)$$

By using the obtained results (32) and the expressions (63)-(64) we get for the scattered field  $\mathbf{E}_{sc}$  the following expression

$$\mathbf{E}_{sc}(\mathbf{r}, t) = \frac{(\varepsilon_{sc} - \varepsilon_h)V}{8\pi^3 c^2} \int \left( \widehat{I} - \frac{\mathbf{q} \otimes \mathbf{q}}{\left( k + \sqrt{\varepsilon_h} \frac{\mathbf{q}\mathbf{v}}{c} \right)^2} \right) \widetilde{\mathbf{E}}_1(\omega) d\omega \times \quad (65)$$

$$\frac{(\omega + \mathbf{q}\mathbf{v})^2 e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}_{01}) - i(\omega+\mathbf{q}\mathbf{v})t}}{q^2 - \left( k + \sqrt{\varepsilon_h} \frac{\mathbf{q}\mathbf{v}}{c} \right)^2} d\mathbf{q},$$

where  $\widetilde{\mathbf{E}}_1(\omega)$  is the field inside the particle and it is

$$\widetilde{\mathbf{E}}_1(\omega) = \widetilde{\mathbf{E}}_{in,1}(\omega) + \frac{(\varepsilon_{sc} - \varepsilon_h)}{c^2} \times \quad (66)$$

$$\int \left( \widehat{I} - \frac{\mathbf{q} \otimes \mathbf{q}}{\left( k + \sqrt{\varepsilon_h} \frac{\mathbf{q}\mathbf{v}}{c} \right)^2} \right) \widetilde{\mathbf{E}}_1(\omega) \widetilde{U}(\mathbf{q}) (\omega + \mathbf{q}\mathbf{v})^2 \frac{d\mathbf{q}}{q^2 - \left( k + \sqrt{\varepsilon_h} \frac{\mathbf{q}\mathbf{v}}{c} \right)^2}.$$

We note that the integral in Equation (65) can be calculated with the help of the stationary phase method for the large distances when  $kR \gg 1$  ( $R \equiv |\mathbf{r} - \mathbf{r}_1(t)|$ ). Integrating both formulae (65) and (66) over  $\mathbf{q}$  we get for the scattered field and the field inside particle  $\widetilde{\mathbf{E}}_1(\omega)$  the following expressions respectively

$$\mathbf{E}_{sc}(\mathbf{r}, t) = \frac{(\varepsilon_{sc} - \varepsilon_h)V}{4\pi c^2 \rho(t)(1 - \beta^2)} \left[ \left( 1 + \beta \frac{R_x(t)}{\rho(t)} \right)^2 - \quad (67)$$

$$\frac{\mathbf{u}}{\rho(t)} \otimes \frac{\mathbf{u}}{\rho(t)} \right] \int \omega^2 \widetilde{\mathbf{E}}_1(\omega) e^{i\omega(\varphi(t)-t)} d\omega,$$

$$\rho(t) \equiv \sqrt{R^2 - \beta^2 R_x^2}, \quad R \equiv |\mathbf{r} - \mathbf{r}_1(t)|, \quad (kR \gg 1).$$

Here the vector  $\mathbf{u}$  is defined as

$$\mathbf{u} \equiv \mathbf{r} - \mathbf{r}_1(t) - \beta^2 \mathbf{R}_\perp + \beta \rho(t) \mathbf{v}/v,$$

and the field  $\widetilde{\mathbf{E}}_1(\omega)$  is calculated via the following formulae

$$\widehat{D}(\omega) \widetilde{\mathbf{E}}_1(\omega) = \widetilde{\mathbf{E}}_{in,1}(\omega), \text{ and } D_{ij} = \delta_{ij} W_j. \quad (68)$$

The coefficient  $\varphi$  is explained in the formula (46), and the coefficients  $W_j$  are

$$W_x(\omega) = 1 - (\varepsilon_{sc} - \varepsilon_h) \left\{ \frac{1}{\varepsilon_h \beta^2} \left[ 1 - \beta^2 + \frac{\beta^2 - 1}{2\beta} \ln \left( \frac{1+\beta}{1-\beta} \right) \right] + \frac{L^2 \omega^2}{2c^2 \beta^2} \left[ \frac{1}{(1-\beta^2)} - \ln \left( \frac{1+\beta}{1-\beta} \right) / 2\beta \right] + \frac{i 2\omega^3 L^3}{9c^3} \frac{\sqrt{\varepsilon_h}}{(1-\beta^2)^2} \right\}, \quad (69)$$

$$W_{y,z}(\omega) = 1 - (\varepsilon_{sc} - \varepsilon_h) \left\{ \frac{1}{\varepsilon_h \beta^2} \left[ \beta^2 - 1/2 + \frac{\beta^2 + 1}{4\beta} \ln \left( \frac{1+\beta}{1-\beta} \right) \right] + \frac{L^2 \omega^2}{2c^2 \beta^2} \left[ \frac{3\beta^2 - 1}{2(1-\beta^2)^2} + \ln \left( \frac{1+\beta}{1-\beta} \right) / 4\beta \right] + \frac{i 2\omega^3 L^3}{9c^3} \frac{\sqrt{\varepsilon_h} (1+\beta^2)}{(1-\beta^2)^3} \right\}, \quad (70)$$

$$\text{and } \beta \equiv \sqrt{\varepsilon_h} \frac{v}{c}. \quad (71)$$

For the static particle, the expressions (69) and (70) transform to the known formula presented, for example, in Bass et al. (2000). We note also that the formula (67) reproduces the one obtained in the vector case (see for example Zutter (1980)).

#### 4.1 The Resonance Frequencies

The formulae (69) and (70) suggest that the dynamic light scattering in the vector case has two resonances defined by two following equations

$$\text{Re } W_x(\omega_{r,x}) = 0, \text{ Re } W_{y,z}(\omega_{r,yz}) = 0. \quad (72)$$

The resonance frequencies are

$$\omega_{r,x} = \frac{\sqrt{2}c}{L \sqrt{\varepsilon_{sc} - \varepsilon_h}} \sqrt{\frac{\beta^2 - \frac{(\varepsilon_{sc} - \varepsilon_h)}{\varepsilon_h} \left[ 1 - \beta^2 + (\beta^2 - 1) \ln \left( \frac{1+\beta}{1-\beta} \right) / 2\beta \right]}{\frac{1}{(1-\beta^2)} - \ln \left( \frac{1+\beta}{1-\beta} \right) / 2\beta}} \quad (73)$$

$$\omega_{r,yz} = \frac{\sqrt{2}c}{L \sqrt{\varepsilon_{sc} - \varepsilon_h}} \sqrt{\frac{\beta^2 + \frac{(\varepsilon_{sc} - \varepsilon_h)}{2\varepsilon_h} \left[ 1 + 2\beta^2 - (\beta^2 + 1) \ln \left( \frac{1+\beta}{1-\beta} \right) / 2\beta \right]}{\frac{3\beta^2 - 1}{2(1-\beta^2)^2} + \ln \left( \frac{1+\beta}{1-\beta} \right) / 4\beta}} \quad (74)$$

The obtained expressions for the resonance frequencies (73) and (74) are not transparent due to complex relations between  $\beta$  and logarithmic function. For the small speeds when  $\beta \ll 1$ , the resonance frequencies  $\omega_r$  of the field scattered by the moving particle are

$$\omega_{r,x} = \omega_{r,0} (1 - \varsigma \beta^2 / 5), \quad \omega_{r,yz} = \omega_{r,0} (1 - 2\varsigma \beta^2 / 5), \quad (75)$$

$$\omega_{r,0} \equiv \frac{c}{L} \sqrt{\frac{2\varepsilon_h + \varepsilon_{sc}}{\varepsilon_h(\varepsilon_{sc} - \varepsilon_h)}}, \quad \varsigma \equiv \frac{(4\varepsilon_{sc} + 5\varepsilon_h)}{(\varepsilon_{sc} + 2\varepsilon_h)}, \quad (\beta \ll 1). \quad (76)$$

We do not consider the resonance width here, because the resonance is broad even for the static particle (Bass et al., 2000).

The expressions (73)-(75) show that as well as in the scalar case, the resonance frequencies decrease with the speed of the particle (we assumed that  $\varepsilon_{sc} > \varepsilon_h$ ). However, in distinction to the scalar case, there are two resonance frequencies of the scattered field in the vector case: in the direction of the particle propagation and in the perpendicular direction. In addition, the formulae (75) shows that ratio of the frequencies  $\omega_{r,x}/\omega_{r,yz}$  grows with the particle's speed as

$$\frac{\omega_{r,x}}{\omega_{r,yz}} = 1 + \zeta\beta^2/5. \quad (77)$$

We note, that the scattered intensities can be discussed in the similar way as it was done for the scalar case in the previous section, and we will not do it here.

## 5. Conclusions

The method describing the wave propagation and scattering in the medium filled with the small moving particles has been proposed. The explicit analytical solution was presented for the field scattered by the particles moving with the constant speed.

As an example, the light scattered by small moving sphere is calculated in the scalar approximation and in the vector case.

It was found that in the scalar approximation there is one resonance frequency of the scattered light, and the frequency decreases with the speed of the particle when the optical contrast of the particle is positive. The resonance width, however, can decrease or increase its value depending on the amount of the optical contrast of the moving particle.

It was also found that in the vector case there are two resonance frequencies: one in direction of the movement of the particle and another one in the direction transverse to the movement. Both resonance frequencies decrease with the speed of the moving particle when the optical contrast of the particle is positive.

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