# Intraocular Lens Implant with Mirrors and Intraocular Three Lenses Implant with a Changed Curvature Radius 

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#### Abstract

Two different intraocular lens implants are suggested for age macula degeneration (AMD) eye disease that causes the loss of central vision. An intraocular Lens implant that includes two mirrors with a continuity change in their curvature radius and an intraocular hyperboloid surface lens implant closed by two lenses also with a continuity change in their curvature radiuses and includes an additional lens between the two. Thus, both eyes can see an increased image of a near central object projected on the retina, removed from the degenerated macula and also see the peripheral objects images decreasing continuity according to the continuity change in the curvature radiuses of the mirrors and lenses.


Keywords: curvature radius, mirror, lens, focus, intraocular implant, retina, image

## 1. Introduction

AMD eye disease causes loss of central vision. To prevent this loss, an intraocular telescope lens implant was suggested (Lipshitz Isaac \& Gross Yosy, 2002). Articles about medium-term results in AMD patients by using such implants, were written (Primo, 2010; Hudson et al., 2006; Alio et al., 2004). An article about a double intraocular implant for visual rehabilitation of patients with macular diseases that is similar to the intraocular telescope lens implant were written (Orzalest et al., 2007).
This implant increases the central object's image but hides the peripheral vision that might interrupt the central object's image. In order to improve it, we suggested an intraocular lens implant with two mirrors (Ruchvarger \& Lipshitz, 2005). This implant increases the central object's image, removes it from the degenerated macula and does not hide the peripheral vision. Articles about medium-term results results in AMD patients by using a similar implant were written (Amar Agarwal, 2012) and (Agarwal et al., 2008).
An article that compares between medical results of the intraocular implant was also written (Singer, Amir, \& Herro, 2012). This implant, increases the central image and suddenly decreases the peripheral vision that causes a strange vision. Therefore, the objective of this paper is to suggest two new intraocular lens implants. An intraocular lens implant includes two mirrors with a continuity change in their curvature radiuses and an intraocular hyperboloid surface lens implant closed by two lenses with a continuity change in their curvature radiuses and includes an additional lens between the two. By the two suggested intraocular implants, both eyes can see an increased image of a central object, removed from the degenerated macula and also see peripheral objects images decreasing continuity. The derivation of the Equations in this paper including Appendix A, are originally, based on lenses and mirrors optical geometry and on mathematical principles (Edwards \& Penney, 1998).

## 2. The Suggested Intraocular Lens Implant with Parabolic Mirrors Surfaces

In order to increase the central object's image, removed from the macula and decreasing continuity the peripheral vision, the mirrors curvature radius has to be increased continuity from the center to its contours which fits parabolic surfaces.
The intraocular lens implant with mirrors contains two parabolic mirrors 1 and 2 that are shown in Figure 1. An example of the curvature horizontal and vertical radiuses of parabolic mirrors is given by

$$
\begin{gather*}
y=x^{2}+z^{2}, \quad z=0, y=f(x)=x^{2}, \quad y^{\prime}=f^{\prime}(x)=2 x, \quad y^{\prime \prime}=2 \\
r_{x}=r_{h}=\frac{\left(1+\left(f^{\prime}(x)\right)^{2}\right)^{\frac{3}{2}}}{f^{\prime \prime}(x)}=\frac{\left(1+4 x^{2}\right)^{\frac{3}{2}}}{2}  \tag{1}\\
x=0, y=f(z)=z^{2}, \quad r_{z}=r_{v}=\frac{\left(1+4 z^{2}\right)^{\frac{3}{2}}}{2} \tag{2}
\end{gather*}
$$

Thus, the curvature radius is a function of the object's distance from the central eye axis. Equations (1) and (2) are developed according to Equation (22) in Appendix A.


Figure 1. Horizontal or vertical middle cross-section of an eye with an intraocular lens implant consists of two mirrors

## 3. Intraocular Hyperboloid Surface Lens Implant Contains Ellipsoidal Surface Lenses

In order to increase the central object's image, we need two lenses and an additional lens for overturning the image on the retina. The three lenses are placed in an hyperboloid surface. The intraocular hyperboloid surface lens implant is closed by two lenses and includes an additional lens between the two, as shown in Figure 2.


Figure 2. Horizontal or vertical middle cross-section of an eye with intraocular three lenses implant placed in a closed hyperboloid surface

According to Figure 2, the central object's image with the top point A should be increased without hiding the image of the peripheral side or far object with the top point B. Therefore, each lens that closes the hyperboloid surface, should be planed with a continues changed focus distance and therefore, a continues changed curvature radius along its height and wide.
$f=\frac{r}{2}$ is the connection between the focus distance and the lens's radius.
The light rays Equations of the horizontal and vertical cross-sections are obtained according to Figure 3, where $w$ is the object's wide, h is the object's height, 1 is the object's distance from the Lens and f is the focus distance. The relevant Equations are obtained as follows

$$
\begin{align*}
& \left\{\begin{array}{l}
y=-\frac{w}{l} x \\
y=-\frac{w}{f} x+w
\end{array}\right\} \Rightarrow y=\frac{l}{f} y+w \Rightarrow y=-\frac{w \cdot f}{l-f}=\operatorname{image}(w), \quad x=\frac{f \cdot l}{l-f}  \tag{3}\\
& \left\{\begin{array}{l}
z=-\frac{h}{l} x \\
z=-\frac{h}{f} x+h
\end{array}\right\} \Rightarrow z=\frac{l}{f} z+h \quad \Rightarrow \quad z=-\frac{h \cdot f}{l-f}=\operatorname{image}(h), \quad x=\frac{f \cdot l}{l-f}  \tag{4}\\
& l \gg f \Rightarrow x=f \quad, \quad y=-\frac{w \cdot f}{l}=\operatorname{image}(w) \quad, \quad z=-\frac{h \cdot f}{l}=\operatorname{image}(h) \tag{5}
\end{align*}
$$

Figure 3. Horizontal and vertical Crosse-section object's images created by light rays
Thus, object's image size is a direct function of the lens's focus. Therefore the focus point for the central and near top point object's image should be further from the lens than that for the side and farther one.
Thus, lenses with a changed curvature radius along their height and wide cross sections, should be ellipsoidal surfaces as shown in Figure 4. For example, the curvature radiuses of the horizontal and vertical lens's middle cross sections of the following ellipsoidal surface, are developed as follows

$$
\begin{align*}
& x^{2}+\frac{y^{2}}{a^{2}}+\frac{z^{2}}{a^{2}}=1, a>1 \quad, \quad z=0, y=f(x)= \pm a \sqrt{1-x^{2}} \quad, \quad y^{\prime}=f^{\prime}(x)=\frac{\mp a x}{\sqrt{1-x^{2}}} \\
& \left(1+\left(f^{\prime}(x)\right)^{2}\right)^{\frac{3}{2}}=\left(\frac{1+\left(a^{2}-1\right) x^{2}}{1-x^{2}}\right)^{\frac{3}{2}}, \quad y^{\prime \prime}=f^{\prime \prime}(x)=\frac{\mp a \sqrt{1-x^{2}} \mp \frac{a x^{2}}{\sqrt{1-x^{2}}}}{1-x^{2}}=\frac{\mp a}{\left(1-x^{2}\right)^{\frac{3}{2}}}  \tag{6}\\
& r_{y}=r_{h}=\left|\frac{\left(1+\left(f^{\prime}(x)\right)^{2}\right)^{\frac{3}{2}}}{f^{\prime \prime}(x)}\right|=\frac{\left(1+\left(a^{2}-1\right) x^{2}\right)^{\frac{3}{2}}}{a}=\frac{\left(a^{4}-\left(a^{2}-1\right) y^{2}\right)^{\frac{3}{2}}}{a^{4}}=2 f_{h}=2 f
\end{align*}
$$

$$
\begin{equation*}
y=0, z=f(x)= \pm a \sqrt{1-x^{2}} \quad, \quad r_{z}=r_{v}=\frac{\left(a^{4}-\left(a^{2}-1\right) z^{2}\right)^{\frac{3}{2}}}{a^{4}}=2 f_{v}=2 f \tag{7}
\end{equation*}
$$



Figure 4. Ellipsoid surface lens and its curvature radius as function of lens's depth, wide and height

The derivation of the curvature radius Equation is given in detail in Appendix A.
Equations (6) and (7) represent the curvature radius of the first ellipsoidal lens.
According to these Equations, the curvature radius is a direct function of the lens's depth x and decreases with the wide and height of the lens, as shown in Figure 4.
The wide and height of the object's images on the eyes retina are obtained according to Equations (5)-(7). The first lens creates image $1(\mathrm{w})$ and image $1(\mathrm{~h})$, the second lens creates image $2(\mathrm{w})$ and image $2(\mathrm{~h})$ and the last lens creates image 3(w) and image 3(h). The derivation of image 3(w) and image 3(h), on the eyes retina, is given as follows is the distance between the first lens and the second one, $d_{1,2}$ where

$$
\begin{gather*}
\text { image } 1(w)=\frac{-w\left(a^{4}-\left(a^{2}-1\right) y^{2}\right)^{\frac{3}{2}}}{2 a^{4} l}, \quad \text { imagel }(h)=\frac{-h\left(a^{4}-\left(a^{2}-1\right) z^{2}\right)^{\frac{3}{2}}}{2 a^{4} l}  \tag{8}\\
\operatorname{image} 2(w)=\frac{\operatorname{imagel}(w) \cdot f_{2}}{d_{1,2}-x}=\frac{\text { imagel }(w) \cdot f_{2}}{d_{1,2}-f} \quad, \quad \text { image } 2(h)=\frac{\text { imagel }(h) \cdot f_{2}}{d_{1,2}-f}  \tag{9}\\
\operatorname{image} 3(w)=\frac{\operatorname{image} 2(w)}{d_{2,3}-f_{2}} \cdot f_{3} \quad, \quad \text { image } 3(h)=\frac{\text { image } 2(h)}{d_{2,3}-f_{2}} \cdot f_{3}  \tag{10}\\
f_{3}=\frac{\left(a_{3}^{4}-\left(a_{3}^{2}-1\right) z^{2}\right)^{\frac{3}{2}}}{2 a_{3}^{4}}=\frac{\left(a_{3}^{4}-\left(a_{3}^{2}-1\right) y^{2}\right)^{\frac{3}{2}}}{2 a_{3}^{4}} \tag{11}
\end{gather*}
$$

$f_{2}$ is the constant focus distance of the second lens and
$d_{2,3}$ is the distance between the second lens and the third last one.
The lenses and the distances between the lenses, should be planed according to

$$
\begin{equation*}
0<d_{1,2}-f_{\max }<1 \quad, \quad 0<d_{2,3}-f_{2}<1 \quad \text { and } \quad f_{2}>1 \tag{12}
\end{equation*}
$$

Equation (11) represents the changed focus distance of the ellipsoidal third and last lens according to Equations (6) and (7) where the ellipsoidal Equation is

$$
\begin{equation*}
x^{2}+\frac{y^{2}}{a_{3}^{2}}+\frac{z^{2}}{a_{3}^{2}}=1 \quad \text { and } \quad 1<a_{3}<a_{1}=a \tag{13}
\end{equation*}
$$

Thus, according to Equations (6)-(13), the image of the wide and height of an object near the central axis of vision in the eyes are developed as follows

$$
\begin{gather*}
f_{\max }=\lim _{y \rightarrow 0, z \rightarrow 0} f=\frac{a^{2}}{2} \quad, \quad f_{3 \max }=\lim _{y \rightarrow 0, z \rightarrow 0} f_{3}=\frac{a_{3}^{2}}{2}  \tag{14}\\
\operatorname{imag}(w)=\lim _{y \rightarrow 0} \operatorname{imag} 3(w)=-\frac{w \cdot a^{2} \cdot f_{2} \cdot a_{3}^{2}}{2 l \cdot\left(2 d_{1,2}-a^{2}\right) \cdot\left(d_{2,3}-f_{2}\right)}  \tag{15}\\
\operatorname{imag}(h)={\operatorname{limimag} 3(h)=-\frac{h \cdot a^{2} \cdot f_{2} \cdot a_{3}^{2}}{2 l \cdot\left(2 d_{1,2}-a^{2}\right) \cdot\left(d_{2,3}-f_{2}\right)}}^{\operatorname{imag}} \tag{16}
\end{gather*}
$$

The general images through one lens with a constant radius is obtained by Equation (5) as follows $R=a$

$$
\begin{equation*}
\operatorname{image}(w)=-\frac{w \cdot f}{l}=-\frac{w \cdot a}{2 l} \quad, \quad \operatorname{image}(h)=-\frac{h \cdot a}{2 l} \tag{17}
\end{equation*}
$$

By Equations (12)-(17), we obtain that

$$
\begin{equation*}
\frac{a}{2 l} \ll \frac{a^{2} \cdot f_{2} \cdot a_{3}^{2}}{2 l \cdot\left(2 d_{1,2}-a^{2}\right)\left(d_{2,3}-f_{2}\right)} \tag{18}
\end{equation*}
$$

According to Equation (18) the suggested intraocular lenses implant, increases the central object's image and removes it from the generated macula and according to Equations (6)-(13), the image of the wide and height of a peripheral object decreases continuity as a function of its distance from the center.

## 4. Conclusions

By the two suggested intraocular lens implants, both eyes can see an increased image of a central object removed from the degenerated macula and also see the peripheral objects images decreasing continuity and not suddenly. It can be obtained, according to the continuity change in the curvature radiuses of the mirrors and the lenses of the two intraocular lens implants that are suggested in this paper. The advantages of the lens implant with mirrors compared to the hyperboloid lens implant are the possibilities to change implant size and place according to the degenerated place at the patient's macula. The limitation of this implant is to chose mirror's material that do not damage the eyes. The advantages of the hyperboloid lens implant compared to the lens implant with mirrors are materials that do not damage the eyes and the implant is similar to the cataract implants. The limitation of this implant might be much more place in the lens eyes compared to the lens implant with mirrors.
Each of the two suggested intraocular implants, might prevent the loss of central vision caused by eye disease and also have the peripheral vision decreasing continuity in both eyes. The surgery for these intraocular implants is the same as a cataracts surgery. Not every eyes parameter has been taken into account in the Equations, but the two suggested intraocular lens implants can be improved according to the main ideas. Thus, the innovations and contributions of this paper are as follows:
1). The continuity change in the curvature radiuses of the mirrors and lenses in the suggested implants that allows both eyes to see an increased image of a central object and the peripheral vision, eliminating a sudden change of the peripheral vision that decreases continuity and causes a normal view.
2). The derivations of mathematical Equations for the continuity change of the curvature radius connected with the derivations of geometrical optics, are shown to support the results.

## Referents

Agarwal, A., Lipshitz, I., Jacob, A. et al. (2008). Mirror Telescopic Lens for AMD. Jurnal of Cataract and Refractive Surgery, 34.
Alio, J. L., Mulet, E. M., Roize-Moreno, J. M. et al. (2004). Intraocular Telescopic Lens Evaluated for Patients with AMD. Jurnal of Cataract and Refractive Surgery, 30.
Amar Agarwal. (2012). Eyeworld.
Henry Edwards, C., \& David, E. Penney. (1998). Calculus with Analytic Geometry. University of Georgia, Atlanta.

Hudson, H. L., Lans, Heier, J. S. et al. (2006). Implantable Miniature Telescopes for the Treatment of Visual Acuuity Resulting from AMD. Ophthalmology, 113.
Isaac, L., \& Gross, Yosef. (2002). Intraocular Lens Implant. Canadian Patent Number, 2241635.
Orzalest, N., Pierrottet, C. O., Zenoni, S. et al. (2007). The IOL-VIP.A Double Intraocular Implant for Visual Rehabilitation of AMD Patients. Ophthalmology, 114.
Primo, S. A. (2010). Implantation Miniature Telescope. American Optometric.
Ruchvarger, Haya, \& Lipshitz, Isaac. (2005). Intraocular Lens Implant with Mirrorors. US Patent Number 6902577.

Singer, M. A., Amir, N., \& Herro, A. (2012). Improving Quality of Life Patients with AMD Focus on Miniature Ocular Implants. Clinical Ophthalmology.

## Appendix A. The Derivation of a Curvature Radius at an Arbitrary Graph Point

The derivation of the curvature radius according to Figure 5, is given as follows

$$
\begin{align*}
& d l=r \cdot \lim _{\delta \rightarrow 0} \delta=\lim _{\Delta l \rightarrow 0} \Delta l=\lim _{\Delta x \rightarrow 0} \sqrt{\Delta x^{2}+\Delta y^{2}}=\sqrt{d x^{2}+d y^{2}}=\sqrt{1+\left(\frac{d y}{d x}\right)^{2}} d x=  \tag{19}\\
& =\sqrt{1+\left(f^{\prime}(x)\right)^{2}} d x \\
& \beta=\alpha+\delta \Rightarrow \delta=\beta-\alpha, \quad \tan (\delta)=\tan (\beta-\alpha)=\frac{\tan (\beta)-\tan (\alpha)}{1+\tan (\beta) \cdot \tan (\alpha)}  \tag{20}\\
& \tan (\alpha)=f^{\prime}(x), \quad \tan (\beta)=f^{\prime}(x+\Delta x), \quad \lim _{\delta \rightarrow 0} \frac{\delta}{\tan (\delta)}=1 \\
& =\frac{\lim _{\delta \rightarrow 0} \delta=\lim _{\delta \rightarrow 0} \tan (\delta)=\lim _{\Delta x \rightarrow 0} \frac{f^{\prime}(x+\Delta x)-f^{\prime}(x)}{1+f^{\prime}(x+\Delta x) \cdot f^{\prime}(x)}=\lim _{\Delta x \rightarrow 0} \frac{f^{\prime}(x+\Delta x)-f^{\prime}(x)}{1+f^{\prime}(x) \cdot f^{\prime}(x)}=}{1+\left(f^{\prime}(x)\right)^{2}} \cdot \lim _{\Delta x \rightarrow 0} \frac{f^{\prime}(x+\Delta x)-f^{\prime}(x)}{\Delta x} \cdot \Delta x=\frac{f^{\prime \prime}(x) \cdot d x}{1+\left(f^{\prime}(x)\right)^{2}}  \tag{21}\\
& \quad r=\frac{d l}{\lim _{\delta \rightarrow 0} \delta}=\frac{\sqrt{1+\left(f^{\prime}(x)\right)^{2}} d x \cdot\left(1+\left(f^{\prime}(x)\right)^{2}\right)}{f^{\prime \prime}(x) \cdot d x}=\frac{\left(1+\left(f^{\prime}(x)\right)^{2}\right)^{\frac{3}{2}}}{f^{\prime \prime}(x)}
\end{align*}
$$



Figure 5. The curvature radius of a function's graph at an arbitrary point

