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Interaction of a System of Four Particles through a Random Unique Source

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Abstract

To this purpose, one uses the result that quantum phenomena in the Euclidean formulation of the theory are due to a stochastic space-time background interaction, whose essence is the time derivative of the Wiener process. The problem of calculating both the transition probability, the path integral for the systems of four particles and factorization solution of Fokker-Planck equation are then solved. The transition probability solution of Fokker-Planck equation factorizes into a first component describing the system at its ground state and a second component characterizing its transition dynamics. The path integral for these system are then solved.

Keywords: wiener process, Fokker-Planck equation, path integral, transition probability density, theorem of factorization

1. Introduction

Understanding the process of classical Euclidian theory is one of the major challenges in the last twenty years in the fields of Brownian motion dynamics and quantum mechanics.

This interest can be explained by the fact that there is a close relationship between Brownian motion and quantum mechanics (Beilinson, 1959, 1982; Beilinson & Leal, 1993; Beilinson & Massou, 1996; Feyman & Gibs, 1965; Gelfand & Vilenkin, 1961; Glimm & Jeffe, 1987; Kac, 1959). Indeed, the solution of the time-dependent Schrodinger equation:

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = \hat{H}\Psi(x,t) \tag{1.1}$$

can be obtained from the Bloch equation

$$\frac{\partial Z(x,t)}{\partial t} = \hat{H} Z(x,t) \tag{1.2}$$

through analytic continuation of $Z(x,t)$, relative to variable t, up to the imaginary axis. Formally, it means the substitution of t by it and thus one gets the transition $Z(x,it) = \Psi(x,t)$.

It is known (Beilinson & Leal, 1993) that the strong interrelation between the Brownian motion problems and those of the quantum mechanics allows a simplified numerical solution of concrete quantum mechanics problems. Instead of solving numerically in a considerably simpler manner with a further time-analytical extension of the obtained results. In the Euclidean quantum mechanics (and, therefore, also in usual quantum mechanics), the quantum nature of the particles can be related, not with the particle itself, but with the stochastic space-time derivative of the Wiener process, used in the right hand side of Jacobi conjugate equation in classic Euclidean mechanics. This explains why these equations become stochastic.

In this work we study such a limit problem, when the correlation is absolute between particles. We first consider a system of three stochastic equations

$$\begin{cases} \dot{x} + \frac{\partial S}{\partial x} = \dot{\varphi} \\ \dot{y} + \frac{\partial R}{\partial y} = \dot{\varphi} \\ \dot{z} + \frac{\partial Q}{\partial z} = \dot{\varphi} \end{cases} \tag{1.3}$$

$\varphi(\tau)$ is the Wiener process with the measure (Gelfand & Vilenkin, 1961; Glimm & Jeffe, 1987).

$$d_W\varphi(\tau) = exp\{-\int_0^t \dot{\varphi}^2(\tau)d\tau\} \prod_{\tau=0}^t \frac{d\varphi(\tau)}{\sqrt{\pi d\tau}} \tag{1.4}$$

With the functions

$$S(x_0, 0; x, t) = \frac{m\omega}{2sh\omega t}[(x^2 + {x_0}^2)ch\omega - 2xx_0],$$

$$R(y_0, 0; y, t) = \frac{M\beta}{2sh\beta t}[(y^2 + {y_0}^2)ch\beta - 2yy_0] \tag{1.5}$$

$$Q(z_0, 0; z, t) = \frac{\mu\gamma}{2sh\gamma t}[(z^2 + {z_0}^2)ch\gamma - 2zz_0]$$

Associated to the corresponding classical Euclidean oscillators by time analytic continuation of the real axis by imaginary one. M, m and μ are the masses of oscillators; ω, β *and* γ their frequencies.

In Euclidean Quantum mechanics, the problem we used to tackle involves three different independent Wiener processes $\dot{\varphi}_1$, $\dot{\varphi}_2$ and $\dot{\varphi}_3$ describing a system of noninteracting quantum oscillators in the form:

$$\begin{cases} \dot{x} + \frac{\partial S}{\partial x} = \dot{\varphi}_1 \\ \dot{y} + \frac{\partial R}{\partial y} = \dot{\varphi}_2 \\ \dot{z} + \frac{\partial Q}{\partial z} = \dot{\varphi}_3 \end{cases} \tag{1.6}$$

At the opposite, oscillator characteristics that are described by (1.3) and (1.1), interact each to other only through a single space-time function $\dot{\varphi}(\tau)$. This work aims to investigate this specific space-time nature of interaction between quantum oscillators.

To a single Hamiltonian operator in Euclidean Quantum mechanics correspond an infinite number of stochastic equations, involving enormous computation difficulties, like equations (1.3). In this work we propose to consider another system of stochastic equations equivalent to the previous one and involving the same point of stochastic space-time background for the quantum oscillators as:

$$\begin{gathered} \dot{x}(\tau) + \omega x(\tau) = \dot{\varphi}(\tau) \\ \dot{y}(\tau) + \beta y(\tau) = \dot{\varphi}(\tau) \\ \dot{z}(\tau) + \gamma z(\tau) = \dot{\varphi}(\tau) \\ \dot{\aleph}(\tau) + \lambda\aleph(\tau) = \dot{\varphi}(\tau) \end{gathered} \tag{1.7}$$

Where ω, β, γ *and* λ are frequencies of oscillators.

This article is organized as follows. In section 2, a brief description of the four particle case based on the equations (1.7) in given to set up the transition probability, corresponding Fokker-Planck equation the path integral and the factorization theorem involved.

2. Four Particle Case

2.1 The Transition Probability and the Fokker-Planck Equation

Let now consider a system four oscillators possessing the masses m_1, m_2, m_3, and m_4 and frequencies $\omega, \beta, \gamma, \lambda$ located at the same point of the stochastic space-time background accordingly to the following system of equations:

$$\begin{cases} \dot{x} + \omega x = \dot{\varphi} \\ \dot{y} + \beta y = \dot{\varphi} \\ \dot{z} + \gamma z = \dot{\varphi} \\ \dot{\aleph} + \lambda\aleph = \dot{\varphi} \end{cases} \tag{2.1.1}$$

and to the measure (1.4) of the Wiener process.

Using Duhamel integration method, the solutions of (8) can be settled up:

$$x(t) - x_0(t)e^{-\omega t} = \int_0^t \left[2\delta(t-s) - \omega e^{-\omega(t-s)}\right]\varphi(s)ds$$

$$y(t) - y_0(t)e^{-\beta t} = \int_0^t \left[2\delta(t-s) - \beta e^{-\beta(t-s)}\right]\varphi(s)ds \tag{2.1.2}$$

$$z(t) - z_0(t)e^{-\gamma t} = \int_0^t \left[2\delta(t-s) - \gamma e^{-\gamma(t-s)}\right]\varphi(s)ds$$

$$\aleph(t) - \aleph_0(t)e^{-\lambda t} = \int_0^t \left[2\delta(t-s) - \lambda e^{-\lambda(t-s)}\right]\varphi(s)ds$$

Such that the probability density of simultaneous realizations of the values

$$x(t) - x_0(t)e^{-\omega t}\ ,\ y(t) - y_0(t)e^{-\beta t}, z(t) - z_0(t)e^{-\gamma t} \text{ and } \aleph(t) - \aleph_0(t)e^{-\lambda t}$$

of these functional coincides with the transition probability associated to (2.1.1) as:

$$W(x_0, y_0, z_0, \aleph_0, 0; x, y, z, \aleph, t) = \int_c \delta\left\{\left(x(t) - x_0(t)e^{-\omega t}\right) - \int_0^t \left[2\delta(t-s) - \omega e^{-\omega(t-s)}\right]\varphi(s)ds\right\} \times$$

$$\delta\left\{\left(y(t) - y_0(t)e^{-\beta t}\right) - \int_0^t \left[2\delta(t-s) - \beta e^{-\beta(t-s)}\right]\varphi(s)ds\right\} \times \delta\left\{\left(z(t) - z_0(t)e^{-\gamma t}\right) - \int_0^t [2\delta(t-s) -$$

$$\gamma e^{-\gamma(t-s)}]\varphi(s)ds\} \times \delta\left\{\left(\aleph(t) - \aleph_0(t)e^{-\lambda t}\right) - \int_0^t \left[2\delta(t-s) - \lambda e^{-\lambda(t-s)}\right]\varphi(s)ds\right\} d_w\varphi(s) \tag{2.1.3}$$

With the following initial condition

$$\varphi(0) = 0$$

Following the integration process (Beilinson & Massou, 1996; Massou & Olatunji, 2002), one can rewrite

$$W(x_0, y_0, z_0, \aleph_0, 0; x, y, z, \aleph, t) = exp\left\{\frac{A}{B}\right\}(\pi^4 B)^{-1/2} \tag{2.1.4}$$

Where

$$A = \begin{vmatrix} 0 & (x - x_0 e^{-\omega t}) & (y - y_0 e^{-\beta t}) & (z - z_0 e^{-\gamma t}) & (\aleph - \aleph_0 e^{-\lambda t}) \\ (x - x_0 e^{-\omega t}) & a_{11} & a_{12} & a_{13} & a_{14} \\ (z - z_0 e^{-\gamma t}) & a_{21} & a_{22} & a_{23} & a_{24} \\ (y - y_0 e^{-\beta t}) & a_{31} & a_{32} & a_{33} & a_{34} \\ (\aleph - \aleph_0 e^{-\lambda t}) & a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

$$B = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

With the diagonal elements obtained as:

$$a_{11} = \int_0^t \left[\int_\tau^t [2\delta(t-s) - \omega e^{-\omega(t-s)}]ds\right]^2 d\tau = \int_0^t \left(1 - 1 + e^{-\omega(t-s)}\right)^2 d\tau = \frac{1 - e^{-2\omega t}}{2\omega}$$

$$a_{22} = \int_0^t \left[\int_\tau^t [2\delta(t-s) - \beta e^{-\beta(t-s)}]ds\right]^2 d\tau = \int_0^t \left(1-1+e^{-\beta(t-s)}\right)^2 d\tau = \quad = \frac{1-e^{-2\beta t}}{2\beta}$$

$$a_{33} = \int_0^t \left[\int_\tau^t [2\delta(t-s) - \gamma e^{-\gamma(t-s)}]ds\right]^2 d\tau = \int_0^t \left(1-1+e^{-\gamma(t-s)}\right)^2 d\tau = = \frac{1-e^{-2\gamma t}}{2\gamma}$$

$$a_{44} = \int_0^t \left[\int_\tau^t [2\delta(t-s) - \lambda e^{-\lambda(t-s)}]ds\right]^2 d\tau = \int_0^t \left(1-1+e^{-\lambda(t-s)}\right)^2 d\tau = = \frac{1-e^{-2\lambda t}}{2\lambda}$$

The off diagonal elements are symmetric and involve the cross product of integrals as

$$a_{12} = a_{21} = \int_0^t \left\{\left[\int_\tau^t [2\delta(t-s) - \beta e^{-\beta(t-s)}]ds\right] \times \left[\int_\tau^t [2\delta(t-s) - \omega e^{-\omega(t-s)}]ds\right]\right\} d\tau = \int_0^t e^{-(\omega+\beta)(t-s)} d\tau$$

$$= \frac{1-e^{-(\omega+\beta)t}}{\omega+\beta}$$

$$a_{13} = a_{31} = \int_0^t \left\{\left[\int_\tau^t [2\delta(t-s) - \gamma e^{-\gamma(t-s)}]ds\right] \times \left[\int_\tau^t [2\delta(t-s) - \omega e^{-\omega(t-s)}]ds\right]\right\} d\tau = \int_0^t e^{-(\omega+\gamma)(t-s)} d\tau$$

$$= \frac{1-e^{-(\omega+\gamma)t}}{\omega+\gamma}$$

$$a_{14} = a_{41} = \int_0^t \left\{\left[\int_\tau^t [2\delta(t-s) - \lambda e^{-\lambda(t-s)}]ds\right] \times \left[\int_\tau^t [2\delta(t-s) - \omega e^{-\omega(t-s)}]ds\right]\right\} d\tau = \int_0^t e^{-(\omega+\lambda)(t-s)} d\tau$$

$$= \frac{1-e^{-(\omega+\lambda)t}}{\omega+\lambda}$$

$$a_{23} = a_{32} = \int_0^t \left\{\left[\int_\tau^t [2\delta(t-s) - \beta e^{-\beta(t-s)}]ds\right] \times \left[\int_\tau^t [2\delta(t-s) - \gamma e^{-\gamma(t-s)}]ds\right]\right\} d\tau = \int_0^t e^{-(\gamma+\beta)(t-s)} d\tau$$

$$= \frac{1-e^{-(\gamma+\beta)t}}{\gamma+\beta}$$

$$a_{42} = a_{24} = \int_0^t \left\{\left[\int_\tau^t [2\delta(t-s) - \beta e^{-\beta(t-s)}]ds\right] \times \left[\int_\tau^t [2\delta(t-s) - \lambda e^{-\lambda(t-s)}]ds\right]\right\} d\tau = \int_0^t e^{-(\lambda+\beta)(t-s)} d\tau$$

$$= \frac{1-e^{-(\lambda+\beta)t}}{\lambda+\beta}$$

$$a_{34} = a_{43} = \int_0^t \left\{\left[\int_\tau^t [2\delta(t-s) - \gamma e^{-\gamma(t-s)}]ds\right] \times \left[\int_\tau^t [2\delta(t-s) - \lambda e^{-\lambda(t-s)}]ds\right]\right\} d\tau = \int_0^t e^{-(\lambda+\gamma)(t-s)} d\tau$$

$$= \frac{1-e^{-(\lambda+\gamma)t}}{\lambda+\gamma} \tag{2.1.5}$$

From (2.1.4), after some calculations we obtain the following relations:

$$\lim_{t\to 0} \frac{\overline{x-x_0}}{t} = \omega x_0$$

$$\lim_{t\to 0}\frac{\overline{y-y_0}}{t}=\beta y_0$$

$$\lim_{t\to 0}\frac{\overline{\aleph-\aleph_0}}{t}=\lambda\aleph_0$$

$$\lim_{t\to 0}\frac{\overline{z-z_0}}{t}=\gamma z_0$$

$$\lim_{t\to 0}\frac{\overline{(x-x_0)^2}}{t}=\lim_{t\to 0}\frac{\overline{(y-y_0)^2}}{t}=\lim_{t\to 0}\frac{\overline{(z-z_0)^2}}{t}=\lim_{t\to 0}\frac{\overline{(\aleph-\aleph_0)^2}}{t}=\frac{1}{4}$$

$$\lim_{t\to 0}\frac{\overline{(x-x_0)(y-y_0)}}{t}=\lim_{t\to 0}\frac{\overline{(x-x_0)(z-z_0)}}{t}=\lim_{t\to 0}\frac{\overline{(y-y_0)(z-z_0)}}{t}=$$

$$\lim_{t\to 0}\frac{\overline{(\aleph-\aleph_0)(y-y_0)}}{t}=\lim_{t\to 0}\frac{\overline{(x-x_0)(\aleph-\aleph_0)}}{t}=\frac{1}{2} \tag{2.1.6}$$

So, the searched Fokker-Planck equation takes the form:

$$\frac{\partial W}{\partial t}-\omega\frac{\partial}{\partial x}(xW)-\beta\frac{\partial}{\partial y}(yW)-\gamma\frac{\partial}{\partial z}(zW)-\lambda\frac{\partial}{\partial \aleph}(\lambda\aleph)=\frac{1}{4}\frac{\partial^2 W}{\partial x^2}+\frac{1}{4}\frac{\partial^2 W}{\partial y^2}+\frac{1}{4}\frac{\partial^2 W}{\partial z^2}+\frac{1}{4}\frac{\partial^2 W}{\partial \aleph^2}+\frac{1}{2}\frac{\partial^2 W}{\partial x\partial y}+\frac{1}{2}\frac{\partial^2 W}{\partial x\partial z}+$$

$$\frac{1}{2}\frac{\partial^2 W}{\partial x\partial \aleph}+\frac{1}{2}\frac{\partial^2 W}{\partial y\partial z}+\frac{1}{2}\frac{\partial^2 W}{\partial y\partial \aleph}+\frac{1}{2}\frac{\partial^2 W}{\partial z\partial \aleph} \tag{2.1.7}$$

2.2 The Path Integral

Since (2.1.4) is a function of causality, then the following relation is verified:

$$W\left(x_0,y_0,z_0,N_0,0;x,y,z,N,t\right)=\int_{-\infty}^{+\infty}\prod_{j=1}^{n}W\left(x_{j-1},y_{j-1},z_{j-1,}N_{j-1},t_{j-1},x_j,y_j,z_j,N_j\right)\times$$
$$\times\, dx_1dy_1dz_1dN_1\cdots dx_{n-1}dy_{n-1}dz_{n-1}dN_{n-1} \tag{2.2.1}$$

We suppose that ($x_n=x,\ y_n=y,\ z_n=z,\ N_n=N,\ t_j=j\frac{t}{n},\ t_n=t$)

Using the Taylor power series expansion (Beilinson & Leal, 1993; Beilinson & Massou, 1996) and (Gelfand & Vilenkin, 1961), we write the determinants A and B occurring in the expression (2.1.4) as:

$$B=b_6(\Delta t)^6-b_7(\Delta t)^7+0(\Delta t)^8 \tag{2.2.2}$$

With

$$b_6=\frac{1}{6!}\{24[(2\beta+\lambda-\gamma)+(\gamma+\beta)(\lambda-\beta)]+12(\beta-\omega)$$
$$+3[(\beta+\omega)(\omega+\gamma)(\lambda-\omega)-(\lambda+\omega)(\beta+\omega)(\gamma-\omega)+(\lambda+\omega)(\gamma+\omega)(\beta-\omega)]$$
$$-(\omega+\gamma)(\omega+\beta)(\lambda-\omega)+(\omega+\gamma)(\omega+\lambda)(-\beta^2-\omega^2+2\omega\beta)-2\omega(\omega+\lambda)(\gamma-\beta)$$
$$+(\omega+\lambda)(\beta+\omega)(\gamma-\omega)\}$$

$$b_7=-\frac{1}{7}[-\beta^3-\beta^2(\gamma+2\lambda)-\lambda^2(\beta+\gamma+1)+5\lambda-3\beta\gamma-\gamma-24\lambda^3+3(\beta+\lambda)^3+18(\beta+\lambda)^3$$
$$-24\omega^3(\beta+\lambda)^2-24\beta^3(\omega+\gamma)^2-24\gamma^3(\beta+\lambda)^2]$$

$$A_j \approx (\Delta t)^3 \left\{ \left[x_j - x_{j-1}\left(1 - \omega\Delta t + \frac{\omega^2(\Delta t)^2}{2}\right)\right]^2 \times k_1 \right.$$
$$+ \left[x_j - x_{j-1}\left(1 - \omega\Delta t + \frac{\omega^2(\Delta t)^2}{2}\right)\right]\left[y_j - y_{j-1}\left(1 - \beta\Delta t + \frac{\beta^2(\Delta t)^2}{2}\right)\right] \times k_2$$
$$+ \left[x_j - x_{j-1}\left(1 - \omega\Delta t + \frac{\omega^2(\Delta t)^2}{2}\right)\right]\left[z_j - z_{j-1}\left(1 - \gamma\Delta t + \frac{\gamma^2(\Delta t)^2}{2}\right)\right] \times k_3$$
$$+ \left[x_j - x_{j-1}\left(1 - \omega\Delta t + \frac{\omega^2(\Delta t)^2}{2}\right)\right]\left[\aleph_j - \aleph_{j-1}\left(1 - \lambda\Delta t + \frac{\lambda^2(\Delta t)^2}{2}\right)\right] \times k_3$$
$$+ \left[y_j - y_{j-1}\left(1 - \beta\Delta t + \frac{\beta^2(\Delta t)^2}{2}\right)\right]^2 \times k_4 + \left[z_j - z_{j-1}\left(1 - \gamma\Delta t + \frac{\gamma^2(\Delta t)^2}{2}\right)\right]^2 \times k_5$$
$$\left. + \left[\aleph_j - \aleph_{j-1}\left(1 - \lambda\Delta t + \frac{\lambda^2(\Delta t)^2}{2}\right)\right]^2 \times k_6 \right\} \tag{2.2.3}$$

The coefficients $k_i \quad (i = 1, \ldots\ldots, 6)$ are given in table below.

Using (2.2.2) and (2.2.3) in (2.1.4), we have :

$$exp\left\{\sum_{j=1}^{n-1} \frac{A_{j+1}+A_j}{B_j}\right\} =$$

$$exp\left\{\sum_{j=1}^{n-1}\left[\frac{(x_{j+1}-x_j)-(x_j-x_{j-1})+\omega\Delta t(x_j-x_{j-1})-\omega^2\frac{\Delta t^2}{2}(x_j-x_{j-1})}{(\Delta t)^3}\right]^2 \times \frac{(\Delta t)^3\times k_1}{(b_6+b_7\Delta t)}\right\} \times exp\left\{\left[\frac{\left[x_{j+1}-x_j+\omega(\Delta t)x_j-\omega^2\frac{\Delta t^2}{2}x_j\right]}{\Delta t} \times \right.\right.$$
$$\left. \frac{[x_j-x_{j-1}+\omega(\Delta t)x_{j-1}-\omega^2\frac{\Delta t^2}{2}x_{j-1}]}{\Delta t} \times \frac{(\Delta t)^3 k_2}{(\Delta t)^4(b_6+b_7\Delta t)}\right] \times exp\left\{\sum_{j=1}^{n-1}\left[\frac{(y_{j+1}-y_j)-(y_j-y_{j-1})+\beta\Delta t(y_j-y_{j-1})-\beta^2\frac{\Delta t^2}{2}(y_j-y_{j-1})}{(\Delta t)^3}\right]^2 \times \right.$$
$$\left.\frac{(\Delta t)^3\times k_3}{(b_6+b_7\Delta t)}\right\} \times exp\left\{\left[\frac{\left[y_{j+1}-y_j+\beta(\Delta t)y_j-\beta^2\frac{\Delta t^2}{2}y_j\right]}{\Delta t} \times \frac{[y_j-y_{j-1}+\beta(\Delta t)y_{j-1}-\beta^2\frac{\Delta t^2}{2}y_{j-1}]}{\Delta t} \times \frac{(\Delta t)^3 k_4}{(\Delta t)^4(b_6+b_7\Delta t)}\right]\right\}\Bigg) \times$$
$$exp\left\{\sum_{j=1}^{n-1}\left[\frac{(z_{j+1}-z_j)-(z_j-z_{j-1})+\gamma\Delta t(z_j-z_{j-1})-\gamma\frac{\Delta t^2}{2}(z_j-z_{j-1})}{(\Delta t)^3}\right]^2 \times \frac{(\Delta t)^3\times k_5}{(b_6+b_7\Delta t)}\right\} \times exp\left\{\left[\frac{\left[z_{j+1}-z_j+\gamma(\Delta t)z_j-\gamma^2\frac{\Delta t^2}{2}z_j\right]}{\Delta t} \times \right.\right.$$
$$\left.\left.\frac{[z_j-z_{j-1}+\gamma(\Delta t)z_{j-1}-\gamma^2\frac{\Delta t^2}{2}z_{j-1}]}{\Delta t} \times \frac{(\Delta t)^3 k_6}{(\Delta t)^4(b_6+b_7\Delta t)}\right]\right\} \times exp\left\{\sum_{j=1}^{n-1}\left[\frac{(\aleph_{j+1}-\aleph_j)-(\aleph_j-\aleph_{j-1})+\lambda\Delta t(\aleph_j-\aleph_{j-1})-\lambda^2\frac{\Delta t^2}{2}(\aleph_j-\aleph_{j-1})}{(\Delta t)^3}\right]^2 \times \right.$$
$$\left.\frac{(\Delta t)^3\times k_1}{(b_6+b_7\Delta t)}\right\} \times$$
$$exp\left\{\left[\frac{\left[\aleph_{j+1}-\aleph_j+\lambda(\Delta t)\aleph_j-\lambda^2\frac{\Delta t^2}{2}\aleph_j\right]}{\Delta t} \times \frac{[\aleph_j-\aleph_{j-1}+\lambda(\Delta t)\aleph_{j-1}-\lambda^2\frac{\Delta t^2}{2}\aleph_{j-1}]}{\Delta t} \times \frac{(\Delta t)^3 k_2}{(\Delta t)^4(b_6+b_7\Delta t)}\right] + \right.$$
$$\left. 0(\Delta t)\right\} \tag{2.2.4}$$

Passing now to the limit $\Delta t \to 0$, we obtain:

$$W(x_0, y_0, z_0, \aleph_0, 0; x, y, z, \aleph, t)$$

$$= \int_{cx}\int_{cy}\int_{cz}\int_{C\aleph} exp\left\{\frac{1}{b_6}\int_0^t c_1(\ddot{x}+\omega\dot{x}-\omega^2 x)^2 + c_2(\ddot{y}+\beta\dot{y}-\beta^2 y)^2 + c_3(\ddot{z}+\gamma\dot{z}-\gamma^2 z)^2\right.$$

$$\left. + c_4(\ddot{\aleph}+\lambda\dot{\aleph}-\lambda^2\aleph)^2 d\tau\right\}\delta[(\dot{x}+\omega x)-(\dot{y}+\beta y)]\times\delta[(\dot{x}+\omega x)-(\dot{z}+\gamma z)]\times\delta[(\dot{x}+\omega x)$$

$$-\left(\dot{\aleph}+\lambda\aleph\right)]\times\delta[(\dot{y}+\beta y)-(\dot{z}+\gamma z)]\times\delta[(\dot{y}+\beta y)-\left(\dot{\aleph}+\lambda\aleph\right)]\times\delta[(\dot{z}+\gamma z)$$

$$-\left(\dot{\aleph}+\lambda\aleph\right)]\times\prod_{\tau=0}^{t}\frac{dx(\tau)dy(\tau)dz(\tau)d\aleph(\tau)}{|b_6|^{1/2}\pi^2(d\tau)^3}$$

$$x(0)\equiv x_0\ ;y(0)\equiv y_0;\ \ z(0)\equiv z_0;\ \aleph(0)\equiv\aleph_0$$
$$x(t)\equiv x\ ;\ \ y(t)\equiv y\ \ ;z(t)\equiv z\ ;\ \aleph(t)\equiv\aleph$$

The k_i's values.

Table

$$k_1 = (1-\gamma)(1-\lambda)(1-\beta)-(1-\beta)[1-(\gamma+\lambda)]^2$$

$$k_2 = (1-\gamma)(1-\lambda)[1-(\omega+\beta)]^2$$

$$k_3 = [1-(\omega+\lambda)][1-(\gamma+\beta)][1-(\beta+\lambda)]$$

$$k_4 = (1-\omega)(1-\lambda)[1-(\beta+\gamma)]^2$$

$$k_5 = (1-\omega)[1-(\beta+\lambda)][1-(\gamma+\lambda)]$$

$$k_6 = [1-(\omega+\gamma)][1-(\omega+\lambda)]^2$$

We have for the normalization condition probability transition:

$$\prod_{j=1}^{n}(\pi^4 B) = \prod_{j=1}^{n}\left\{\pi^4 b_6(\Delta t)^6\left[1-\frac{b_7}{b_6}\Delta t\right]+0(\Delta t)^8\right\} =$$

$$\pi^4(\Delta t)^6 b_6\prod_{j=1}^{n}\left[\left\{\left\{1-\frac{b_7}{b_6}.\frac{t}{n}\right\}\right\}^{\frac{b_6 n}{b_7 t}}\right]^{\frac{b_7 t}{b_6}}\rightarrow\exp\left(-\frac{b_6}{b_7}\right)t \tag{2.2.5}$$

$$\text{if }\ \Delta t\rightarrow 0$$

Finally, the fundamental solution of (2.1.7) writes:

$$W(x_0, y_0, z_0, \aleph_0, 0; x, y, z, \aleph, t) = \int_{cx}\int_{cy}\int_{cz}\int_{C\aleph} exp\left\{\frac{1}{b_6}\int_0^t c_1(\ddot{x}+\omega\dot{x}-\omega^2 x)^2 + c_2(\ddot{y}+\beta\dot{y}-\beta^2 y)^2 +\right.$$

$$\left. c_3(\ddot{z}+\gamma\dot{z}-\gamma^2 z)^2 + c_4(\ddot{\aleph}+\lambda\dot{\aleph}-\lambda^2\aleph)^2 d\tau\right\} exp\left(-\frac{b_6}{b_7}t\right)\delta[(\dot{x}+\omega x)-(\dot{y}+\beta y)]\times\delta[(\dot{x}+\omega x)-(\dot{z}+$$

$$\gamma z)]\times\delta[(\dot{x}+\omega x)-\left(\dot{\aleph}+\lambda\aleph\right)]\times\delta[(\dot{y}+\beta y)-(\dot{z}+\gamma z)]\times\delta[(\dot{y}+\beta y)-\left(\dot{\aleph}+\lambda\aleph\right)]\times\delta[(\dot{z}+\gamma z)-$$

$$\left(\dot{\aleph}+\lambda\aleph\right)]\times\prod_{\tau=0}^{t}\frac{dx(\tau)dy(\tau)dz(\tau)d\aleph(\tau)}{|b_6|^{1/2}\pi^2(d\tau)^3} \tag{2.2.6}$$

$$x(0)\equiv x_0\ ;y(0)\equiv y_0;\ \ z(0)\equiv z_0;\ \aleph(0)\equiv\aleph_0$$
$$x(t)\equiv x\ ;\ \ y(t)\equiv y\ \ ;z(t)\equiv z\ ;\ \aleph(t)\equiv\aleph$$

2.3 The Theorem of Factorization

The probability transition (2.2.6) can be rearranged and factorized as:

$$W(x_0, y_0, z_0, \aleph_0, 0; x, y, z, \aleph, t) = \frac{\tilde{Z}(x,y,z,\aleph)}{\tilde{Z}(x_0,y_0,z_0,\aleph_0)} . Z(x_0, y_0, z_0, \aleph_0, 0; x, y, z, \aleph, t) \quad (2.3.1)$$

with,

$$\tilde{Z}(x, y, z, \aleph) = exp\left\{\frac{-1}{b_6}[k_1\omega^3x^2 + k_2\beta^3y^2 + k_3\gamma^3z^2 + k_4\lambda^3\aleph^2]\right\}$$

$$\tilde{Z}(x_0, y_0, z_0, \aleph_0) = exp\left\{-\frac{1}{b_6}[k_1\omega^3x_0^2 + k_2\beta^3y_0^2 + k_3\gamma^3z_0^2 + k_4\lambda^3\aleph_0^2]\right\}$$

and

$$Z(x_0, y_0, z_0, \aleph_0, 0; x, y, z, \aleph, t) = \int_{cx} \int_{cy} \int_{cz} \int_{c\aleph} exp\left\{\int_0^t \frac{1}{b_6}\left[(\ddot{x} + \omega\dot{x})^2 - 2\omega^2 x\ddot{x} + \omega^4x^2 + (\ddot{y} + \beta\dot{y})^2 - 2\beta^2y\ddot{y} + \beta^4y^2 + (\ddot{z} + \gamma\dot{z})^2 - 2\gamma^2z\dot{z} + \gamma^4z^2 + (\ddot{\aleph} + \lambda\dot{\aleph})^2 - 2\lambda^2\aleph\ddot{\aleph} + \lambda^4\aleph^2\right]\right\} d\tau \, exp\left(-\frac{b_6}{b_7}t\right) \delta[(\dot{x} + \omega x) - (\dot{y} + \beta y)] \times \delta[(\dot{x} + \omega x) - (\dot{z} + \gamma z)] \times \delta\left[(\dot{x} + \omega x) - \left(\dot{\aleph} + \lambda\aleph\right)\right] \times \delta[(\dot{y} + \beta y) - (\dot{z} + \gamma z)] \times \delta\left[(\dot{y} + \beta y) - \left(\dot{\aleph} + \lambda\aleph\right)\right] \times \delta\left[(\dot{z} + \gamma z) - \left(\dot{\aleph} + \lambda\aleph\right)\right] \times \prod_{\tau=0}^{t} \frac{dx(\tau)dy(\tau)dz(\tau)d\aleph(\tau)}{|b_6|^{1/2}\pi^2(d\tau)^3} \quad (2.3.2)$$

$$x(0) \equiv x_0 \;\; ; y(0) \equiv y_0; \;\; z(0) \equiv z_0; \; \aleph(0) \equiv \aleph_0$$
$$x(t) \equiv x \; ; \;\; y(t) \equiv y \;\; ; z(t) \equiv z \;\; ; \; \aleph(t) \equiv \aleph$$

The integrals on (2.3.2) are calculated with respect to all the continuous paths with fixed extremities. Substituting relation (2.3.1) in the Fokker-Plank equation (2.1.7), one can easily show that z is the solution so the Bloch equation

$$\frac{\partial z}{\partial t} = \hat{H}Z \quad ; \quad \hat{H}\tilde{Z} = 0$$

With $\hat{H}$ given by the expression:

$$\hat{H} = \frac{1}{4}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial \aleph^2}\right] + \frac{1}{2}\frac{\partial^2}{\partial x\partial y} + \frac{1}{2}\frac{\partial^2}{\partial x\partial Z} + \frac{1}{2}\frac{\partial^2}{\partial x\partial \aleph} + \frac{1}{2}\frac{\partial^2}{\partial y\partial Z} + \frac{1}{2}\frac{\partial^2}{\partial y\partial \aleph} + \frac{1}{2}\frac{\partial^2}{\partial Z\partial \aleph} + C_1(\alpha_1x + \beta_1y + \gamma_1z + \lambda_1\aleph)\frac{\partial}{\partial x} + C_2\,(\alpha_2x + \beta_2y + \gamma_2z + \lambda_2\aleph)\frac{\partial}{\partial y} + C_3(\alpha_3x + \beta_3y + \gamma_3z + \lambda_3\aleph)\frac{\partial}{\partial z} + C_4(\alpha_4x + \beta_4y + \gamma_4z + \lambda_4\aleph)\frac{\partial}{\partial \aleph} + C_1'\omega^4x^2 + C_2'\beta^4y^2 + C_3'\gamma^4z^2 + C_4'\lambda^4\aleph^2 \quad (2.3.3)$$

The coefficients $C_i; \alpha_i; \beta_i; \gamma_i; \lambda_i, i = \overline{1,4}$ occurring in (2.3.3) are given as follows.

Equivalently, $\tilde{z}$ is the solution of the corresponding time-reversed Bloch equation:

$$\frac{\partial \tilde{z}}{\partial t} = -\widehat{H}\tilde{Z}.$$

The specific interaction of quantum particles considered here, which takes place whenever particles stay close enough to one another for a sufficiently long time interval (a realization of such situation probably being particles confined into an atomic nucleus), can be illustrated by the interaction between four quantum oscillators with a potential energy

$$V = \frac{1}{2}\omega^2x^2 - \frac{1}{2}\omega + \frac{1}{2}\beta^2y^2 - \frac{1}{2}\beta + \frac{1}{2}\gamma^2z^2 - \frac{1}{2}\gamma + \frac{1}{2}\lambda^2\aleph^2 - \frac{1}{2}\lambda \; ,$$

$$(m_1 = m_2 = m_3 = m_4 = \hbar = 1)$$

Conversely, in the usual quantum theory, these oscillators, in general, do not interact.

Under the conditions assumed in this work, an interaction arises between the oscillators that are due to the stochastic space-time background. In this way, $x, y, z, \aleph$ are not to be considered as normal coordinates, thus leading to a shifting of their normal frequencies.

3. Conclusion

The quantum problem of three and four particles in a small region of dimension comparable to that of an existing background interaction is considered.

Using the theorem of the factorization of the solution of the Fokker-Planck equation, we obtained the Hamilton's operator $\widehat{H}$ who cancels a function of state stationary $\tilde{Z}(x, y, z, \aleph)$.

The first component of the solution of Fokker-Planck equation describing the system at its ground state and a second component characterizing its transition dynamics.

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