Electromagnetism with Nanodoped Charges and Currents

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Abstract

We analyze the solutions of Maxwell's equations in a 2D-medium with nano doped static and moving charges and currents, doping being made of delta Dirac dots.

Keywords: Nanodot, Nanocurrent, TM field, Hankel function

1. Introduction

The blossoming of nanotechnology (Royal Society and Royal Academy of Engineering, 2011) and of nanomaterials (Bucci, 2010; Dai, 2002; Y. Zhang, P. L. Lang, & R. Zhang, 2009; C. Brosseau, 2007) during these last years has generated a flow of experimental and theoretical works in different domain of physics, chemistry, biology with interesting new results. And. of importance is the electromagnetic field behaviour, light specially, in these materials (C.Buzea,et al., 2007; A. Rinkewich, et al., 2003; P. Hillion, 2010) particularly for 2D-nano-structures.

The solutions of Maxwell's equations were previously analyzed (P. Hillion, 2010, where further references can be found), in absence of charges and currents, in a dielectric endowed with a nanodoped permittivity. Here instead, we suppose a homogeneous isotropic medium with nanodoped charges and currents so that, in a 2D space to comply with 2D-nano structures, only TM fields are concerned and in addition, the doping is made of delta Dirac dots. We consider two situations according that the charges are static or moving. In both cases, the concentration in nanodots is assumed small enough no to modify the electric properties of the host medium at microwave and optical frequencies.

The purpose of this work was to check how the presence of nanodoped charges and currents transforms the solutions of Maxwell's equations. This paper is organized as follows: suppo-sing first the presence of static charges, Sec.2 deals with the TM field when one of the two components J_x, J_z, of the current is null, then the same analysis is performed in absence of charges. The case where both currents are nonnull is discussed in Appendix A. Sec.3 is con-cerned with similar situations when charges and currents are moving with a small velocity $|v| << c$ which requires the use of the Laplace transform whose definition and main properties are summarized in Appendix B. Finally, a discussion is provided in Sec.4

2. TM Field with Nanodoped Charges and Currents

In a 2D-space where the electromagnetic field **E**, **B**, **D**, **H** with $\mathbf{D} = \varepsilon\mathbf{E}$, $\mathbf{B} = \mu\mathbf{H}$ depends only on the coordinates x,z, as well as the charge ρ and current **J**, this field being in addition harmonic with $\exp(i\omega t)$, the Maxwell equations :

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$$\nabla\wedge\mathbf{E} + \partial_t\mathbf{B} = 0, \qquad \nabla.\mathbf{B} = 0, \qquad \nabla\wedge\mathbf{H} - \partial_t\mathbf{D} = J, \qquad \nabla.\mathbf{D} = \rho \tag{0}$$

reduce to

$$\begin{aligned} &-\partial_z E_y + i\omega\mu H_x = 0 && a) \\ &\partial_x E_y + i\omega\mu H_z = 0 && b) \\ &\partial_z E_x - \partial_x E_z + i\omega\mu H_y = 0 && c) \\ &\partial_x H_x + \partial_z H_z = 0 && d) \end{aligned} \tag{1}$$

$$\begin{aligned} &-\partial_z H_y - i\omega\varepsilon E_x = J_x && a) \\ &\partial_x H_y - i\omega\varepsilon E_z = J_z && b) \\ &\partial_z H_x - \partial_x H_z - i\omega\varepsilon E_y = J_y && c) \\ &\varepsilon(\partial_x E_x + \partial_z E_z) = \rho && d) \end{aligned} \tag{2}$$

we get from (2a,b)

$$-i\omega\varepsilon(\partial_x E_x + \partial_z E_z) = \partial_x J_x + \partial_z J_z \tag{3}$$

so that acccording to (2d) and (3)

$$\partial_x J_x + \partial_z J_z = -i\omega\rho \tag{4}$$

The Maxwell equations (1), (2) divide into two sets corresponding to the TE field (H_x,H_z,E_y) and to the TM field (E_x,E_z,H_y) with a different behaviour.

Assuming $J_y = 0$ since J_y does not intervene in Eq.(4), the component E_y of the TE field, H_x, H_z being obtained from Eqs.(1a,b), satisfies the Helmholtz equation

$$(\Delta + k^2)\, E_y = 0 \quad , \quad \Delta = \partial_x^{\,2} + \partial_z^{\,2} \quad , \quad k^2 = \omega^2\varepsilon\mu \tag{5}$$

with well known solutions so that we may discard the TE field from this analysis.

Now, the H_y component of the TM field with E_x, E_z obtained from (2a,b), satisfies the non-homogeneous Helmholtz equation

$$(\Delta + k^2)\, H_y = \partial_x J_z - \partial_z J_x \tag{6}$$

in which J_x, J_z, have to fulfill the constraint (4).

A situation different from that of the TE field, because the effective permittivity is a tensor

$$\varepsilon_x = \varepsilon + 4i\pi\sigma_x/\omega \quad , \quad \varepsilon_z = \varepsilon + 4i\pi\sigma_z/\omega \quad , \quad \varepsilon_y = \varepsilon \tag{7}$$

with the conductivity σ satisfying the relations`

$$J_x = \sigma_x E_x \quad , \quad J_z = \sigma_z E_z \tag{7a}$$

making the medium anisotropic for TM fields.

2.1 Doping with charged nanodots

The charge density with amplitude q is assumed to have the expression

$$\rho = -4\pi q \sum_m \delta(z - mh) \sum_n \delta(x - na) \tag{8}$$

in which m, n are integers, h, a, some length and δ the Dirac distribution.

So, the charged dots are uniformly distributed along the x-axis of the $z - mh = 0$ planes.

The constraint (4) is also assumed to be fulfilled with J_z or J_x null so that, according to (4) and (8), we have with the antiderivative operator ∂_x^{-1} (the antiderivative of a function f is a function F whose derivative is equal to f, also called primitive)

$$J_z = 0 \quad , \quad J_x = 4i\pi q\omega \sum_m \delta(z - mh) \sum_n \partial_x^{-1}\delta(x - na) \tag{9}$$

$$\partial_x^{-1}\delta(x - na) = \int_{na}^{x} \delta(t - na)\, dt$$

$$= U(x - na) - U(0) \tag{9a}$$

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in which U is the unit step function. Similarly :

$$J_x = 0 \quad , \quad J_z = 4i\pi q\omega \sum_m \partial_z^{-1}\delta(z - mh) \sum_n \delta(x - na) \tag{10}$$

$$\partial_z^{-1}\delta(z - mh) = U(z - mh) - U(0) \tag{10a}$$

The two possibilities $J_z = 0$ and $J_x = 0$ are successively discussed.

2.1.1 First mode $J_z = 0$

Applying the derivative operator ∂_x to Eq.(6) gives

$$(\Delta + k^2)\partial_x H_y = -\partial_z\partial_x J_x \tag{11}$$

in which according to (9)

$$\partial_x J_x = 4i\pi q\omega \sum_m \delta(z - mh) \sum_n \delta(x - na) \tag{12}$$

Then, we look for $\partial_x H_y$ in the form

$$\partial_x H_y = i\omega q \sum_{n,m} \partial_z \psi_{n,m} \tag{13}$$

Substituting (13) into (11) and taking into account (12) give the equation fulfilled by $\psi_{n,m}$

$$(\Delta + k^2)\, \psi_{n,m} = -4\pi\, \delta(z - mh)\, \delta(x - na) \tag{14}$$

with, as solution, the Green's function of the 2D-Helmholtz equation (Morse & Feshbach, 1953)

$$\psi_{n,m}(x,z) = i\pi\, H_0^1(kP_{n,m}) \quad , \quad P_{n,m} = [(x-na)^2 + (z-mh)^2]^{1/2} \tag{15}$$

in which H_0^1 is the Hankel function of the first kind. Then, according to (13) and (15)

$$\partial_x H_y = -\,\omega\pi q \sum_{n,m} \partial_z H_0^1(kP_{n,m})$$

$$= \omega\pi q k \sum_{n,m} (z-mh)\, H_1^1(kP_{n,m}) / P_{n,m} \tag{16}$$

where $-H_1^1$ is the derivative of H_0^1. And, introducing the function

$$R_{n,m}(x,z) = (z-mh)\, H_1^1(kP_{n,m}) / P_{n,m} \tag{17}$$

we get from (16)

$$H_y(x,z) = \quad \omega\pi q k \sum_{n,m} \partial_x^{-1} R_{n,m}(x,z) \tag{18}$$

in which ∂_x^{-1} is the antiderivative operator {see (9)}

$$\partial_x^{-1} R_{n,m}(x,z) \quad = \int_{na}^{x} R_{n,m}(t,z)\, dt \tag{18a}$$

Once H_y known, the electric components E_x, E_z of the TM fleld are obtained from the Max-well equations (2a,b). Now, with $J_z = 0$, we get at once from (2b) $\partial_x H_y = i\omega\varepsilon\, E_z$ so that, accor-

ding to (18) and (18a)

$$\varepsilon E_z = -i\pi q k \sum_{n,m} \quad R_{n,m} \tag{19}$$

The E_x component is a bit more intricate. We get from (3),

$$\partial_x(\omega\varepsilon E_x \; - iJ_x) = -\omega\varepsilon \partial_z E_z \tag{20}$$

with according to (19)

$$\omega\varepsilon \partial_z E_z \quad = i\omega\pi q k \sum_{n,m} \quad \partial_z R_{n,m} \tag{20a}$$

so that

$$\omega\varepsilon E_x \; - iJ_x = i\omega\pi q k \sum_{n,m} \quad \partial_x^{-1} \partial_z R_{n,m} \tag{21}$$

$$\partial_x^{-1} \partial_z R_{n,m} = \int_{na}^{x} \; \partial_z R_{n,m}(t,z)\, dt \tag{21a}$$

which achieves to determine the TM field.

Now, while the z-component of the conductivity σ is null, we get from (21) for σ_x taking into account (7a)

$$(\omega\varepsilon/\sigma_x - i) J_x = i\omega\pi q k \sum_{n,m} \quad \partial_x^{-1} \partial_z R_{n,m} \tag{22}$$

Substituting J_x/σ_x and E_x into (21) supplies σ_x .

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2.1.2 Second mode $J_x = 0$

Applying ∂_z to Eq.(6) gives

$$(\Delta + k^2)\, \partial_z H_y = \partial_x \partial_z J_z \tag{23}$$

with according to (10)

$$\partial_z J_z = 4i\pi q\omega \sum_m \delta(z - mh) \sum_n \delta(x - na) \tag{24}$$

Then, we look for $\partial_z H_y$ in the form

$$\partial_z H_y = i\omega q \sum_{n,m} \partial_x \phi_{n,m} \tag{25}$$

Substituting (25) into (23) and taking into account (24) give the equation fulfilled by $\phi_{n,m}$

$$(\Delta + k^2)\ \phi_{n,m} = 4\pi\ \delta(z - mh)\ \delta(x - na) \tag{26}$$

with as solutions

$$\phi_{n,m}(x,z) = -i\pi\ H_0^{\,1}(kP_{n,m}) \tag{27}$$

and $P_{n,m}$ given in (15). Then, according to (25) and (27)

$$\begin{aligned} \partial_z H_y &= \omega\pi q \textstyle\sum_{n,m} \partial_x H_0^{\,1}(kP_{n,m}) \\ &= -\omega\pi qk \textstyle\sum_{n,m} (x-na) H_1^{\,1}(kP_{n,m}) / P_{n,m} \end{aligned} \tag{28}$$

Then, introducing the function

$$S_{n,m}(x,z) = (x-na) H_1^{\,1}(kP_{n,m}) / P_{n,m} \tag{29}$$

we get from (28)

$$H_y = \omega\pi qk \textstyle\sum_{n,m} \partial_z^{-1} S_{n,m}(x,z) \tag{30}$$

with

$$\partial_z^{-1} S_{n,m}(x,z) = \int_{mh}^{z} S_{n,m}(x,t)\, dt \tag{30a}$$

Now, when $J_x = 0$, we get from (2a): $\partial_z H_y + i\omega\varepsilon E_x = 0$ which gives according to (28)

$$\varepsilon E_x(x,z) = i\pi qk \textstyle\sum_{n,m} S_{n,m}(x,z) \tag{31}$$

Then, according to (3)

$$\partial_z(\omega\varepsilon E_z - iJ_z) = -\ \omega\varepsilon\partial_x E_x \tag{32}$$

with using (31)

$$\omega\varepsilon\partial_x E_x = i\omega\pi qk \textstyle\sum_{n,m} \partial_x S_{n,m}(x,z) \tag{33}$$

so that

$$\omega\varepsilon E_z - iJ_z = -i\omega\pi qk \textstyle\sum_{n,m} \partial_z^{-1}\partial_x S_{n,m}(x,z) \tag{34}$$

and

$$\partial_z^{-1}\partial_x S_{n,m}(x,z) = \int_{mh}^{z} \partial_x S_{n,m}(x,t)\, dt \tag{34a}$$

Substituting J_z/σ_z and E_z into (34) gives the z-component of the conductivity σ while $\sigma_x = 0$.

2.2 Doping with nano-currents in absence of charges

With $\rho = 0$, the equations (2d), (4) become $\partial_x E_x + \partial_z E_z = 0$ and

$$\partial_x J_x + \partial_z J_z = 0 \tag{35}$$

The nanocurrents satisfying (35) are

$$\begin{aligned} J_x &= 4\pi\omega q \textstyle\sum_{n,m} \delta(z-mh)\ U(x-na) \\ J_z &= -4\pi\omega q \textstyle\sum_{n,m} \delta(x-na)\ U(z-mh) \end{aligned} \tag{36}$$

Then, with $H_y = H_y^{\,1} + H_y^{\,2}$, we get from Eq.(6)

$$\begin{aligned} (\Delta + k^2)\ H_y^{\,1} &= -4\pi\omega q \textstyle\sum_{n,m} \delta'(x-na)\ U(z-mh) \\ (\Delta + k^2)\ H_y^{\,2} &= -4\pi\omega q \textstyle\sum_{n,m} \delta'(z-mh)\ U(x-na) \end{aligned} \tag{37}$$

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in which δ' is the derivative of the Dirac distribution.

Now, we look for $H_y^{\,1}$, $H_y^{\,2}$ (not to be confised with the Hakel functions), in the form

$$H_y^{\,1} = -\omega q \textstyle\sum_{n,m}\psi_{n,m} \quad , \quad H_y^{\,2} = -\omega q \textstyle\sum_{n,m}\phi_{n,m} \tag{38}$$

so that according to (37), (38), $\psi_{n,m}$ and $\phi_{n,m}$ are solutions of the Helmholtz equation

$$\begin{aligned} (\Delta + k^2)\psi_{n,m} &= -4\pi\ \delta'(x-na)\ U(z-mh) \\ (\Delta + k^2)\phi_{n,m} &= -4\pi\ \delta'(z-mh)\ U(x-na) \end{aligned} \tag{39}$$

But, let $G_{n,m}$ be the solution of the equation

$$(\Delta + k^2) G_{n,m} = -4\pi\ \delta(z-mh)\ \delta(x-na) \tag{40}$$

$G_{n,m} = i\pi\ H_0^{\,1}(P_{n,m})$ with $P_{n,m}$ given by (15) is the Green's function of the 2D-Helmholtz equa-tion intems of the Hankel function $H_0^{\,1}$ and according to (39), (40)

$$\psi_{n,m} = \delta_x\delta_z^{-1} G_{n,m} = i\pi\, \delta_x\delta_z^{-1} H_0^1(P_{n,m}) \tag{41a}$$

$$\phi_{n,m} = \delta_z\delta_x^{-1} G_{n,m} = i\pi\, \delta_z\delta_x^{-1} H_0^1(P_{n,m}) \tag{41b}$$

Substituting (41) into (38), we get according to (17) and (29) with the functions R_{mn}, S_{mn}

$$H_y^1 = -i\pi\omega q \quad \sum_{n,m} \delta_z^{-1} S_{n,m}$$
$$H_y^2 = -i\pi\omega q \quad \sum_{n,m} \delta_x^{-1} R_{n,m} \tag{42}$$

$\delta_x^{-1} R_{n,m}$ and $\delta_x^{-1} S_{n,m}$ being supplied by (18a) and (30a).

Finally, the components E_x, E_z of the electric field are obtained from Eqs.(2a,b)

$$i\omega\varepsilon E_x = -J_x - i\pi\omega q \sum_{n,m}[S_{n,m} + \partial_z\partial_x^{-1}R_{n,m}]$$
$$i\omega\varepsilon E_z = -J_z + i\pi\omega q \sum_{n,m}[R_{n,m} + \partial_x\partial_z^{-1}S_{n,m}] \tag{43}$$

satisfying the divergence equation $\partial_x E_x + \partial_z E_z = 0$.

Remark 1

The concentration of nanodots has been supposed small enough to left unchanged the elec-tric properties of the medium. A less drastic assumption is obtained from the Maxwell-Garnett theory (P. Hillion, 2010; Tsang & Kong, 2001; Mallet, Guerin, & Santenac, 2005) in which, the permittivity $\underline{\varepsilon}$ of a composite material is

$$(\underline{\varepsilon} - \varepsilon_1)(\underline{\varepsilon} + 2\varepsilon_1)^{-1} = f(\varepsilon_2 - \varepsilon_1)(\varepsilon_2 + 2\varepsilon_1)^{-1} \tag{44}$$

f is the fillling factor of inclusion (volume fraction) in the host material (here the dielectric), ε_1, ε_2 corresponding to the host and to the charged nanodots respectively. We get from (44)

$$\underline{\varepsilon} = \varepsilon_1(1+2\alpha f)(1-\alpha f)^{-1} \qquad , \qquad \alpha = (\varepsilon_2 - \varepsilon_1)(\varepsilon_2 + 2\varepsilon_1)^{-1} \tag{45}$$

with for $f \ll 1$ the $0(f^2)$ approximation

$$\underline{\varepsilon} = \varepsilon_1(1+3\alpha f) \tag{45a}$$

So, with the Maxwell-Garnett theory, one would have to change ε into the approximation (45a) of $\underline{\varepsilon}$ in the effective permittivity (7) which has not an important effect. It is implicitly assumed that the real part of the effective permittivity $\varepsilon_j = \varepsilon + 4i\pi\sigma_j/\omega$, j = 1,2,3 is positive but, in some frequency ranges, that we discard in this work, the real part becomes negative so

that the doped dielectric behaves as a metamaterial (left handed). A recent monography (Sarychev & Shalaev, 2008) is

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devoted to electromagnetism in metamaterials.

Remark 2

As noticed in Secs.3.1, 3,2, $G(x,z,x_0,z_0) = i\pi H_0^1(kP)$, $P^2 = (x-x_0)^2+(z-z_0)^2$ is the Green function of the 2D Helmholtz equation

$$(\Delta+k^2)\, G(x,z,x_0,z_0) = -4\pi\delta(x-x_0)\,\delta(z-z_0)$$

so that the solution of the inhomogeneous equation with the charge density $\rho(x,z)$: $(\Delta+k^2)\psi(x,z) = \rho(x,z)$ is

$$\psi(x,z) = \iint_{-\infty}^{\infty} G(x,z,x_0,z_0)\, \rho(x_0,z_0) \tag{46}$$

Then, in a 2D-medium with charged inclusions, we get for $J_z = 0$ according to Eq.(11)

$$\partial_x H_y = -i\omega \iint_{-\infty}^{\infty} G(x,z,x_0,z_0)\partial_{z0}\, \rho(x_0,z_0) \tag{47}$$

and according to (23) for $J_x = 0$, $\partial_z H_y = i\omega \iint_{-\infty}^{\infty} G(x,z,x_0,z_0)\partial_{x0}\, \rho(x_0,z_0)$ so that if $\rho(z)$ has the expansion $\rho(z) = -4\pi \sum_{n,m} \alpha_{n,m}\, \delta(z-mh)\, \delta(x-na)$ where $\alpha_{n,m}$ is the amplitude of the

$\delta(z-mh)\,\delta(x-na)$ mode, we could generalize the previous results obtained for nanodot compo-sites to materials with inclusions of a more structured form.

Remark 3

The solutions of Maxwell's equations for the charged nanodots (8), uniformly distributed in 2D-space, are obtained in the form of infinite series such as (16) and (30). So, these calcu-

lations are formal since nothing is known on the convergence of these series and it could be

more realistic to work with a non-uniform distribution of nanodots, for instance

$$\rho = -4\pi q \sum_m \quad 1/n!\ 1/m!\delta(z - mh) \sum_n \delta(x - na) \tag{48}$$

with a better supposed convergence.

The constraint (4) has been assumed fulfilled with J_z or J_x null. Calculations, a bit more ela-borated when this condition is not satisfied, are performed in Appendix A.

Remark 4

To launch a TM wave in a 2D-medium endowed with the charged nanodots (8), we could illu-

minate the space with an harmonic plane wave whose electric field E_0 is in oz or ox direction. In the first case, Eq.(19) becomes $\varepsilon E_z(x,z) = \varepsilon E_0 - i\pi qk \sum_{n,m} R_{n,m}(x,z)$ so that since Eq. (2b) gives $\partial_x H_y = i\omega\varepsilon E_z$

$$\partial_x H_y(x,z) = i\omega\varepsilon E_0 + \pi\omega qk \sum_{n,m} R_{n,m}(x,z) \tag{49}$$

while according to Eq.(21) the component E_x is left unchanged.

In the second case we get from (31) $\varepsilon E_x(x,z) = \varepsilon E_0 + i\pi qk \sum_{n,m} S_{n,m}(x,z)$ and, since

$\partial_z H_y = -i\omega\varepsilon E_x$, it comes

$$\partial_z H_y(x,z) = -i\omega\varepsilon E_0 + \pi\omega qk \sum_{n,m} S_{n,m}(x,z) \tag{49a}$$

and now according to (34), E_z is left unchanged.

Similarly, to launch a TM wave in a material with nanocurrents, the medium would be illumi-

nated with an harmonic plane wave, the components E_s, E_s satisfying $|E_x| = |E_z| = E_0$ so that the relations (43) become

$$i\omega\varepsilon E_x = i\omega\varepsilon E_0 - J_x - i\pi\omega q \sum_{n,m}[S_{n,m} + \partial_z\partial_x^{-1}R_{n,m}]$$

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$$i\omega\varepsilon E_z = -i\omega\varepsilon E_0 - J_z + i\pi\omega q \sum_{n,m}[R_{n,m} + \partial_x\partial_z^{-1}S_{n,m}] \tag{50}$$

Computational works would have still to be performed to check these results.

3. TM Fields with Moving Nanodoped Charges and Currents

We consider a 2D - medium in which is moving, at a constant low velocity ($v << c$), a flow of charged nanodots characterized by the time dependent charge

$$\rho(x,z\,;t) = 4\pi q/v_x v_z\ \delta(t - x/v_x)\ \delta(t - z/v_z) \tag{51}$$

δ is the Dirac distribution, v_x, v_z the components of the velocity in x,z directions and q the charge. To get time dependent solutions of Maxwell's equations in presence of the charged flow (51), we work with the Laplace transform (B.Van der Pol & H.Bremmer, 1959)

$$f(p) \equiv L\{F(t)\} = p \int_{-\infty}^{\infty} \exp(-pt)\ F(t)\ dt \tag{52}$$

The symbolic variable p is supposed real and the main useful properties of the transform (52)

are summarized in Appendix B where it is proved that the Laplace transform χ of ρ is

$$\chi(x,z\,;p) \equiv L\{\rho(x,z\,;t)\} = -(4\pi qp/v_x v_z) \exp[-p(x/v_x + z/v_z)/2]\ \delta(x/v_x - z/v_z) \tag{53}$$

and, the Laplace transform of the fields **E**, **H**, **D**, **B** are represented by small letters **e**,**h**,**d**,**b**.

We proceed as in Sec.(2) with the Laplace transformed Maxwell equations according to B(1,a) and (B.10a) in Appendix B

$$\begin{aligned} -\partial_z e_y + p\mu h_x &= 0 && a)\\ \partial_x e_y + p\mu h_z &= 0 && b)\\ \partial_z e_x - \partial_x e_z + p\mu h_y &= 0 && c)\\ \partial_x h_x + \partial_z h_z &= 0 && d) \end{aligned} \tag{54}$$

$$\begin{aligned} -\partial_z h_y - p\varepsilon e_x &= j_x && a)\\ \partial_x h_y - p\varepsilon e_z &= j_z && b)\\ \partial_z h_x - \partial_x h_z - p\varepsilon e_y &= j_y && c)\\ \varepsilon(\partial_e e_x + \partial_z e_z) &= \chi && d) \end{aligned} \tag{55}$$

and we get from (55a,b)

$$-p\varepsilon(\partial_x e_x + \partial_z e_z) = \partial_x j_x + \partial_z j_z \tag{56}$$

so that according to (55d) and (56)

$$\partial_x j_x + \partial_z j_z = -p\chi \tag{57}$$

The Maxwell equations (54), (55) divide into two sets:TE field (h_x,h_z,e_y) and TM field (e_x,e_z,h_y). Assuming $j_y = 0$ since j_y does not intervene in Eq.(57), the component e_y of the TE field, supplying h_x, h_z, from (54a,b), satisfies the Laplace transform of the 2D wave equation

$$(\Delta - p^2n^2)\, e_y = 0 \quad , \quad \Delta = \partial_x^{\,2} + \partial_z^{\,2} \quad , \quad n^2 = \varepsilon\mu \quad , \tag{58}$$

with evident solutions of amplitude f(p) : $e_y = f(p) \exp[pn(x \sin\theta + y \cos\theta)]$ so that we may discard the TE field. Note that pn has the L^{-1} dimension.

Now, the h_y component of the TM field, supplying e_x,e_z from (55a,b) is solution of the La-place transformed nonhomogeneous wave equation in which j_x, j_z satisfy the constraint (57).

$$(\Delta - p^2n^2)\, h_y = \partial_x j_z - \partial_z j_x \tag{59}$$

As in Sec.(2) the effective permittivity for the TM field becomes a tensor with components

$$\varepsilon_x = \varepsilon - 4\pi\sigma_x/p \quad , \quad \varepsilon_z = \varepsilon - 4i\pi\sigma_z/\omega p \quad , \quad \varepsilon_y = \varepsilon \tag{60}$$

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the conductivity σ satisfying the relations generalizing the Ohm's law.

$$j_x = \sigma_x\, e_x \quad , \quad j_z = \sigma_z\, e_z \tag{60a}$$

The constraint (57) is assumed fulfilled either with j_z or j_x null and we discuss successiely both situations.

3.1 First mode : $j_z = 0$

Then, we have according to (51) and (57)

$$\partial_x j_x = (4\pi q p^2/v_x v_z) \exp[-p(x/v_x + z/v_z)/2]\, \delta(x/v_x - z/v_z) \tag{61}$$

with $\tau = x/v_x - z/v_z$ and $\partial_x = 1/v_x\partial_\tau$ this expression becomes

$$\partial_\tau j_x = A_x \exp(-pz/v_z)\, p^2 \exp(-p\tau/2)\, \delta(\tau) \quad , \quad A_x = 4\pi q/v_z \tag{62}$$

so that ∂_τ^{-1} denoting the antiderivative operator

$$\begin{aligned} j_x &= A_x p^2 \exp(-pz/v_z)\partial_\tau^{-1} \exp(-p\tau/2)\, \delta(\tau) \\ &= A_x p^2 \exp(-pz/v_z)\, [\exp(-p\tau/2)\, U(\tau) \quad + \\ &\qquad p/2 \int U(\tau) \exp(-p\tau/2)\, d\tau] \end{aligned} \tag{63}$$

in which U(τ) is the Heaviside function.

Now, using the relation $f(\tau)\, \delta'(\tau) = -f'(\tau)\, \delta(\tau)$, we get from (62) $\partial_\tau^{\,2} j(x) = 0$ that is $\partial_x^{\,2} j_x = 0$.

This last relation together with $j_z = 0$, reduces the Laplacian equation (59) to $(\Delta - p^2n^2)h_y = 0$ so that using (55b), we get for the e_z, h_y, components of the TM field the equations

$$(\Delta - p^2n^2)e_z = 0 \quad a) \quad , \quad \partial_z h_y - p\varepsilon e_z = 0 \quad b) \tag{64}$$

while, according to (55a), the component e_x is with j_x supplied by (63)

$$p\varepsilon e_x = -\partial_z h_y - j_x \tag{65}$$

Eq.(64a) has the elementary solutions in which E is a constant amplitude and f(p) an arbitrary function of p

$$e_z = E\, f(p) \exp[pn\, (x \sin\theta + z \cos\theta)] \tag{66a}$$

and, according to (64b)

$$h_y = E\, (\varepsilon /n \sin\theta\,)\, f(p) \exp[pn\, (x \sin\theta + z \cos\theta)] \tag{66b}$$

Substituting (63) and (66b) into (65) gives

$$\begin{aligned} e_x = e_1+e_2+e_3 \quad &, \quad e_1 = -E\, \varepsilon \coth\theta\, f(p) \exp[pn\, (x \sin\theta + z \cos\theta)] \\ e_2 &= -A_x\, \varepsilon^{-1}\, p\, f(p) \exp[-p(x/v_x + z/v_z)/2])\, U(\tau) \\ e_3 &- A_x\, \varepsilon^{-1}\, p^2\, f(p)/2 \exp[-p(x/v_x + z/v_z)/2]) \int U(\tau)\, d\tau \end{aligned} \tag{67}$$

Then, F(t) being the inverse Laplace transform of f(p), we get using the relations (B.2) and (B.10) of Apendix B, the inverse Laplace transforms of (66a,b) and (67)

$$\begin{aligned} E_z &= E\, F[t + n\, (x \sin\theta + z \cos\theta)] \\ H_y &= E\, (\varepsilon /n \sin\theta)\, F[t + n\, (x \sin\theta + z \cos\theta)] \end{aligned} \tag{68}$$

and

$$E_x = E_1 + E_2 + E_3 \quad , \quad E_1 = -E\ \varepsilon \coth\theta\ F[t+n(x \sin\theta + z\cos\theta)]$$

$$E_2 = \ -A_x\ \varepsilon^{-1}\ \partial_t F[t-(x/v_x + z/v_z)/2]\ U(\tau)$$

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$$E_3 = A_x\ \varepsilon^{-1}/2\ L^{-1}\{\int U(\tau)\ p^2 f(p)\ \exp[-p(x/v_x + z/v_z)/2]\ d\tau\} \tag{69}$$

E_3 has to been obtained numerically.

For the effective permittivity, $\varepsilon_z = \varepsilon_y = \varepsilon$ according to (60) and using (B.11)

$$\eta_x(t) = \varepsilon - 4\pi \int_{-\infty}^{t} \Sigma_x(u)\ du \qquad p > 0 \quad ; \quad \eta_x(t) = \varepsilon + 4\pi \int_{t}^{\infty} \Sigma_x(u)\ du \qquad p < 0 \tag{*}$$

in which $\eta_x(t)$, $\Sigma_x(t)$ are the inverse Laplace transform of $\varepsilon_x(p)$ and $\sigma_x(p)$. Then, multiplying (60a) by p and using (B.10a), (B.12) give the simple relation

$$\partial_t J_x = U(t) \int_0^t \Sigma_x(u)\ E_x(t-u)\ du \tag{**}$$

3.2 Second mode $j_x = 0$

Proceeding similarly, we now get from (53) and (57)

$$\partial_z j_z \ = (4\pi q p^2/v_x v_z)\ \exp[-p(x/v_x + z/v_z)/2]\ \delta(x/v_x - z/v_z) \tag{70}$$

that we write with τ

$$\partial_\tau j_z = A_z \exp(-px/v_x)\ p^2 \exp(p\tau/2)\ \delta(\tau) \quad , \quad A_z = 4\pi q/v_x \tag{70a}$$

so that

$$j_z = A_z p^2 \exp(-px/v_x)\ \partial_\tau^{-1} [\exp(-p\tau/2)\ \delta(\tau)]$$

$$= A_z p^2 \exp(-px/v_x)\ [\exp(-p\tau/2)\ U(\tau) \ +$$

$$p/2 \int U(\tau) \exp(p\tau/2)\ d\tau] \tag{71}$$

and we get from (70a) $\partial_z^2 j_z = 0$.

So, taking into account this last relation together with $j_x = 0$, reduces the Laplace equation (59) to $(\Delta - p^2 n^2)h_y = 0$ and using (55a), we get for the e_x, h_y, components of the TM field the equations

$$(\Delta - p^2 n^2)e_x = 0 \qquad a) \quad , \quad \partial_z h_y + p\varepsilon e_x = 0 \qquad b) \tag{72}$$

while, according to (55b), the component e_z is with j_x supplied by (71)

$$p\varepsilon e_z = \partial_x h_y - j_z \tag{73}$$

Eq.(72a) has the elementary solutions

$$e_x = E\ f(p)\ \exp[pn\ (x \sin\theta + z\cos\theta)] \tag{74a}$$

and, according to (72b)

$$h_y = -\ E\ (\varepsilon/n \cos\theta)\ f(p)\ \exp[pn\ (x \sin\theta + z\cos\theta)] \tag{74b}$$

Substituting (71) and (74b) into (73) gives

$$e_x = e_1 + e_2 + e_3 \quad , \quad e_1 = -E\ \varepsilon\ \text{tang}\theta\ f(p)\ \exp[pn\ (x\sin\theta + z\cos\theta)]$$

$$e_2 = -\ A_z \varepsilon^{-1}\ p\ f(p) \exp[-p(x/v_x + z/v_z)/2])\ U(\tau)$$

$$e_3 - A_z \varepsilon^{-1}\ p^2 f(p)/2\ \exp[-p(x/v_x + z/v_z)/2]) \int U(\tau)\ d\tau \tag{75}$$

with the inverse Laplace transforms

$$E_x = F[t + n\ (x\sin\theta + z\cos\theta)]$$

$$H_y = -E(\varepsilon/n \cos\theta)\ F[t + n\ (x\sin\theta + z\cos\theta)] \tag{76}$$

and

$$E_x = E_1 + E_2 + E_3 \quad , \quad E_1 = -E\ \varepsilon\ \text{tang}\theta\ F[t+n(x\sin\theta + z\cos\theta)]$$

$$E_2 = -\ A_z\ \varepsilon^{-1}\ \partial_t F[t - (x/v_x + z/v_z)/2])\ U(\tau) \tag{77a}$$

while

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$$E_3 = A_z\ \varepsilon^{-1}/2\ L^{-1}\ \{\int U(\tau)\ p^2 f(p)\ \exp[-p(x/v_x + z/v_z)/2])\}\ d\tau \tag{77b}$$

requires also a numerical integration.

The effective permittivity has now the components $\varepsilon_x = \varepsilon_y = \varepsilon$ and $\varepsilon_z = \varepsilon - 4\pi\sigma_z/p$. Changing the subscript x into z in (*), (**) gives the inverse Laplace transform ε_z in terms of $\sum_z E_z$.

The results of the Sections 2.1 and 2.2 may be extended to the generai solution of the wave equation (14a) (resp.22a) in which the amplitude E depends on θ

$$e_{x,z} = \int_{-\pi}^{\pi} E_{x,z}(\theta)\ f(p)\ \exp[pn\ (x \sin\theta + z \cos\theta)] \quad (78)$$

with of course more intricate calculations.

The particular function $f(p) = 2\pi(p+i\omega)/\omega\ \delta(p^2/\omega^2+1)$ with inverse Laplace transform $\exp(i\omega t)$ supplies harmonic plane waves for the TM field.

4. Discussion

The analytical expressions of the TM field obtained here are somewhat formal since based on the antiderivatives ∂_x^{-1} , ∂_z^{-1}. So, this work is a first step for computational algorithms dedu-ced from those developed to solve Maxwell's equations and which would provide more exact results. Nevertheless, the four remarks at the end of Sec.2 allow to discern the main features of electromagnetism in doped materials with charged nanodots and currents, justifying the analytical expressions.

Let us also note that only the component $S_z(x,z) = c/4\pi\ E_x(x,z)\ H_y(x,z)$ of the vector represen-ting the TM energy flow is non null with according to (2a) for static charges and currents

$$E_z = 1/i\omega\varepsilon\ (J_x + \partial_x H_y) \quad (79)$$

so that

$$S_z(x,z) = c/4\pi\varepsilon\ (J_x H_y + H_y \partial_x H_y) \quad (80)$$

with for instance H_y, J_x, given by (18) and (22) or by (30) and $J_x = 0$. The component S_z is a bit more intricate for moving charges and currents since one has to use the expressions (68), (69) of H_y and E_x when $J_z = 0$ and (76), (77) when $J_x = 0$,

Then, we are now concerned with the possibility to generate the flow of nanodots (51) made of static charges in a moving medium. It has been proved (Montigny & Rousseau, 2006; Rousseau, 2008) that the Galilean covariance of electromagnetism in media moving at low velocities imposes for the charge ρ an *electric limit*

$$\rho(x',z';t') = \rho(x,z;t)\ , \qquad t' = t \quad , \quad x' = x - v_x t, \quad z' = z - v_z t. \quad (81)$$

So, taking into account this result, the flow (51) could be generated by static charges $\delta(x)\delta(z)$ in a medium moving at a small velocity **v** with components v_x, v_z.

$$\rho(x,z;t) = 4\pi/v_x v_y \sum_k q_k\ \delta(t - x_k/v_x)\ \delta(t - y_k/v_y) \quad (82)$$

In addition, the Laplace transform appears as an efficient tool to look for the solutions of Maxwell equations when one has to deal with more structured nano flows, the Dirac distri-butions being replaced by some functions $F(t - x/v_x, t - y/v_y)$. Of course, no analytical solution

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will generally be available but there now exists powerful numerical codes to get the solutions of the non homogeneous wave equation and to perform the inverse Laplace transform of these solutions.

The analysis of TM fields has been made easier by supposing the constraint (57) satisfied with one of the two components j_x, j_z null. When this condition is not fulfilled, calculations would become more intricate but no more interesting results would be probably obtained. To link nanodots with delta Dirac pulses seems rather natural from a theoretical viewpoint and

suggests, since quantum mechanical effects become important at this scale, that the tech-niques of quantum electrodynamics, largely used in Optics (Gryndberg, Aspect & Fabre, 2010; G. New, 2011) could supersede the clas-sical analysis used here. In addition, this link is of a great practical interest in processes cha-racterized by a fundamental dimension large at the nanoscale. For instance, nanopills are mo-re and more used to cure some health problems because they reach just the point to be cured but it is important to follow the track of the nanodoped flows not only inside but also outside the human body and, this could be made easier by the detection of the electromagnetic field generated by nanocharges.

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Appendix A

The constraint (4) is now assumed fulfilled with non-null J_x and J_z. Then, changing q into q_1 and q_2 with $q_1+q_2=$ q in (9), (10), the currents J_x, J_z become

$$J_x = 4i\pi q_1\omega \sum_{m,n} \delta(z - mh)\, \partial_x^{-1}\delta(x - na)$$

$$J_z = 4i\pi q_2\omega \sum_{m,n}\partial_z^{-1}\, \delta(z - mh)\, \delta(x - na) \qquad (A.1)$$

while with $H_y = H_y^1 + H_y^2$, {not to be confused with Hankel functions} Eq.(6) implies

$$(\Delta + k^2)\, \partial_x H_y^1 = - \partial_z\partial_x J_x \quad , \quad (\Delta + k^2)\, \partial_z H_y^2 = \partial_x\partial_z J_x \qquad (A.2)$$

so that according to (18) and (30)

$$H_y^1(x,z) = \omega\pi q_1 k \sum_{n,m} \partial_x^{-1} R_{n,m}(x,z)$$

$$H_y^2(x,z) = \omega\pi q_2 k \sum_{n,m}\partial_z^{-1} S_{n,m}(x,z) \qquad (A.3)$$

Now, the components E_x, E_z are supplied by Eqs.(2a,b)

$$i\omega\varepsilon E_x = - J_x - \partial_z H_y \quad , \quad i\omega\varepsilon E_z = - J_z + \partial_x H_y \qquad (A.4)$$

and taking into account (A.3) since $H_y = H_y^1 + H_y^2$

$$i\omega\varepsilon E_x(x,z) = - J_x(x,z) - \omega\pi k \sum_{n,m} (q_1\partial_z\partial_x^{-1} R_{n,m}(x,z) + q_2 S_{n,m}(x,z)]$$

$$i\omega\varepsilon E_z(x,z) = - J_x(x,z) + \omega\pi k \sum_{n,m} [q_1 R_{n,m}(x,z) + q_2\partial_x\partial_z^{-1} S_{n,m}(x,z)] \qquad (A.5)$$

Appendix B: Laplace transform (Van der Pol & Bremmer, 1959)

The Laplace transform L and its inverse L^{-1} have the formal definitions

$$f(p = L\{F(t)\} = = p \int_{-\infty}^{\infty} \exp(-pt)\, F(t)\, dt \qquad a)$$

$$F(t) = L^{-1}\{f(p)\} = 1/2\pi i \int_{c-i\infty}^{c+i\infty} \exp(pt)\, f(p)/p\, dp \qquad b) \qquad (B.1)$$

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and for real p as supposed here, they actually exist for p and c in some real intervall (α. β).

Two of their main properties are

$$L\{F(\lambda t)\} = f(p/\lambda) \quad , \quad L\{F(t+\lambda)\} = \exp(\lambda p)\, f(p) \qquad a)$$

implying

$$L\{F(at-b)\} = \exp(-pb/a)\, f(p/a) \qquad b) \qquad (B.2)$$

Another property used in this work concerns the product of two functions

$$L\{F_1(t)F_2(t)\} = (p/2\pi i) \int_{c-i\infty}^{c+i\infty} f_1(s)/s\, f_2(p-s)/(p-s)\, ds \qquad (B.3)$$

with

$$\alpha_1 < p < \beta_1 \quad , \quad \alpha_2 < p < \beta_2 \quad , \quad \alpha_1+c < p < \beta_1+c \quad (\alpha_2+c < p < \beta_2+c) \qquad (B.3a)$$

The relation (B.3) is applied here when $F_1(t)$ and $F_2(t)$ are two Dirac distributions whose La-
place transform is

$$L\{\delta(t)\} = p \qquad (B.4)$$

giving according to (B.2b)

$$L\{\delta(x-v_x t)\} = p/v_x \exp(-px/v_x) \qquad (B.5)$$

Then, taking into account (B.3)

$$L\{\delta(x-v_x t)\}\ \delta(z-v_z t)\} = (p/2\pi i v_x v_z) \int_{c-i\infty}^{c+i\infty} \exp(-sx/v_x) \exp[-(p-s)z/v_z]\, ds$$

$$= \quad (p/2\pi i v_x v_z) \exp(-pz/v_z) \quad \int_{c-i\infty}^{c+i\infty} \exp[s(z/v_z - x/v_x)]\, ds \qquad (B.6)$$

but, according to (B.1b) and (B.4) with $\tau = x/v_x - z/v_z$

$$1/2\pi i \int_{c-i\infty}^{c+i\infty} \exp[s(z/v_z - x/v_x)]\, ds = \delta(\tau) \qquad (B.7)$$

Substituting (B.7) into (B.6) gives

$$L\{\delta(x-v_x t)\}\ \delta(z-v_z t)\} = (p/v_x v_z) \exp(-pz/v_z)\, \delta(\tau) \qquad (B.8)$$

So, the Laplace transform of the charged flow (51) $\rho = = 4\pi q/v_x v_z\, \partial(t- x/v_x)\, \delta(\tau)$ together with (B.5) is

$$\chi(x,z\ ;p) = L\{\rho(x,z\ ;t)\} = -(4\pi qp/v_x v_z) \exp(-pz/v_z)\, \delta(\tau) \qquad (B.9a)$$

and using the property of the Dirac distribution, we also have

$$\chi(x,z\ ;p) = -(4\pi qp/v_x v_z) \exp(-px/v_x)\, \delta(\tau) \qquad (B9.b)$$

with finally

$$\chi(x,z\ ;p) = -(4\pi qp/v_x v_z) \exp[-p(x/v_x + z/v_z)\, \delta(\tau) \qquad (B.9c)$$

The Laplace transform of the t-derivative of a function is

$$L\{\partial_t F(t)\} = p\, f(p) \qquad (B.10a)$$

a relation used to get the Laplace transform of the Maxwell equations.

At the opposite, to perform the inverse Laplace transform of some expressions the following relation in which U is the unit step function is useful

$$L\{U(t)\} = 1 \qquad (B.10b)$$

Two more properties are needed to manage conductivity and effective permittivity

$$L^{-1}\{f(p)/p)\} = \int_{-\infty}^{t} F(t)\, dt \qquad p > 0 \quad , \quad = \ -\int_{t}^{\infty} F(t)\, dt \qquad p < 0 \qquad (B.11)$$

and $\alpha < p < \beta$ while for $\max(\alpha_1,\alpha_2) < p < \beta$

$$L^{-1}\{f_1(p)f_2(p)/p)\} = U(t) \int_0^t F_1(u)F_2(t-u)\, du \qquad (B.12)$$

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