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Complex Limiting Velocity Expressions as Likely Characteristics of Dark Matter Particles

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Abstract

Many astrophysical and cosmological observations suggest that the matter in the universe is mostly of the dark matter type whose behavior goes beyond the Standard Model description. Hence it is justifiable to take a drastically different approach to the dark matter particles which is here done through the bicubic equation of limiting particle velocity formalism. The bicubic equation discriminant D in this undertaking satisfy $D \succeq 0$ determined by the congruent parameter z satisfying $z^2 \succeq 1$, where formally $z(m) = 3\sqrt{3}mv^2/2E$, with m, v, and E being respectively, particle mass, velocity and energy. Also nonlinearly related to the the particle congruent parameter z is the particle congruent angle α . These two dimensionless parameters z and α simplify expressions in the bicubic equation limiting particle velocity formalism when evaluating the three particle limiting velocities, c_1, c_2 and c_3, (primary, obscure and normal) in terms of the ordinary particle velocity, v. Corresponding to these limiting velocities one then deduces, with equal values, dark matter particle energies $E(c_1)$, $E(c_2)$ and $E(c_3)$. The exemplary values of the congruent parameters are in these regions, $1 \preceq z \prec 3\sqrt{3}$ $/2$ and $\pi/2 \succeq \alpha \succeq \pi/3$. Already within these ranges of congruent parameters, the bicubic formalism yields for squares of particle limiting velocities that c_1^2 and c_2^2 are complex conjugate to each other, $c_1^{2*} = c_2^2$,and that c_3^2 is real. The imaginary portions of c_1^2 and c_2^2 do not change the realities of numerically equal to each other dark matter energies $E(c_i), i = 1, 2, 3$. In fact, real $E(c_{1,2})$ energies can be equally evaluated with $c_{1,2}^2$ or Re $c_{1,2}^2$ or even with $Imc_{1,2}^2$ so that in new notation, $E\left(c_{1,2}^2\right) = E\left(Rec_{1,2}^2\right) = E\left(Imc_{1,2}^2\right) = E\left(c_3^2\right)$ all with the same real values. However, in these notations, the real particle momenta are $\overrightarrow{p}\left(\left(Rec_{1,2}^2\right)\right.$ and $\overrightarrow{p}\left(\left(c_3^2\right)\right.$, defined with respective energies and, while in similar forms , numerically are different from each other.

Keywords: dark matter particles, astrophysics

1. Introduction

Many observations in astrophysical, cosmological, as well as, in electro-weak particle physics, show that a large portion of matter, either luminous or non-luminous, exhibits different kind of behaviors, which people simply call dark matter, as argued recently by Buchmueller, Doglioni, and Wong (2017), among others. These, so called dark matter particles do not seem to be successfully described so far by other physical models such as, for example, the Standard Model despite its large content. Here, based on the bicubic equation limiting particle velocity formalism as developed in 'Soln (2014, 2015, 2016, 2017, 2018a, 2018b, 2019). One is trying to understand dark matter particles with the formalism that is rather quite different from other descriptions. An explicit difference that follows from the bicubic formalism is the fact that for three squares of limiting particle velocities, primary c_1^2 and obscure c_2^2 are not only complex but also complex conjugate to each other, while normal c_3^2 is real. The bicubic equation for particle limiting velocity, c, with global particle mass, m, velocity v and energy E, we take it to be in the same form as in 'Soln (2014, 2019),

$$\left(\frac{c^2}{v^2}\right)^3 - \left(\frac{E}{mv^2}\right)^2\left(\frac{c^2}{v^2}\right) + \left(\frac{E}{mv^2}\right)^2 = 0. \quad (1.1)$$

In our approach the cubic equation discriminant D depending on the congruent parameter, $z(m)$, is of the following global form 'Soln (2014, 2015, 2016, 2017, 2018a, 2018b, 2019) with restrictions as indicated,

$$D(m) = \left(\frac{27}{8}\right)^2 \frac{1}{z^4\left(m\right)}\left(1 - \frac{1}{z^2\left(m\right)}\right) \succeq 0, \tag{1.2}$$

$$E = \frac{3\sqrt{3}mv^2}{2z(m)}, z^2\left(m\right) \succeq 1. \tag{1.3}$$

In Section 2 the exact solutions for squares of limiting velocities c_1^2, c_2^2 and c_3^2 , respectively of the primary, obscure and normal dark matter particles are given. Connecting the convenient squares of usual particle velocities, $v^2(c_i)$ with c_i^2, $i = 1, 23$, one can express the real particle energies in exact forms in terms of the newly introduced congruent angle $\alpha(m)$. These energies,despite involving imaginary portions, are all real and of the same value.

Section 3 is devoted to summarizing the results and speculative assumptions. Evaluations of $c_{1,2}^2\left(m\right)$ and $c_3^2\left(m\right)$, respectively for congruent angle $\alpha \succeq \pi/2$ are compared with evaluations for $\alpha \preceq \pi/2$ and as well with some experimental data. In this Section also the squares of limiting velocity solutions $c_{1,2}^2\left(m\right)$ are applied to the dark matter particle believed to be a sterile neutrino (Ng et al., 2019) plus suggesting other physics attributes for lighter dark matter particles.

2. Exact Limiting Velocity Solutions

With the help of (1.2) and (1.3) the solutions of (1.1), rather than in forms from ('Soln, 2019), are now equivalently in these forms,

$$c_{1,2}^2\left(m\right) = \frac{3\left[1 \pm i\sqrt{3}\cos(\alpha(m))\right]v^2}{2z(m)\sin(\alpha(m))} = Rec_{1,2}^2\left(m\right) + iImc_{1,2}^2\left(m\right), \tag{2.1}$$

$$Rec_{1,2}^2\left(m\right) = \frac{3v^2}{2z(m)\sin(\alpha(m))},\ Imc_{1,2}^2\left(m\right) = \pm\frac{3\sqrt{3}v^2}{2z(m)}ctn(\alpha(m)), \tag{2.2}$$

$$c_3^2\left(m\right) = -\frac{3v^2}{z\left(m\right)\sin(\alpha(m))}. \tag{2.3}$$

In the physics evaluations with limiting velocities c_1 , c_2 and c_3,associated respectively with primary, obscure and normal dark matter particles, a very useful parameter is the congruent angle $\alpha(m)$ nonlinearly related to the congruent parameter $z(m)$ ('Soln, 2019) with $z^2(m) \succeq 1$, $\alpha\left(m\right) \preceq \pi/2$,

$$\alpha\left(m\right) = 2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}\left(\frac{1}{z\left(m\right)}\right)\right)\right)^{\frac{1}{3}}, \tag{2.4}$$

$$\frac{1}{z\left(m\right)} = \sin\left[2\tan^{-1}\left(\tan\left(\frac{\alpha\left(m\right)}{2}\right)\right)^3\right]. \tag{2.5}$$

Both of them $z(m)$ and $\alpha(m)$, as we shall see shortly,have evolutionary roles on physical quantities. As long as the quantities such as E, m and v together with $z\left(m\right)$ and $\alpha\left(m\right)$ are real , from (2. 1,2) and (2.3), one notices that $c_1^2\left(m\right)$ and $c_2^2\left(m\right)$ are complex conjugate to each other and $c_3^2\left(m\right)$ is negative and real. When the congruent angle $\alpha\left(m\right) = \pi/2$, all $c_i^2\left(m\right), i = 12, 3$ are real. Next, with real $z\left(m\right)$ and $\alpha\left(m\right)$,from solutions (2. 1,2) and (2. 3), one notices the zero sum rule for squares of limiting velocities ('Soln, 2016) which indicates the correctness of c^2 solutions.

At this point, we wish to show the similarities and differences between our expression for energy from (1. 3) and the so called relativistic expression (see Griffiths, 2008). To see that, we seek to change our congruent parameter $z\left(m\right)$ into $z(m, relative)$ so that our energy from (1.3) becomes $E(relative)$:

$$E(relative) = \frac{3\sqrt{3}mv^2}{2z(m, relative)} = \frac{mv^2}{\beta^2\left(1-\beta^2\right)^{\frac{1}{2}}}, \tag{3.1}$$

$$z(m,relative)=\frac{3\sqrt{3}}{2}\beta^2\left(1-\beta^2\right)^{\frac{1}{2}},\beta=\frac{v}{c},\tag{3.2}$$

$$z(m,relative)\lesssim 1,\beta\leq 1;\ z(m,relative)\simeq 1,\beta\simeq 0.8-0.84,\alpha\left(m\right)\simeq\frac{\pi}{2}\ .\tag{3.3}$$

It is apparent that the parameter $z(m.relative)$ in (3. 2,3) is applicable to subluminal particles with luminal limiting velocity c. However, as we shall see shortly, in general $z\left(m\right)$ together with $\alpha\left(m\right)$ from ((2. 1,2,3,4,5), may apply to subluminal, luminal or even superluminal particles because there are no prior restrictions on values of limiting velocities, which may be from subluminal to superluminal values. The main reason for that is that when $z\left(m\right)$ is expressed in terms of the congruent angle $\alpha\left(m\right)$ there is more flexibility than when expressed in terms of β as in in (3.2). This flexibility is needed particularly if dark matter particle is superluminal and as such does not cover the same energy region as subluminal one. In other words, one should be aware that superluminal dark matter particle energies while calculable from (1.3) cannot be derived from (3.1) with β related to $z(m,relative)$ as in (3.2) which is the main reason for introducing the bicubic equation particle limiting velocity formalism in (2. 1,2,3,4,5) forms.

The inter-related congruent parameters $z(m)$ and $\alpha\left(m\right)$ play interchangeable roles as evolution parameters. In Table 1 we give some of their related numerical values, consistent with (2.4) and (2.5), which come very useful when evaluating some physical quantities, such as the energy from (1.3), for example.

Table 1. Some numerics of congruent parameters $\alpha\left(m\right)$ and $z(m)$

$$\begin{pmatrix}\alpha\left(m\right): & \frac{\pi}{2} & \frac{\pi}{2.25} & \frac{\pi}{2.3} & \frac{\pi}{2.5} & \frac{\pi}{2.75} & \frac{\pi}{3} & \frac{\pi}{3.25}\\ \frac{1}{z(m)}: & 1 & 0.876 & 0.835 & 0.669 & 0.496 & 0.371 & 0.283\\ z(m): & 1 & 1.142 & 1.198 & 1.495 & 2.016 & 2.694 & 3.531\end{pmatrix}$$

With the real quantities from the last raw in Table 1, it is evident how congruent parameter $z(m)$ differs from $z(m,relative)$, particularly since one may have $z(m)\succ 1$ with limiting velocity still being the velocity of light c. To continue in new different directions, we show how the squared dark matter particle velocity, v^2, is directly related to primary,obscure $Rec_{1,2}^2\left(\alpha\left(m\right)\right)$, primary,obscure $Imc_{1,2}^2\left(\alpha\left(m\right)\right)$, the whole primary, obscure $c_{1,2}^2\left(\alpha(m)\right)$ and to normal $c_3^2\left(\alpha(m)\right)$, respectively for each of the primary, obscure and normal dark matter particles. Taking into account that v^2 and $z(m)$ are real quantities, specifically from (2. 1,2) and (2. 3) we derive the relations between particle usual and limiting velocities, necessary in derivation of dark matter particle energies. We start by inverting limiting velocity solutions (2. 1,2) and (2. 3),

$$c_{1,2}^2=Rec_{1,2}^2+iImc_{1,2}^2,\tag{4.1}$$

$$c_{1,2}^2:\frac{v^2}{z(m)}=\frac{2\sin\left(\alpha(m)\right)c_{1,2}^2\left[1\mp i\sqrt{3}\cos\left(\alpha(m)\right)\right]}{3\left[1+3\cos^2\left(\alpha(m)\right)\right]}\equiv\frac{v^2}{z(m)}(c_{1,2}^2),\tag{4.2}$$

$$c_{1,2}^2:\frac{v^2}{z(m)}=\frac{2\sin\left(\alpha(m)\right)}{3\left[1+3\cos^2\left(\alpha(m)\right)\right]}[Rec_{1,2}^2\pm\sqrt{3}\cos\left(\alpha(m)\right)Imc_{1,2}^2$$
$$+i\left(Imc_{1,2}^2\mp\sqrt{3}\cos\left(\alpha(m)\right)Rec_{1,2}^2\right)]\equiv\frac{v^2}{z(m)}(c_{1,2}^2),\tag{4.3}$$

$$Reality:i\left(Imc_{1,2}^2\mp\sqrt{3}\cos\left(\alpha(m)\right)Rec_{1,2}^2\right)=0,\tag{4.4}$$

$$c_{1,2}^2:\frac{v^2}{z(m)}=\frac{2\sin\left(\alpha(m)\right)}{3\left[1+3\cos^2\left(\alpha(m)\right)\right]}[Rec_{1,2}^2\pm\sqrt{3}\cos\left(\alpha(m)\right)Imc_{1,2}^2]$$
$$\equiv\frac{v^2}{z(m)}(c_{1,2}^2),\tag{4.5}$$

$$Rec_{1,2}^2:\frac{v^2}{z(m)}=\frac{2}{3}\sin\left(\alpha(m)\right)Rec_{1,2}^2\equiv\frac{v^2}{z(m)}(Rec_{1,2}^2),\tag{4.6}$$

$$Imc_{1,2}^2:\frac{v^2}{z(m)}=\pm\frac{2}{3\sqrt{3}}\tan\left(\alpha(m)\right)Imc_{1,2}^2\equiv\frac{v^2}{z(m)}(Imc_{1,2}^2),\tag{4.7}$$

$$c_3^2 : \frac{v^2}{z(m)} = -\frac{\sin(\alpha(m))}{3} c_3^2 \equiv \frac{v^2}{z(m)} (c_3^2). \tag{4.8}$$

Due to the reality condition (4. 4), relation (4.3) shrinks to (4.5)) from which (4. 6) and (4. 7) follow so that with (4. 8) we have complete limiting velocity presentations for evaluating respective primary, obscure and normal dark matter particle energies. To this effect, it is worthwhile to see how the values of the congruent angle $\alpha(m)$ from Table 1., according to the limiting velocity solutions (2. 1,2) and (2. 3), may affect such calculations,

$$z(m) = 1, \alpha(m) = \frac{\pi}{2}$$

$$: c_{1,2}^2 = Rec_{1,2}^2 = \frac{3}{2} v^2, Imc_{1,2}^2 = 0, c_3^2 = -3v^2, \tag{4.9}$$

$$z(m) = 1.495, \alpha(m) = \frac{\pi}{2.5}$$

$$: c_{1,2}^2 = 1.055 v^2 \pm i 0.565 v^2, c_3^2 = -2.1 v^2. \tag{4.10}$$

The dark matter particle energies labeled by specific limiting velocities, according to (1. 3) together with relations (4. 1-10) will follow. We start with the exemplary general expression:

$$E\left(c_{1,2,3}^2(m)\right) = \frac{3\sqrt{3}m}{2} \cdot \frac{v^2}{z(m)} (c_{1,2,3}^2(m)). \tag{5}$$

Next, in the fashion of (5), we deduce from (4. 2) the following dark matter particle energies for the primary (c_1) and obscure (c_2) dark matter particles:

$$\begin{aligned} E(c_{1,2}^2(m)) &= \frac{3\sqrt{3}m}{2} \cdot \frac{v^2}{z(m)} (c_{1,2}^2(m)) \\ &= \sqrt{3} m \sin(\alpha(m))\, c_{1,2}^2(m) \frac{\left[1 \mp i\sqrt{3}\cos(\alpha(m))\right]}{\left[1 + 3\cos^2(\alpha(m))\right]} \\ &= \frac{\sqrt{3} m 3 v^2 \left[1 \pm i\sqrt{3}\cos(\alpha(m))\right] \left[1 \mp i\sqrt{3}\cos(\alpha(m))\right]}{2z(m)\left[1 + 3\cos^2(\alpha(m))\right]} \\ & \left(= \frac{3\sqrt{3} m v^2}{2z(m)}\right). \end{aligned} \tag{5.1,2}$$

In this evaluation, the reality condition is automatically taken into account without specifying it in the course of evaluation with the solutions of limiting velocities (4.1) -(4. 7). This is so, since from the limiting velocity solutions (2. 1,2) for the primary (c_1) and obscure (c_2) particles, one has implicitly that $Imc_{1,2}^2 = \pm\sqrt{3}\cos(\alpha(m))\, Rec_{1,2}^2$. Similarly we can evaluate the energy of the normal (c_3) particle from (4. 8),

$$E(c_3^2(m)) = \frac{3\sqrt{3}m}{2} \cdot \frac{v^2}{z(m)} (c_3^2) = -\frac{\sqrt{3} m \sin(\alpha(m))\, c_3^2}{2} \left(= \frac{3\sqrt{3} m v^2}{2z(m)}\right). \tag{5.3}$$

All three of these, numerically equal, energies are real despite the fact that (primary, obscure) dark matter particle's squares of limiting velocities, $c_{1,2}^2$,are complex. Just knowing $Rec_{1,2}^2$ or $Imc_{1,2}^2$ one can still find out the corresponding energies of which each, as we shall see, equals the energy appearing in (5. 1,2) . Specifically, for $Rec_{1,2}^2$ and $Imc_{1,2}^2$ from respectively (4. 6) and (4. 7) we write

$$\begin{aligned} E(Rec_{1,2}^2(m)) &= \frac{3\sqrt{3}m}{2} \cdot \frac{v^2}{z(m)} (Rec_{1,2}^2) \\ &= \sqrt{3} m \sin(\alpha(m))\, Rec_{1,2}^2 \left(= \frac{3\sqrt{3} m v^2}{2z(m)}\right), \end{aligned} \tag{5.4}$$

$$E(Imc_{1,2}^2(m)) = \frac{3\sqrt{3}m}{2} \cdot \frac{v^2}{z(m)}(Imc_{1,2}^2)$$
$$= \pm m\ \tan\alpha(m)\ Imc_{1,2}^2 \left(= \frac{3\sqrt{3}mv^2}{2z(m)}\right). \tag{5.5}$$

The common value of energy $3\sqrt{3}mv^2/2z(m)$ in (5. 1-5) follows from solutions for (primary, obscure) $c\ _{1,2}^2(m)$, $Rec_{1,2}^2(m)$ and $Imc_{1,2}^2(m)$ in (2. 1,2) and for (normal) $c\ _3^2(m)$ (2.3) limiting velocity squares. The easiest way to demonstrate that is to take specific congruent parameters with unspecified dark matter particle mass and velocity and inserting them into (5. 1-5) :

$$\alpha(m) = \frac{\pi}{2.5},\ z(m) = 1.495$$
$$: Rec_{1,2}^2 = 1.055v^2, Imc_{1,2}^2 = \pm 0.565v^2, c_3^2 = -2.1v^2,$$
$$E(c_{1,2}^2(m)) \simeq 1.738mv^2, E(Rec_{1,2}^2(m)) \simeq 1.738mv^2,$$
$$E(Imc_{1,2}^2(m)) \simeq 1.739mv^2, E(c_3^2(m)) \simeq 1.729mv^2. \tag{5.6}$$

One may see the energy as a sacred quantity, as all these energy expressions give the same value even from $Imc_1^2(m)$).This particularly so as the zero sum rule for squares of limiting velocities holds for both real and imaginary portions.

$$Rec_1^2(m)) + Rec_2^2(m)) + c_3^2 = 0,\ Imc_1^2(m)) + Imc_2^2(m)) = 0. \tag{5.7}$$

As long as the congruent parameters satisfy $z(m) \neq 1$ and $\alpha(m) \neq \pi/2$, dark matter particle energies and momenta appear not to be expressible in the Lorentzian like forms as in 'Soln (2019). However, as also in 'Soln (2019), they are expressible in more general usual forms. All the numerically equal energy expressions, including $E(Imc_{1,2}^2(m))$, are real, indicating energy as fundamental quantity in physics. The particle momentum is more a quantity of convenience, as its real value is associated with $Rec_1^2(m))$ or c_3^2 but not also with $Imc_1^2(m))$.

The dark matter particle momenta are defined with their energies, preferably in such a way , as to be real in values. With that in mind, we write down the particle momenta for primary, obscure and normal dark matter particles,

$$\overrightarrow{p}(c_{1,2}^2) = \overrightarrow{p}(Rec_{1,2}^2) = \frac{E(c_{1,2}^2(m))\overrightarrow{v}}{Rec_{1,2}^2} = \sqrt{3}m\overrightarrow{v}\sin(\alpha(m)), \tag{6.1,2}$$
$$\overrightarrow{p}(c_3^2) = \frac{E(c_3^2(m))\overrightarrow{v}}{(-c_3^2(m))} = \frac{\sqrt{3}}{2}m\overrightarrow{v}\sin(\alpha(m)). \tag{6.3}$$

For primary,obscure dark matter particles with $c_{1,2}$ limiting velocities, in the definitions we use (primary, obscure) $Rec_{1,2}^2(m)$ with $E(Rec_{1,2}^2(m))$ (numerically equal to $E(c_{1,2}^2(m))$) to define the equal value momenta of which each in form is very similar to normal particle momentum but double in value.

3. Discussion with Example Applications and Conclusion

One notices interesting things happening to dark matter particles once the congruent parameter $z(m) \neq 1$ and congruent angle $\alpha(m) \neq \pi/2$. As one sees , the nonlinearly connected dimensionless congruent parameter $z(m)$ and dimensionless congruent angle $\alpha(m)$ are essential in evaluating not only all forms of particle energies but also of linear particle momenta. In fact, as we can see from (3. 1,2,3), they have similar roles in evaluating particle energy E as does the relative velocity β from Special Relativity. For one thing, the Lorentzian like form is not favored by either the dark matter particle energy or the momentum as seen respectively in each of (5.1-5.7) and (6.1-6.3). The most amazing things are that different forms of dark matter particle complex limiting velocity-squares (primary, obscure) $c_{1,2}^2(m)$, (primary, obscure) $Rec_{1,2}^2(m)$ and (primary, obscure) $Imc_{1,2}^2(m)$ (2. 1,2) separately yield the same value dark matter energy $E(c_{1,2}^2(m))$,while the real $c_3^2(m)$ (2. 3) yields $E(c_3^2(m)$ (numerically equal to $E(c_{1,2}^2(m))$). Similarly, one has the expressions for dark matter particle momenta, $\overrightarrow{p}(c_{1,2}^2) = \overrightarrow{p}(Rec_{1,2}^2)$ from $E(c_{1,2}^2(m))$ together with (primary, obscure) $Rec_{1,2}^2(m)$; while normal dark matter particle momentum $\overrightarrow{p}(c_3^2)$ follows in a usual way from limiting velocity-square $c_3^2(m)$ and the energy $E(c_3^2(m))$.

Recently Ng et al.(2019), from studying dark matter from NuSTAR M31 observations have put forward well-motivated dark sterile neutrino dark matter candidate, denoted as χ,which radioactively can decay into mono-energetic photon, γ plus active neutrino ν, $\chi \longrightarrow \gamma+\nu$. Here, we wish to move on from the congruent parameters values of $z(m)=1$ and $\alpha(m)=\frac{\pi}{2}$, as in 'Soln (2019), to different values with $z(m)\neq 1$ and $\alpha(m)\neq\frac{\pi}{2}$. In doing so, also here as in 'Soln (2019) with Ng et al.(2019), we shall accept that the dark sterile neutrino mass satisfies $m_\chi \succeq 12keV/c^2$, with c the velocity of light . Continuing with the example of of the dark sterile neutrino candidate, we shall demonstrate the evolutionary qualities of congruent parameters $z(m_\chi)$ and $\alpha(m_\chi)$ by changing them in succession as $z(m_\chi)=1, 1.2$ and 1.5 with $\alpha(m_\chi)=\pi/2, \pi/2.3$ and $\pi/2.5$. These demonstrations will show that for $m_\chi \simeq 12keV/c^2$ the dark sterile neutrino energy will change in decreasing order as $E(m_\chi)\simeq 20.8, 20.3$ and 20 keV . With more details we have

$$\begin{aligned}
m_\chi &\simeq 12\ keV/c^2, Rec_{1,2}^2=c^2,\\
(2.2) &: v^2=\frac{2}{3}z(m_\chi)\sin(\alpha(m_\chi))\ Re(c_{1,2}^2=c^2),\\
(2.3) &: c_3^2\left(m_\chi\right)=-\frac{3v^2}{z\left(m_\chi\right)\sin(\alpha(m_\chi))}=-2c^2,\\
(5.4) &: E(Re(c_{1,2}^2(m_\chi))=E(c_{1,2}^2(m_\chi))=\sqrt{3}m_\chi\sin(\alpha(m_\chi))c^2=\frac{3\sqrt{3}m_\chi v^2}{2z\left(m_\chi\right)},\\
(5.3) &: E(c_3^2(m_\chi))=\frac{\sqrt{3}}{2}m_\chi\sin(\alpha(m_\chi))(-c_3^2\left(m_\chi\right))=\frac{3\sqrt{3}m_\chi v^2}{2z\left(m_\chi\right)}.
\end{aligned}$$

$$\begin{aligned}
\alpha(m_\chi) &= \frac{\pi}{2}, z(m_\chi)\ =1, v^2=\frac{2}{3}c^2\\
&: E(Re_{1,2}^2)=E(c_3^2(m_\chi))\simeq\ 20.8keV\simeq 2.6\ m_\chi v^2,\\
\alpha(m_\chi) &= \frac{\pi}{2.3}, z(m_\chi)\ =1.2, v^2=0.78c^2\\
&: E(Re_{1,2}^2)=E(c_3^2(m_\chi))\simeq\ 20.3keV\simeq 2.2\ m_\chi v^2,\\
\alpha(m_\chi) &= \frac{\pi}{2.5}, z(m_\chi)=1.5, v^2=0.95c^2\\
&: E(Re_{1,2}^2)=E(c_3^2(m_\chi))\simeq\ 20k\ eV\simeq 1.73m_\chi v^2.
\end{aligned}$$

Similar analyses one can perform for dark sterile neutrino momenta, but restricting to just the real ones (6.1,2) and (6.3).

In conclusion, the fact that through the bicubic equation limiting particle velocity formalism the real particle energy can be equally well evaluated from the complex, real or imaginary particle limiting velocity-squared expression, makes the particle energy exceptional and important,almost, a sacred quantity, particularly as relations (5. 6) shows explicitly in succession numerically equal real values $E\left(c_{1,2}^2\right)=E\left(Rec_{1,2}^2\right)=E\left(Imc_{1,2}^2\right)$ $=E\left(c_3^2\right)$ for each velocity-squared, complex c_1^2, complex c_2^2 and real c_3^2. What this shows that one should not automatically discard complex or imaginary quantities in physics as their contents may support real physical quantities, such as energy.

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